

Content of CMSC/MATH 858R Ramsey Theory and its “Applications”

Overview: Ramsey Theory is a branch of combinatorics having to do with colorings and patterns. Here are three sample theorem:

1. For all 2-colorings of the edges of the complete graph on 50 nodes, there are 5 nodes so that all the edges between them are the same color.
2. For all 2-colorings of the natural numbers there exists arbitrarily long monochromatic arithmetic sequences (arithmetic sequences are equally spaced, like 11,14,17,20,23,27).
3. For all 2-colorings of $N \times N$ (the infinite grid) there exists four points that form a square that have the same color.

In this course we state and prove many such theorems and also “apply” them— to other parts of math and to TCS.

1. **The infinite Ramsey Theorem** APPLICATION to Proving Programs correct, well-quasi ordering, Canonical Ramsey Theorem. APPLICATION to Geometry.
2. **The finite Ramsey Theorems** Upper and lower bounds on the Ramsey Numbers. “APPLICATIONS” to lower bounds on various models of computation, the Erdos-Szekeres theorem in geometry, logic, Sociology, History.
3. **The Large Ramsey Theorem** “APPLICATION” to logic. (Thats a cheat as you will see.)
4. **Van Der Waerden’s Theorem** Multidim VDW theorem, upper and lower bounds on VDW numbers. “APPLICATION” to Number Theory, “APPLICATION” to Multiparty Comm Complexity, “APPLICATION” to Diag-queens problem.
5. **Roth’s Theorem for $k = 3$** (the combinatorial proof by Szemerédi).
6. **Grid Colorings**
7. **Rado’s theorem**
8. **Polynomial VDW theorem** ”APPLICATION” to graph theory.
9. **Time permitting we’ll do the Hales-Jewitt theorem or Euclidean Ramsey Theory or both.**