BILL, RECORD LECTURE!!!!

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Lower Bounds on $R(k)$

Exposition by William Gasarch

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How good are there upper bounds (UB)?
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We compare our LBs to the UB 2^{2k-1} for convenience.

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To show that $R(k) \geq f(k)$ we need to construct a coloring

$\mathrm{COL}\colon \binom{f(k)}{2} \to [2]$ such that there is no homog set of size k.

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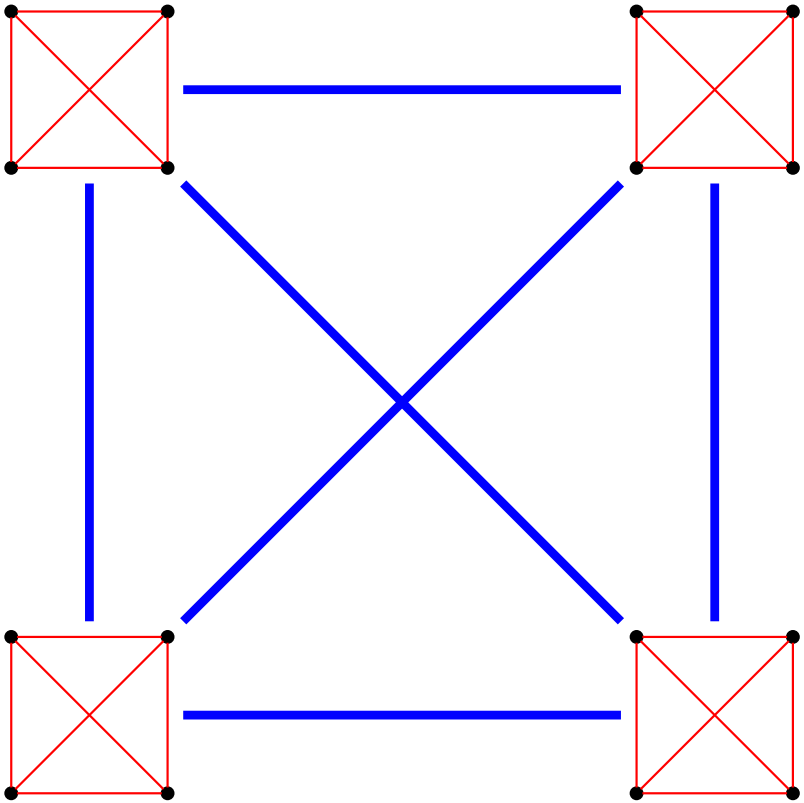
We first give an example, on the next slide.

Example: The $k = 5$ Case



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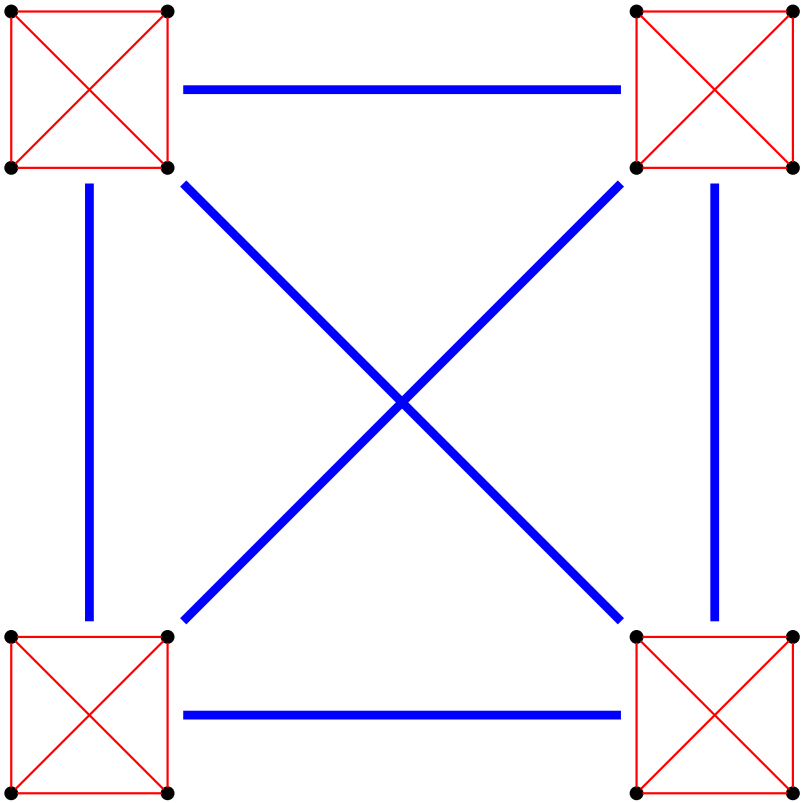

