

Content of AMSC 698D/CMSC 858R/MATH 608
Ramsey Theory and its “Applications”
<http://www/cs.umd.edu/~gasarch/858/S13/S13.html>

Overview: Ramsey Theory is a branch of combinatorics having to do with colorings and patterns. Here is a sample theorem: *for all 2-colorings of the natural numbers there exists arbitrarily long monochromatic arithmetic sequences (arithmetic sequences are equally spaced, like 11,14,17,20,23,27).* In this course we state and prove many such theorems and also “apply” them.

1. **The infinite and finite Erdos-Szekeres theorem on monotone sequences** APPLICATION to Lower bounds on Branching Programs.
2. **The infinite Ramsey Theorem** APPLICATION to Proving Programs correct, well-quasi ordering, Logic (Ramsey’s original motivation!). Canonical Ramsey Theorem. APPLICATION to lower bounds on Parallel Models of Computation.
3. **The finite Ramsey Theorems** Upper and lower bounds on the Ramsey Numbers. APPLICATIONS to lower bounds on various models of computation, the Erdos-Szekeres theorem in geometry, Sociology, History.
4. **The Large Ramsey Theorem** APPLICATION: A natural(?) example of something in Logic.
5. **Van Der Waerden’s Theorem** Multidim VDW theorem, upper and lower bounds on VDW numbers. APPLICATION to Number Theory, Communication Complexity, and the Diag-queens problem.
6. **Roth’s Theorem for $k = 3$** (the combinatorial proof by Szemerédi).
7. **Grid Colorings** APPLICATION: A good example for lower bounds in Tree Resolution.
8. **Rado’s theorem**
9. **Hales-Jewitt Theorem** APPLICATION to communication complexity.
10. **Polynomial VDW theorem** APPLICATION to graph theory.