Homework 8, Morally Due Tue April 28, 2020, 3:30PM
COURSE WEBSITE: `http://www.cs.umd.edu/~gasarch/858/S18.html`
THIS HW IS TWO PAGES LONG!!!!!!!!!!!!

Note: In this homework, $R(a,b)$ refers to the 2-ary asymmetric Ramsey numbers: $R(a,b)$ is the least n such that every 2-coloring of K_n has a mono RED K_a or a mono BLUE K_b. Similarly, $R(a,b,c)$ is the 2-ary asymmetric Ramsey with *three* colors: $R(a,b,c)$ is the least n such that every 3-coloring of K_n has a monoc RED K_a or a mono BLUE K_b or a mono GREEN K_c.

1. (0 points) What is your name? Write it clearly.
2. (30 points) We never defined $R(1,b)$ or $R(a,1)$. Define it so that the inequality
$$R(a,b) \le R(a-1,b) + R(a,b-1)$$
holds. Does the definition also make intuitive sense?
3. (30 points) Let $R(a,b,c)$ be the least n such that, for all 3-colorings of the edges of K_n, there exists either a RED K_a, a BLUE K_b, or a GREEN K_c.
   (a) Give an upper bound on $R(2,b,c)$ in terms of R on two variables.
   (b) **Recall** In class we proved
   $$R(2,b) = b \text{ and } R(a,2) = a$$
   $$R(a,b) \le R(a-1,b) + R(a,b-1).$$
   We used these two relations to get upper bounds on $R(a,b)$. We did this by viewing the size of the input (a,b) as $a+b$. With that viewpoint we are able to get upper bounds on $R(a,b)$ by using upper bounds on smaller pairs.
   In this problem you will use this approach for $R(a,b,c)$.
   The size of (a,b,c) is $a+b+c$. State and prove an inequality that upper bounds $R(a,b,c)$ in terms of R on smaller triples. (for example, it could be $R(a,b,c) \le R(\lfloor \sqrt{a} \rfloor, b-1, c) + R(\lfloor a/2 \rfloor, 2, 2)$ — but its not).

(c) Use parts (a) and (b) to determine reasonable upper bounds on $R(2,2,2)$, $R(2,2,3)$, $R(2,3,3)$, and $R(3,3,3)$.

4. (40 points) The following is a corollary of VDW's theorem that we will cover later in the class

   VDW Theorem For all k there exists $W = W(k)$ such that for all 2-coloring of $[W]$ there exists a, d such that

$$a, a+d, \ldots, a+(k-1)d$$

   are all the same color.

   (This is called a *monochromatic arithmetic progression of length* k which we abbreviate *mono* k*-AP*.

   Use the Prob Method to get a LOWER BOUND on $W(k)$.