Homework 1, Morally Due Tue Feb 6, 2018

COURSE WEBSITE: http://www.cs.umd.edu/~gasarch/858/S18.html
(The symbol before gasarch is a tilde.)

1. (5 points) What is your name? Write it clearly. Staple your HW. When is the midterm tentatively scheduled (give Date and Time)? If you cannot make it in that day/time see me ASAP.

2. (25 points)

   (a) (10 points) Prove that for every c, for every c coloring of $\binom{\mathsf{N}}{2}$, there is a homogenous set USING a proof similar to what I did in class.

   (b) (10 points) Prove that for every c, for every c coloring of $\binom{\mathsf{N}}{2}$, there is an infinite homogenous set USING induction on c.

   (c) (0 points) Which proof do you like better? Which one do you think gives better bound when you finitize it?

   SOLUTION TO PROBLEM TWO

   a) We omit.

   b) For $c = 1$ this is trivial.

   Assume $c \ge 2$ and that theorem is true for $c - 1$. Given a c-coloring

   $$COL : \binom{\mathsf{N}}{2} \rightarrow [c]$$

   define

   $COL'(x, y)$ to be

   (a) $COL(x, y)$ if $COL(x, y) \in [c - 2]$.

   (b) $c - 1$ if $COL(x, y) \in \{c - 1, c\}$

   Note that

   $$COL' : \binom{\mathsf{N}}{2} \rightarrow [c - 1].$$

   Apply the Induction hypothesis to it. There are two cases.

   (a) There exists an infinite homog set of color one of $\{1, \ldots, c - 2\}$. Then you are done!

(b) There exists an infinite homog set of color $c-1$. Let this set be A. Note that if $x, y \in A$ and $COL'(x, y) = c-1$ then $COL(x, y) \in \{c-1, c\}$. So we do not have a homogenous set yet. But now define

$$COL'' : \binom{A}{2} \rightarrow \{c-1, c\}$$

by $COL''(x, y) = COL(x, y)$ (we know that these are the only colors that pairs from A can have. Apply the IN with $c = 2$ to get a homog set.

NOTE: We went from c to $c-1$. We could have gone from c to two cases of $c/2$ or other combinations.

c) I prefer the proof. I think it leads to better bounds in the finite case but we'll look at that later.

SOLUTION TO PROBLEM TWO

3. (20 points) State and prove (rigorously) the c-color a-ary Ramsey Theorem. Your statement should start out *for all* $a \geq 1$, *for all* $c \geq 1$, ... The proof should be by induction on a with the base case being $a = 1$.

SOLUTION TO PROBLEM THREE

Omitted- very similar to what we did in class.

END OF SOLUTION TO PROBLEM THREE

4. (25 points)

   (a) Look up a proof of the Bolzano-Wierstrauss Theorem and present it in your own words.

   (b) THINK ABOUT: Is it similar to the proof of Ramsey's theorem?

   (c) LISTEN TO the one of the many rap songs about the BW theorem:

   www.youtube.com/watch?v=dfO18klwKHg

   (There is also a link on the website.)

   What did you think of it?

5. (25 points) State and prove a theorem with the XXX filled in.

   For every coloring (any number of colors) of $XXX(n)$ points there is EITHER: (a) a set of n that are all colored the same, or (b) a set of n points that are all colored differently. However!- there IS a coloring of $XXX(n) - 1$ points such that there is NEITHER: (a) a set of n that are all colored the same, or (b) a set of n points that are all colored differently.

   SOLUTION TO PROB FIVE

   $XXX(n) = (n-1)^2 + 1$.

   Let COL be a coloring of $(n-1)^2 + 1$ points. There are cases:

   (a) $\ge n$ colors are used. Then we are done.

   (b) $\le n-1$ colors are used. Then some color is used $\left\lceil \frac{(n-1)^2+1}{n-1} \right\rceil = n$ times so we are done.

   NOW I need to show there IS a coloring of $(n-1)^2$ points with neither an n-homog or n-rainbow set. Break into $n-1$ blocks of $n-1$ each. Color each block the same color, but each block different colors.

   END OF SOLUTION TO PROBLEM FIVE