BILL, RECORD LECTURE!!!!

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Solutions to Optional Project

Exposition by William Gasarch-U of MD

VDW Problem

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Assume $VDW(1,\omega)$, $VDW(2,\omega)$, $VDW(3,\omega)$, $VDW(4,\omega)$.

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Prove $VDW(5,2)$.

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We finish this on the next few slides.

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Each C_i has **4-AP** also 5th elt in that sequence.

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We assume $\mathrm{COL}(a+4d) = $ **B**.

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Because block are the same color:
$a+D \qquad a+d+D \qquad a+2d+D \qquad a+3d+D$. If
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Because block are the same color:
$a+D \qquad a+d+D \qquad a+2d+D \qquad a+3d+D$. If $\mathrm{COL}(a+4d+D) =$ **B**, DONE
We assume $\mathrm{COL}(a+D+4d) =$ **B**. Rewrite as $\mathrm{COL}(a+4d+D) =$ **B**.

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Similar: $\mathrm{COL}(a+2D+4d) =$ **B** and $\mathrm{COL}(a+3D+4d) =$ **B**.

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More on next slides.

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$a + 4d$ $\quad a + 4d + D$ $\quad a + 4d + 2D$ $\quad a + 4d + 3D$

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We have the following are all **R**.

a $\quad$ $a + d + D$ $\quad$ $a + 2d + 2D$ $\quad$ $a + 3d + 3D$

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If $\mathrm{COL}(a+4D+4d) =$ **B** then first one yields **B** 5-AP.

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We have the following are all **R**.

$a \qquad a + d + D \qquad a + 2d + 2D \qquad a + 3d + 3D$

If $\mathrm{COL}(a + 4D + 4d) =$ **B** then first one yields **B** 5-AP.

If $\mathrm{COL}(a + 4D + 4d) =$ **R** then second one yields **R** 5-AP.

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BILL Yes; however, if you like the final product, why does that matter.

ANDREW I can't judge that until I hear the final product.

BILL Then you shall hear it. And not just the Chorus.

I Like Big Blocks! (Non Chorus)

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We partition N into block so wide

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Encode each block by its own pro-file

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Now look at blocks, and take a seat.

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Now look at blocks, and take a seat.

Ind-uction hums

The gears all click

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Encode each block by its own pro-file

Collapse the colors; compress the style

Pigeon-hole says types repeat

Now look at blocks, and take a seat.

Ind-uction hums

The gears all click

omega-squared—What a trick!

I Like Big Blocks! (Chorus)

I Like Big Blocks! (Chorus)

I like big blocks and I cannot lie!

I Like Big Blocks! (Chorus)

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You other math folks can't deny!

I Like Big Blocks! (Chorus)

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You other math folks can't deny!

With finitely many colors that you choose

I Like Big Blocks! (Chorus)

I like big blocks and I cannot lie!

You other math folks can't deny!

With finitely many colors that you choose

There's a mono AP that you can't refuse.

I Like Big Blocks! (Non Chorus)

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Color red-blue? Or three or more?

I Like Big Blocks! (Non Chorus)

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Doesn't matter– same encore

I Like Big Blocks! (Non Chorus)

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Doesn't matter– same encore

Hide your patterns, try to duck,

I Like Big Blocks! (Non Chorus)

Color red-blue? Or three or more?

Doesn't matter– same encore

Hide your patterns, try to duck,

Regularity brings the luck

I Like Big Blocks! (Non Chorus)

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Hide your patterns, try to duck,

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A Length-k run in one shade

I Like Big Blocks! (Non Chorus)

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Hide your patterns, try to duck,

Regularity brings the luck

A Length-k run in one shade

Its a combinatorial serenade!

I Like Big Blocks! (Final Chorus)

I Like Big Blocks! (Final Chorus)

I like big blocks and I cannot lie!

I Like Big Blocks! (Final Chorus)

I like big blocks and I cannot lie!

Mono APs you can't deny.

I Like Big Blocks! (Final Chorus)

I like big blocks and I cannot lie!

Mono APs you can't deny.

With giant blocks and recursive tracks

I Like Big Blocks! (Final Chorus)

I like big blocks and I cannot lie!

Mono APs you can't deny.

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Van der Waerden's got your back.

I Like Big Blocks! (Final Chorus)

I like big blocks and I cannot lie!

Mono APs you can't deny.

With giant blocks and recursive tracks

Van der Waerden's got your back.
(Applause if deserved.)

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The End

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The Rated XXX Problem

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Find a condition XXX such that the following is true:

If n satisfies condition $XXX(k)$ then
$\forall \mathrm{COL}\colon \binom{[n]}{2} \rightarrow [2]$,
there exists at least 3 mono K_k's.

