Homework 7, Morally Due Tue April 21, 2020, 3:30PM
COURSE WEBSITE: `http://www.cs.umd.edu/~gasarch/858/S18.html`
THIS HW IS TWO PAGES LONG!!!!!!!!!!!!

1. (0 points) What is your name? Write it clearly.

2. (40 points) Find polynomials $X_0(n)$, $X_1(n)$, $X_2(n)$ and $X_3(n)$ so that (1) they are monotone increasing, (2) on input $n \in \mathbb{N}$ the output is in $\mathbb{N}$, and (3) THE FOUR STATEMENTS below are true. The polynomials must be expressed so the coefficients are clear, something like:

$$\frac{87n^2}{13} + \frac{163n}{85} + \frac{17n}{35} - \frac{31}{14}$$

(This is not close to the answer and the answer need not be quadratic.)

You need to DERIVE the answer, not just write down polys which happens to work.

Advice Use Wolfram Alpha.

THE FOUR STATEMENTS:

- Let $n \equiv 0 \pmod 4$. For all 2-colorings of the edges of K_n there exists $\geq X_0(n)$ monochromatic K_3's.
- Let $n \equiv 1 \pmod 4$. For all 2-colorings of the edges of K_n there exists $\geq X_1(n)$ monochromatic K_3's.
- Let $n \equiv 2 \pmod 4$. For all 2-colorings of the edges of K_n there exists $\geq X_2(n)$ monochromatic K_3's.
- Let $n \equiv 3 \pmod 4$. For all 2-colorings of the edges of K_n there exists $\geq X_3(n)$ monochromatic K_3's.

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3. (30 points- 15 points each)

   (a) Give sentence ϕ in the lang of graphs with $SPEC(\phi) = \{100\}$.

   (b) Give sentence ϕ in the lang of graphs with $SPEC(\phi) = \{0, 3, 6, \ldots\}$.

4. (30 points) We are looking at the language of colored $\le$ 3-ary hype-graphs. Hence we have the following predicates:

   - $RED(x)$, $BLUE(x)$. (so single vertices can be colored RED or BLUE or not at all).
   - $RRED(x, y)$, $BBLUE(x, y)$, $GGREEN(x, y)$. (so 2-ary edges colored RRED or BBLUE or GGREEN or not at all).
   - $RRRED(x, y, z)$, $BBBLUE(x, y, z)$. (so 3-ary edges colored RRRED or BBBLUE or not at all).

   So a sample sentence is

   $(\exists x_1, x_2, x_3, x_4)(\forall y)[RED(x_1) \wedge BLUE(x_2) \implies BBBLUE(x_1, x_2, y)]$

   Show that the following is decidable:

   Given a E^*A^* sentence, return spec(ϕ) (which will always be finite or cofinite).

   You can assume the standard hypergraph Ramsey Theorem, but aside from that you must prove it from scratch. Here is our Litmus test: Someone in this class who missed my lecture on this should be able to read your proof and understand it.