Homework 9, Morally Due Tue Apr 24, 2018

1. (40 points) Use the Prob method to get a lower bound on $W(k,c)$.

2. (40 points) In this problem an iso-L is a set of points that form an isosceles L. Formally an iso-L is (x,y), $(x+d,y)$, and $(x,y+d)$ where $x,y,d \in \mathsf{N}$.

   (a) (20 points) Use VDW theorem to show that there exists N such that, for all 2-colorings of $[N] \times [N]$ there exists a monochromatic isosceles L shape. How big is your N? (you may use VDW numbers in the expression).

   (b) (20 points) Use VDW theorem to show that there exists N such that, for all 3-colorings of $[N] \times [N]$ there exists a monochromatic isosceles L shape. How big is your N (you may use VDW numbers in the expression).

   (c) (0 points but think about) Use VDW theorem to show the following. For all c there exists $N(c)$ such that, for all c-colorings of $[N(c)] \times [N(c)]$ there exists a monochromatic isosceles L shape.

3. (20 points) Prove that there is no solution to $x^4+y^4=z^4$ with $x,y,z \in \mathsf{N}$ (This is the $n=4$ case of Fermat's last theorem. You may go to the web or a textbook or your local number theorist for help, but you must write it in your own words and understand what you write.) Did your proof use that the number of primes is infinite?