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Project Re-derive the known obs sets with a SAT Solver and (try to) find the obs set for 5-coloring.

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Game Ramsey Numbers (example): Least n such that player I wins the game on K_n where both players want a K_3.
Look at avoidance game- try to NOT get K_3.

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Caveat I've had some HS students do this before for Graphs. Might want to do it for VDW's Theorem, Poly VDW, Square-Thm, Grid-Thms.

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See next page for variants.

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Combinations of the above. Empirical studies of the above.

Can Versions of VDW, Rado, Can PolyVDW

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Projects Good writeup of the known proof(s) of this, perhaps better bound on the numbers. Possibly empirical results.

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Ramsey Numbers—Can Ramsey? VDW? GW?

Better Writeups Needed

A better writeup of the Conlon-Fox-Sud 3-ary Ramsey Theory Upper Bouns
A better writuep of Shelah's upper bound on Can Ramsey
A better writeup of Shelah's upper bound on poly VDW.

Coloring the Plane with a Ctble number of colors

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Project Read, understand, and write up these results. Then see if you can extend.

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Look at unit cube problem. And beyond.
For all of the above, more colors.

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Survey of L_a-L_b results- build on what Kelin and Chaewoon have done.

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Better bounds in $\mathbb{R}^3$?
smallskip In $\mathbb{R}^3$ try to get mono tetrahedron.

Anti-Ramsey

Survey of the papers I have on Anti-Ramsey.
The papers I wrote up (with co-author) seem tedious- automate?