

BILL, RECORD LECTURE!!!!

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All 2-Coloring Of the Plane have a Red 2-Stick or Blue 3-stick

Exposition by William Gasarch-U of MD

Credit Where Credit is Due

The main result in these slides is due to Szlam (1999).

Chromatic Number of the Plane: Review

In the prior lecture we proved

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Thm $\forall \text{COL}: \mathbb{R}^2 \rightarrow [2] \exists \text{ either a Red } \ell_2 \text{ or a Blue } \ell_2.$

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Answer on next slide

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Case 2 There is no **Blue** ℓ_3 .

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$\text{COL}'(x, y)$ is the least $i \in \{0, 1, 2\}$ such that $\text{COL}(x, y + i) = \mathbf{R}$.

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This is well defined because of the case we are in.

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So $(x_1, y_1 + 1)$ and $(x_2, y_2 + 1)$ are a **Red** ℓ_2 . Done!

Can Prove Result About (ℓ_2, ℓ_4)

Using that the Chromatic Number of the Plane (χ) is ≤ 4 one can easily prove the following:

Thm $\forall \text{ COL: } \mathbb{R}^2 \rightarrow [2] \exists$ either a **Red** ℓ_2 or a **Blue** ℓ_4 .

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Juhasz prove the above theorem without using $\chi \leq 4$. In fact, he prove the theorem in 1979, 39 years before $\chi \leq 4$ was proven.

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We will soon present statements about $\mathbb{R}^2 \rightarrow (\ell_2, \ell_n)$

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Csizmadia-Togh 1994	$\exists X \subseteq \mathbb{R}^2, X = 8$ $\mathbb{R}^2 \not\rightarrow (\ell_2, X)$	NONE

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All of the negative results hold for all $n \geq 2$. Our interest is in the $n = 2$ case.

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Open Problems

$$\mathbb{R}^2 \rightarrow (\ell_3, \ell_n)?$$

All of the negative results hold for all $n \geq 2$. Our interest is in the $n = 2$ case.

Author and Year	Result
Positive Results	
Currier-Moore-Yip, 2024	$\mathbb{R}^2 \rightarrow (\ell_3, \ell_3)$
Negative Results	
Conlon-Wu, 2022	$\mathbb{R}^n \not\rightarrow (\ell_3, \ell_{10^{50}})$
Fuhrer-Toth, 2024	$\mathbb{R}^n \not\rightarrow (\ell_3, \ell_{1177})$
Currier-Moore-Yip, 2024	$\mathbb{R}^n \not\rightarrow (\ell_3, \ell_{20})$
	$\mathbb{R}^n \not\rightarrow (\ell_4, \ell_{18})$
	$\mathbb{R}^n \not\rightarrow (\ell_5, \ell_{10})$

Open Problems

Narrow the gap between $\mathbb{R}^2 \rightarrow (\ell_3, \ell_3)$ and $\mathbb{R}^2 \not\rightarrow (\ell_3, \ell_{20})$.

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Open Problems

Narrow the gap between $\mathbb{R}^2 \rightarrow (\ell_3, \ell_3)$ and $\mathbb{R}^2 \not\rightarrow (\ell_3, \ell_{20})$.

The results of Fuhrer-Togh and Currie-Moore-Yip are messy. For some reasonable values of n find nice proofs that $\mathbb{R}^2 \not\rightarrow (\ell_3, \ell_n)$.

One More Result

Author and Year	Result
Negative Results Erdos et al. See Below	$\mathbb{R}_2 \not\rightarrow (\ell_6, \ell_6)$

One More Result

Author and Year	Result
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Erdős, Graham, Montgomery, Rothchild, Spencer, Strauss.

One More Result

Author and Year	Result
Negative Results Erdos et al. See Below	$\mathbb{R}_2 \not\rightarrow (\ell_6, \ell_6)$

Erdős, Graham, Montgomery, Rothchild, Spencer, Strauss.

The title of the paper is **Euclidean Ramsey Theorems I (1973)** and was (obviously) an early paper on Euclidean Ramsey Theory.

Open Problems about ℓ_2 , ℓ_3

ℓ_2

Open Problems about ℓ_2 , ℓ_3

ℓ_2

Known $\mathbb{R}^2 \rightarrow (\ell_2, \ell_5)$

Open Problems about ℓ_2 , ℓ_3

ℓ_2

Known $\mathbb{R}^2 \rightarrow (\ell_2, \ell_5)$

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Open Problems about ℓ_2 , ℓ_3

ℓ_2

Known $\mathbb{R}^2 \rightarrow (\ell_2, \ell_5)$

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Open Narrow the Gap.

Open Problems about ℓ_2 , ℓ_3

ℓ_2

Known $\mathbb{R}^2 \rightarrow (\ell_2, \ell_5)$

Known $\mathbb{R}^2 \not\rightarrow (\ell_2, \ell_{10^{50}})$

Open Narrow the Gap.

ℓ_3

Open Problems about ℓ_2 , ℓ_3

ℓ_2

Known $\mathbb{R}^2 \rightarrow (\ell_2, \ell_5)$

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Open Narrow the Gap.

ℓ_3

Known $\mathbb{R}^2 \rightarrow (\ell_3, \ell_3)$

Open Problems about ℓ_2 , ℓ_3

ℓ_2

Known $\mathbb{R}^2 \rightarrow (\ell_2, \ell_5)$

Known $\mathbb{R}^2 \not\rightarrow (\ell_2, \ell_{10^{50}})$

Open Narrow the Gap.

ℓ_3

Known $\mathbb{R}^2 \rightarrow (\ell_3, \ell_3)$

Known $\mathbb{R}^2 \not\rightarrow (\ell_3, \ell_{20})$

Open Problems about ℓ_2 , ℓ_3

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Open Narrow the Gap.

ℓ_3

Known $\mathbb{R}^2 \rightarrow (\ell_3, \ell_3)$

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Open Narrow the Gap.

Open Problems about l_4 , l_5

l_4

Open Problems about ℓ_4 , ℓ_5

ℓ_4

Known (not) Do not know any $a \geq 4$ with $\mathbb{R}^2 \rightarrow (\ell_4, \ell_a)$

Open Problems about ℓ_4 , ℓ_5

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Open Problems about ℓ_4 , ℓ_5

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Open Problems about ℓ_4 , ℓ_5

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Might be $\mathbb{R}^2 \not\rightarrow (\ell_4, \ell_4)$ so there is no such a .

Open Problems about ℓ_4 , ℓ_5

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ℓ_5

Open Problems about ℓ_4 , ℓ_5

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Open Problems about l_4 , l_5

l_4

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l_6

Open Problems about l_4, l_5

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Open Get nicer proofs.

Open In some cases get nicer proofs of weaker results.

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Conjectures? None.