Homework 10, Morally Due Tue May 1, 2018

1. (0 points) Give your name! What kind of final will we have? When is it due?

2. (30 points) Give a 2-coloring of N that does NOT have an INFINITE monochromatic Arithmetic progression and prove that it has no such progression.

3. (30 points) Show that for all k, c, s there exists $J = J(k, c, s)$ such that, for all c-colorings of $[J]$, there exists a, d such that the following are all the same color:

$$a, a+d, a+2d, \ldots, a+(k-1)d, sd$$

4. (40 points) A tuple of integers $(a_1, \ldots, a_n)$ is called *awesome* if the following hold:

   For all c, for all c-colorings of N, there exists monochromatic $(e_1, \ldots, e_n)$ such that $\sum_{i=1}^{n} a_i e_i = 0$ (note- the $e_1, \ldots, e_n$ can have repeated elements, and for this problem 0 is not a natural number).

   (a) Show that if SOME subset of the a_i's sum to 0, then $(a_1, \ldots, a_n)$ is awesome.

   (b) Show that if NO subset of the a_i's sum to 0, then $(a_1, \ldots, a_n)$ is NOT awesome.

5. (0 points but you'll want to do this!) Who was the first person to write novelty songs about math. It may have been Tom Lehrer. I have collected on my website four math-songs and two chemistry songs that he wrote. Listen! Enjoy! In class we'll discuss what you liked and didn't like. Even if you know of his work, some of the songs here are obscure in that they are not on any of his albums. On the other hand, how can something be obscure if its on youtube?