Homework 4, Morally Due Tue Feb 20, 2018

1. (0 points) What is your name? Write it clearly. Staple your HW. What type of midterm will there be?

2. (25 points) In class we showed that for all \( n \), \( R(n) \) exists. The proof DID NOT give any bounds on \( R(n) \). Use a similar proof for the following:
   Let \( R(a, c, n) \) be such that for all \( c \)-colorings of \( \{R(a,c,n)\} \) there exists a homogenous set of size \( n \). Show that \( R(a, c, n) \) exists.

3. (25 points) In class we showed that for all \( n \), \( LR(n) \) exists. The proof DID NOT give any bounds on \( R(n) \). Use a similar proof for the following:
   Let \( LR(a, c, n) \) be such that for all \( c \)-colorings of \( \{n, n+1, \ldots, LR(a,c,n)\} \) there exists a large homogenous set.
   Show that \( LR(a, c, n) \) exists.

4. (25 points) Prove the following using some Can Ramsey Theorem:
   (Countable means infinite - some books disagree but they are wrong.)
   If \( X \subseteq \mathbb{R}^3 \) is a countable set of points, no four on the same plane, there exists countable \( Y \subseteq X \) such that every 4-subset of \( Y \) yields a different volume.

5. (25 points) (For his problem assume that there is NO cardinality between countable and the cardinality of the reals.) We say \( |X| = |\mathbb{R}| \) to mean that \( X \) and \( \mathbb{R} \) are the same size, so there is a bijection between them.
   Prove the following using a Maximal Set argument:
   If \( X \subseteq \mathbb{R}^3, |X| = |\mathbb{R}| \), no four on the same plane, there exists \( Y \subseteq \mathbb{R}^3, |Y| = |\mathbb{R}| \), such that every 4-subset of \( Y \) yields a different volume.

6. (0 points but you must do this so we can discuss) On the course website is a link to a review of a book on the Banach-Tarski Paradox. Read the review. Be prepared to discuss if you think the BT paradox is TRUE or FALSE or SOMETHING ELSE. There is no right answer here but I want to know what you think.