752 Midterm for Spring 2026

1. (0 points) What is your name

GO TO NEXT PAGE

2. (25 points)

   Prove the following:

   For every c, for every COL: $\binom{\mathsf{N}}{2} \rightarrow [c]$,

   there is an infinite homog set.

   (There are two proofs. One mimics the proof I did in class for 2-colors. If you use this one then just do the first few stages of the construction, like I do. The other USES Ramsey on 2 colors. Both proofs are acceptable.)

GO TO NEXT PAGE

3. (25 points)

Definition Let $\mathrm{COL}_1 \colon \mathsf{N} \rightarrow [10]$. Let $\mathrm{COL}_2 \colon \binom{\mathsf{N}}{2} \rightarrow [10]$.

Let H be an infinite subset of N.

- H is *super-dooper homog* if there is a color c such that
  $\mathrm{COL}_1 \colon H \rightarrow [10]$ ONLY uses the color c and
  $\mathrm{COL}_2 \colon \binom{H}{2} \rightarrow [10]$ ONLY uses the color c
  (For example, COL_1 colors every element of H RED, and COL_2 colors every pair of elements of H RED.)
- H is *super homog* if there are two colors c_1, c_2 (they could be the same but do not have to be)
  such that
  $\mathrm{COL}_1 \colon H \rightarrow [10]$ ONLY uses the color c_1 and
  $\mathrm{COL}_2 \colon \binom{H}{2} \rightarrow [10]$ ONLY uses the color c_2
  (For example, COL_1 colors every element of H RED, and COL_2 colors every pair of elements of H BLUE.)

End of Definition

And now for the questions!

(a) Show that, for all
$\mathrm{COL}_1 \colon \mathsf{N} \rightarrow [10]$ and $\mathrm{COL}_2 \colon \binom{\mathsf{N}}{2} \rightarrow [10]$,
there exists an infinite super-homog set.

(b) Show that, there exists a
$\mathrm{COL}_1 \colon \mathsf{N} \rightarrow [10]$ and $\mathrm{COL}_2 \colon \binom{\mathsf{N}}{2} \rightarrow [10]$
such that there is no super-duper set.

GO TO NEXT PAGE

(c) (25 points) Let $(X, \preceq_X)$ be a wqo. Let $Y \subseteq X$ such that the following holds:

For all $x \in X$ and $y \in Y$ if $x \preceq y$ then $x \in Y$
(We have said that Y is closed downward.
(Think Y is the set of planar graphs.)

Show that there is a finite obstruction set for Y .

GO TO NEXT PAGE

(d) (25 points) Give a coloring COL: $\binom{\omega+\omega+\omega+\omega}{2}\rightarrow[20]$ so there is no infinite 19-homog set. No proof required.

JAVIER: Here is what I have in mind.

Partition N into four disjoint parts and each omega uses a different one.

Given $x, y \in \omega+\omega+\omega$, we assume $x < y$.

If they are in both in the ith ω then $\mathrm{COL}(x, y) = i$ (So thats four colors)

If x is in the ith omega and y is in the jth omega then color is (i, j). thats 16 colors.