

# BILL, RECORD LECTURE!!!!

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# Ramsey Theory implies Bolzano Weierstrass

Exposition by William Gasarch-U of MD

# Bolzano-Weierstrass Theorem: Ramsey Part

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We assume (1). The case of (2) is similar.

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