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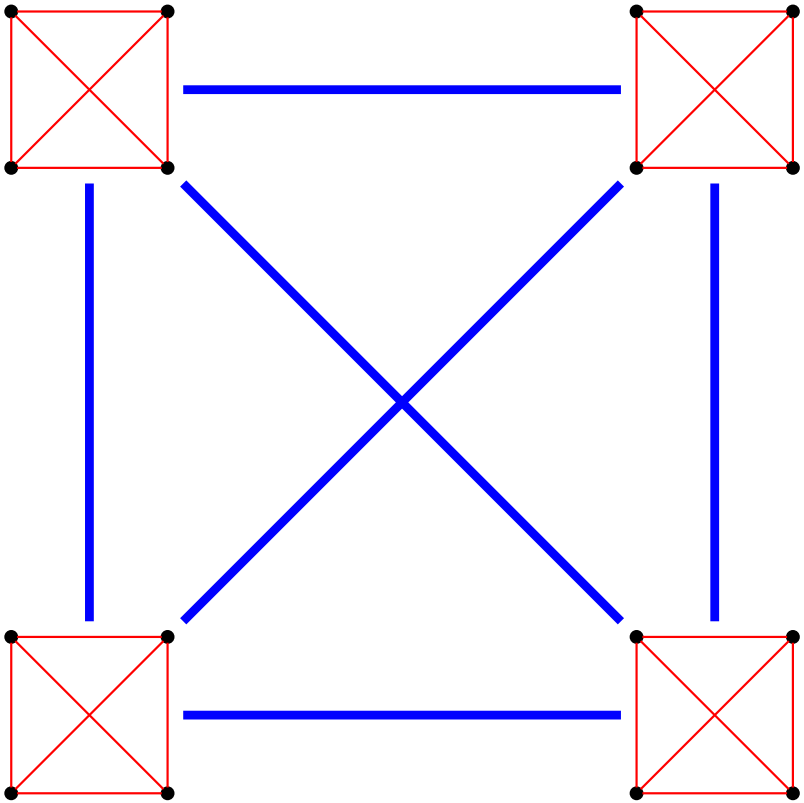

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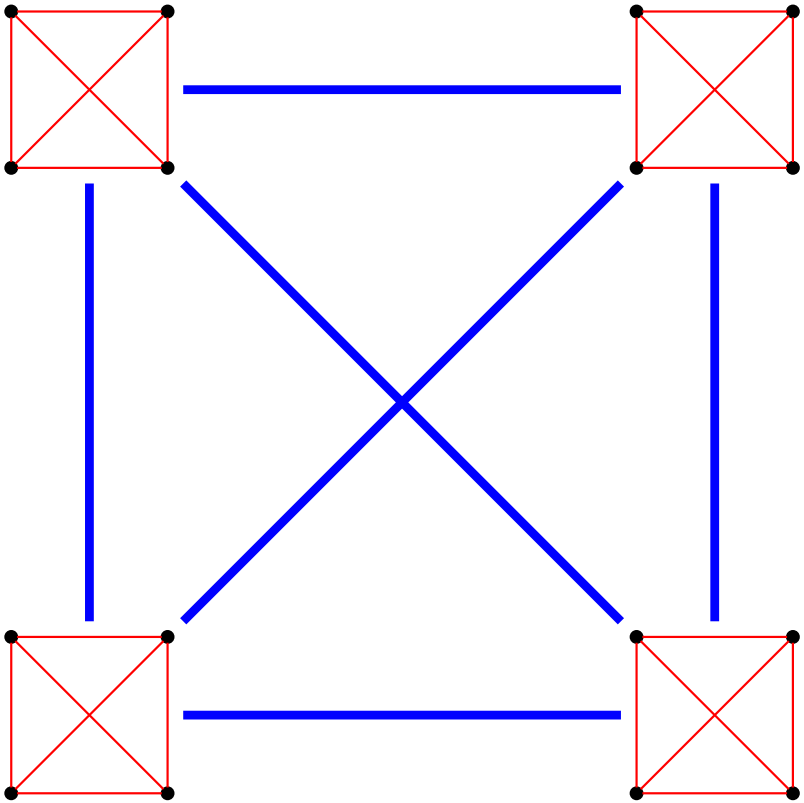

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Can We Do Better?