Ramsey Theory Project. Morally Due 12:30PM May 5, 2026

1. (0 points) What is your name?

GO TO NEXT PAGE

2. (10 points) For this problem you can assume that $R(k) = 2^k$.

   (a) Fill in the following sentence and prove it:

   Let $k \in \mathsf{N}$. If n satisfies condition $XXX(k)$ then, for all $\mathrm{COL}\colon \binom{[n]}{2} \rightarrow [2]$, *there exists at least 3 mono K_k's.*

   Your proof should use a similar technique used in class for K_4.

   (b) (0 point and don't hand in but this will help you with Part 3) Write a program that will, given L, find the least n that satisfies $XXX(L)$.

   (c) (5 points) Run the program for $k = 4$ to 7 give us a table of the following form (I only give the first three rows and the numbers are made up.)

k	least n
4	22
5	27
6	37
7	47

   (d) (5 points) Based on your data, come up with a conjecture for what n is as a function of k.

GO TO NEXT PAGE

3. (20 points) For this problem you may use the following theorem we proved in class:
$\forall$ COL: $\mathsf{N} \times \mathsf{N} \rightarrow [10^{100}]$ $\exists$ infinite $A, B \subseteq \mathsf{N}$ such that
COL: $A \times B \rightarrow [10^{100}]$ only uses 2 colors.

Fill in the c and prove the resulting theorem: (By 'prove' I mean show me the first few steps of the construction and hand-wave the rest.)

1) $\forall$ COL: $\mathsf{N} \times \mathsf{N} \times \mathsf{N} \rightarrow [10^{100}]$ $\exists$ *infinite* $A, B, C \subseteq \mathsf{N}$ *such that*
COL: $A \times B \times C \rightarrow [10^{100}]$ *only uses* c *colors.*

2) $\exists$ COL: $\mathsf{N} \times \mathsf{N} \times \mathsf{N} \rightarrow [c]$ *such that there is no infinite* $A, B, C \subseteq \mathsf{N}$ *such that*
COL: $A \times B \times C \rightarrow [c]$ *only uses* $c - 1$ *colors.*

GO TO NEXT PAGE

4. (20 points) Prove that the following function exists:

$f(n)$ is the least m such that there for every sequence of graphs

$$G_1, G_2, \ldots, G_m$$

where the number of vertices in G_i is $\leq i + n$
there is an $i < j$ with $G_i \preceq_{\text{minor}} G_j$.
You may use the Graph Minor theorem.

GO TO NEXT PAGE

5. (20 points. As of April 14 I have not gone over the material for this problem) You cannot Use Rado's Theorem for this problem. We will be asking you to prove Rado's Theorem in this particular case. You CAN use the Extended VDW theorem.

   Consider the equation

$$E: \quad w + 2x - 3y + z = 0.$$

   Show that there exist a number M such that, for all COL: $[M] \rightarrow [2]$, there exists a mono solution to E with w, x, y, z all distinct.

GO TO NEXT PAGE

6. (20 points. As of April 14 I have not gone over the material for this problem)

   Let $VDW(k, \omega)$ mean that VDW theorem is known for k-APs and ANY number of colors.

   Assume you know VDW theorem for $VDW(1, \omega)$, $VDW(2, \omega)$, $VDW(3, \omega)$, and $VDW(4, \omega)$. Prove VDW theorem for $VDW(5, 2)$.