Solution to the Rated XXX Problem

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List out all subsets of $V = \{1, \ldots, R(k)\}$ of size $R(k) = 2^k$.

$$A_1, A_2, \ldots, A_{\binom{n}{2^k}}.$$

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Since $|A_i| = R(k)$, each A_i has a mono K_k.

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1) Find a mono K_k in A_1. Say its $\{x_1, \ldots, x_k\}$.
2) REMOVE all A_i's that have all of $\{x_1, \ldots, x_k\}$.
There are $\binom{n-k}{2^k-k}$ of there.

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Since $|A_i| = R(k)$, each A_i has a mono K_k.
1) Find a mono K_k in A_1. Say its $\{x_1, \ldots, x_k\}$.
2) REMOVE all A_i's that have all of $\{x_1, \ldots, x_k\}$.
There are $\binom{n-k}{2^k-k}$ of there.
There are $\binom{n}{2^k} - \binom{n-k}{2^k-k}$ left.

Solution to the Rated XXX Problem

List out all subsets of $V = \{1, \ldots, R(k)\}$ of size $R(k) = 2^k$.

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The $A \times B \times C$ Problem

Fill in the c and prove the resulting theorem: (By 'prove' I mean show me the first few steps of the construction and hand-wave the rest.)

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1) $\forall \mathrm{COL}\colon \mathbb{N} \times \mathbb{N} \times \mathbb{N} \rightarrow [10^{100}]$ $\exists$ **infinite** $A, B, C \subseteq \mathbb{N}$ **such that** $\mathrm{COL}\colon A \times B \times C \rightarrow [10^{100}]$ **only uses** c **colors.**

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2) $\exists\ \mathrm{COL}\colon \mathbb{N}\times\mathbb{N}\times\mathbb{N} \rightarrow [c]$ **such that there is no infinite** $A, B, C \subseteq \mathbb{N}$ **such that** $\mathrm{COL}\colon A\times B\times C \rightarrow [c]$ **only uses** $c-1$ **colors.**

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So you end up with 6 colors. So $c = 6$.

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By GMT $\exists$ uptick. So one of the finite sequences has uptick.
Contradiction!

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Show that there exist a number M such that,

for all $\mathrm{COL}\colon [M] \to [2]$, there exists a mono solution to E with w, x, y, z all distinct.

The Solution to The Equations Problem

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We will set $w = a + w'd$, $x = a + x'd$, $y = a + y'd$, $z = d$.

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Prove $VDW(5,2)$.

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View the 2-coloring of N as a $2^{2W(4,2)}$-coloring of the blocks.

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Break N into $N/2W(4,2)$ blocks of size $2W(k,2)$. k later.

Think of $[N]$ as $B_1 B_2 \cdots B_{N/2W(4,2)}$.

View the 2-coloring of N as a $2^{2W(4,2)}$-coloring of the blocks.

Need $N \ge W(4,2)W(4,2^{2W(4,2)})$

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We finish this on the next few slides.

Solution to VDW Problem (cont)

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$C_1 \qquad C_2 \qquad C_3 \qquad C_4 \qquad E$.

Solution to VDW Problem (cont)

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$C_1 \qquad C_2 \qquad C_3 \qquad C_4 \qquad E.$

Each C_i has **4-AP** also 5th elt in that sequence.

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$a \qquad a+d \qquad a+2d \qquad a+3d$. If $\mathrm{COL}(a+4d) =$ **R**, DONE

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We assume $\mathrm{COL}(a+4d) = $ **B**.

Solution to VDW Problem (cont)

We take $N = 2W(4,2)W(4, 2^{2W(4,2)})$.
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Because block are the same color:
$a+D \quad a+d+D \quad a+2d+D \quad a+3d+D$. If
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Solution to VDW Problem (cont)

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Because block are the same color:
$a+D \quad a+d+D \quad a+2d+D \quad a+3d+D$. If
$\mathrm{COL}(a+4d+D) =$ **B**, DONE
We assume $\mathrm{COL}(a+D+4d) =$ **B**. Rewrite as
$\mathrm{COL}(a+4d+D) =$ **B**.

Solution to VDW Problem (cont)

We take $N = 2W(4,2)W(4,2^{2W(4,2)})$.
There is a fifth block in this sequence. Don't know its color.
$C_1 \quad C_2 \quad C_3 \quad C_4 \quad E$.
Each C_i has **4-AP** also 5th elt in that sequence.
Let D be dist between blocks. Can assume the foll are **R**:
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We assume $\mathrm{COL}(a+4d) =$ **B**.

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We assume $\mathrm{COL}(a+D+4d) =$ **B**. Rewrite as
$\mathrm{COL}(a+4d+D) =$ **B**.
Similar: $\mathrm{COL}(a+2D+4d) =$ **B** and $\mathrm{COL}(a+3D+4d) =$ **B**.

