Chandra-Furst-Lipton Inspired me to be a Ramsey Theorist!

by William Gasarch

January 15, 2022

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For this talk we mostly consider $k = 3$.

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Notation $\chi(N)$ is the min number of colors needed to color $\{1,\ldots,N\} \times \{1,\ldots,N\}$ such that there are no monochromatic isosceles L's.

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So we are done! the answer is $\lg(\chi(2^n)$. Or are we?

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Beigel, G, Glenn: `https://www.cs.umd.edu/~gasarch/BLOGBOOK/foreheadserious.pdf`
Foster: `https://www.cs.umd.edu/~gasarch/BLOGBOOK/DONE/foreheadfun.pdf`

Open Questions

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2. $k = 3$ find an easy proof of that $k = 3$ case answer is (say) $\leq \frac{n}{3}$.