BILL, RECORD LECTURE!!!!

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Application! Restricting Domains To Stop Being Onto

Exposition by William Gasarch

February 5, 2026

Credit Where Credit Is Due

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This will not be our concern.

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In either case $f : \mathbb{D} \times \mathbb{D}$ is NOT onto.

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That wasn't stupid, but it was easy.

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Answer on next page.

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Given f, we will use Ramsey's Theorem 3 times.

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Why 3? We will discuss that later.

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Will now look at f restricted to $(x,y) \in H_1 \times H_1$ with $x < y$.

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Not done yet. On H_2 we control $f(x,x)$, $f(x,y)$ with $x < y$.

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Now look at f restricted to $(x,y) \in H_1 \times H_1$ with $x > y$.

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We show that f on $H_3 \times H_3$ is not onto.

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So some color is not used.

Examples

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Why 4? We will discuss this more after we do the Thin Set Theorem for $f(x,y,z)$.