Homework 5, Morally Due Tue Mar 10, 2020, 3:30PM
COURSE WEBSITE: `http://www.cs.umd.edu/~gasarch/858/S18.html`

1. (0 points) What is your name? Write it clearly. Staple your HW. What type of midterm will there be?

2. (35 points) The *Manhattan metric* is the following metric in $\mathbb{R}^2$: the distance between a point $p_1 = (x_1, y_1)$ and $p_2 = (x_2, y_2)$ is $|p_2 - p_1|_1 = |x_2 - x_1| + |y_2 - y_1|$. (Note that this is the same as the the L_1 *norm*)

   Prove the following using the 2-ary Can Ramsey Theorem:

   If $X \subseteq \mathbb{R}^2$ is a countable set of points with the *Manhattan metric*, there exists a countable $Y \subseteq X$ such that every pair of points in Y has a different distance in the Manhattan metric.

3. (35 points) (For this problem assume that there is NO cardinality between countable and the cardinality of the reals.) We say $|X| = |\mathbb{R}|$ to mean that X and $\mathbb{R}$ are the same size, so there is a bijection between them.

   Prove the following using a Maximal Set argument:

   If $X \subseteq \mathbb{R}^3$, $|X| = |\mathbb{R}|$, no four on the same plane, there exists $Y \subseteq X$, $|Y| = |\mathbb{R}|$, such that every 4-subset of Y yields a different volume.

4. (30 points) Prove that for all 2-colorings of the 5×5 grid, there is a monochromatic rectangle.

   Recall that the 5×5 grid is

   $$\{(a, b) : a, b \in \mathbb{N} \text{ and } 1 \leq a, b \leq 5\}$$

   and a *monochromatic rectangle* is a set of four points that are the corners of a rectange, i.e. a set of points

   $$\{(x, y), (x + c, y), (x, y + d), (x + c, y + d)\}$$

   (for some $x, y, c, d \in [5]$) that are all the same color.