Take Home Midterm
Morally Due April 7, DUE DUE-April 9
THIS EXAM IS TWO PAGES!!!!!!!!!!!!!!!

1. (0 points) What is your name? Write it clearly.

2. (30 points) **Definition:** Let X be any set of naturals (it can be finite or infinite) A coloring COL: $\binom{X}{2} \rightarrow \omega$ is *Erika* if $\mathrm{COL}(x, y) \le \min\{x, y\}$. (Note that ω is $\{1, 2, 3, \ldots, \}$, so $COL(1, y) = 1$ always.)

   Consider the following (true) statement which we call STATEMENT.

   If COL is an Erika coloring of $\binom{\mathsf{N}}{2}$ then either (1) there exists an infinite homog set, or (2) there exists an infinite min-homog set.

   (a) (10 points) Prove STATEMENT from the Can Ramsey Theory.

   (b) (10 points) Prove STATEMENT directly, NOT using the Can Ramsey Theory.

   (c) (10 points) Formulate a finite version of STATEMENT. Give a proof of your statement. It DOES NOT have to give bounds on n.

3. (25 points) Prove the following:

   For all k there exist n such that for all Erika colorings $COL : \binom{\{k,\ldots,n\}}{2} \rightarrow \omega$ there exist either (1) a LARGE homog set, or (2) a LARGE min-Homog set. (You need not get a bound on n.)

4. (25 points)

   (a) (15 points) Prove the following. There exists a function f such that the following holds:

   If $T_1, T_2, \ldots, T_{f(k)}$ is a FINITE sequence of trees, where T_i has at most $2^k i$ nodes, there is an uptick.

   For this problem the trees are ordered as $T_1 \le T_2$ if T_1 is a minor of T_2.

   (b) (10 points) Is there some function $g(i, k)$ such that if you replace the $2^k i$ in the first question with $g(i, k)$. the theorem is now false?

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5. (20 points) Let $c_1, c_2, c_3 \in \mathsf{N}$.

Let COL_1 be a c_1-coloring of $\binom{\mathsf{N}}{1}$.

Let COL_2 be a c_2-coloring of $\binom{\mathsf{N}}{2}$.

Let COL_3 be a c_3-coloring of $\binom{\mathsf{N}}{3}$.

A set $H \subseteq \mathsf{N}$ is *Nathan homogeneous* if

COL_1 restricted to $\binom{H}{1}$ is monochromatic, and

COL_2 restricted to $\binom{H}{2}$ is monochromatic, and

COL_3 restricted to $\binom{H}{3}$ is monochromatic.

(a) (10 points) Show that for all c_1, c_2, c_3, for all c_1-colorings of $\binom{\mathsf{N}}{1}$, c_2-colorings of $\binom{\mathsf{N}}{2}$, and all c_3-colorings of $\binom{\mathsf{N}}{3}$, there is an infinite Nathan Homogenous set.

(b) (10 points) State and prove a finite version of part a, with bounds. (You may use the known bounds on Ramsey Numbers.)