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Similar: $\mathrm{COL}(a+2D+4d) =$ **B** and $\mathrm{COL}(a+3D+4d) =$ **B**.
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More on next slides.

Solution to VDW Problem (cont)

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We have the following are all **B**.

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$a + 4d \qquad a + 4d + D \qquad a + 4d + 2D \qquad a + 4d + 3D$

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We have the following are all **R**.

a $\quad$ $a + d + D$ $\quad$ $a + 42 + 2D$ $\quad$ $a + 3d + 3D$

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We have the following are all **B**.

$a + 4d$ $\quad a + 4d + D$ $\quad a + 4d + 2D$ $\quad a + 4d + 3D$

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If $\mathrm{COL}(a + 4D + 4d) =$ **B** then first one yields **B** 5-AP.

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If $\mathrm{COL}(a + 4D + 4d) =$ **B** then first one yields **B** 5-AP.

If $\mathrm{COL}(a + 4D + 4d) =$ **R** then second one yields **R** 5-AP.

The Proof Reminds Me Of

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ANDREW This problem reminds me of another funny moment in class, though it was incomplete.

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BILL What was that?

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BILL Yes; however, if you like the final product, why does that matter.

ANDREW I can't judge that until I hear the final product.

BILL Then you shall hear it. And not just the Chorus.

I Like Big Blocks! (Non Chorus)

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We partition N into block so wide

I Like Big Blocks! (Non Chorus)

We partition N into block so wide

Each Block's got a col-o-ring inside

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Encode each block by its own pro-file

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Collapse the colors; compress the style

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Pigeon-hole says types repeat

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Encode each block by its own pro-file

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Now look at blocks, and take a seat.

I Like Big Blocks! (Non Chorus)

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Ind-uction hums

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Pigeon-hole says types repeat

Now look at blocks, and take a seat.

Ind-uction hums

The gears all click

I Like Big Blocks! (Non Chorus)

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Each Block's got a col-o-ring inside

Encode each block by its own pro-file

Collapse the colors; compress the style

Pigeon-hole says types repeat

Now look at blocks, and take a seat.

Ind-uction hums

The gears all click

omega-squared—What a trick!

I Like Big Blocks! (Chorus)

I Like Big Blocks! (Chorus)

I like big blocks and I cannot lie!

I Like Big Blocks! (Chorus)

I like big blocks and I cannot lie!

You other math folks can't deny!

I Like Big Blocks! (Chorus)

I like big blocks and I cannot lie!

You other math folks can't deny!

With finitely many colors that you choose

I Like Big Blocks! (Chorus)

I like big blocks and I cannot lie!

You other math folks can't deny!

With finitely many colors that you choose

There's a mono AP that you can't refuse.

I Like Big Blocks! (Non Chorus)

I Like Big Blocks! (Non Chorus)

Color red-blue? Or three or more?

I Like Big Blocks! (Non Chorus)

Color red-blue? Or three or more?

Doesn't matter– same encore

I Like Big Blocks! (Non Chorus)

Color red-blue? Or three or more?

Doesn't matter– same encore

Hide your patterns, try to duck,

I Like Big Blocks! (Non Chorus)

Color red-blue? Or three or more?

Doesn't matter– same encore

Hide your patterns, try to duck,

Regularity brings the luck

I Like Big Blocks! (Non Chorus)

Color red-blue? Or three or more?

Doesn't matter– same encore

Hide your patterns, try to duck,

Regularity brings the luck

A Length-k run in one shade

I Like Big Blocks! (Non Chorus)

Color red-blue? Or three or more?

Doesn't matter– same encore

Hide your patterns, try to duck,

Regularity brings the luck

A Length-k run in one shade

Its a combinatorial serenade!

I Like Big Blocks! (Final Chorus)

I Like Big Blocks! (Final Chorus)

I like big blocks and I cannot lie!

I Like Big Blocks! (Final Chorus)

I like big blocks and I cannot lie!

Mono APs you can't deny.

I Like Big Blocks! (Final Chorus)

I like big blocks and I cannot lie!

Mono APs you can't deny.

With giant blocks and recursive tracks

I Like Big Blocks! (Final Chorus)

I like big blocks and I cannot lie!

Mono APs you can't deny.

With giant blocks and recursive tracks

Van der Waerden's got your back.

I Like Big Blocks! (Final Chorus)

I like big blocks and I cannot lie!

Mono APs you can't deny.

With giant blocks and recursive tracks

Van der Waerden's got your back.
(Applause if deserved.)

How Much Time?

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ANDREW How much time did you spend on that?

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The End