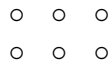


## COLORING GRIDS

### 1 Grids, Colorings, and Rectangles

The  $2 \times 3$  grid is the following



An  $a \times b$  grid is similar. We will refer to the  $a \times b$  grid as  $(a, b)$ .

**definition**

1. A  $c$ -coloring of  $a \times b$  is a function  $COL$  that maps all points in the  $a \times b$  grid to  $\{1, \dots, c\}$ .
2. A  $c$ -coloring  $COL$  of  $a \times b$  is  $R$ -free-coloring if there is no rectangle with all four corners the same color.
3. A  $c$ -coloring  $COL$  of  $a \times b$  is  $L$ -free-coloring if there is no L-shape (three points of a rectangle) with all three corners the same color.

### 2 Questions About R-free 2-Colorings of Grids

1. Find the number  $b$  such that  $(2, b)$  cannot be R-free 2-colored but  $(2, b - 1)$  can be.
2. Find the number  $b$  such that  $(3, b)$  cannot be R-free 2-colored but  $(3, b - 1)$  can be.
3. Find the number  $b$  such that  $(4, b)$  cannot be R-free 2-colored but  $(4, b - 1)$  can be.
4. Find the number  $b$  such that  $(5, b)$  cannot be R-free 2-colored but  $(5, b - 1)$  can be.
5. Come up with a statement like “ $(a, b)$  can be R-free 2-colored iff XXX”.

### 3 More Questions about Grid Colorings

1. Show that there is an  $n$  such that NO 10-coloring of  $n \times n$  is R-free.
2. Show that, for all  $c$ , there is an  $n$  such that NO 10-coloring of  $n \times n$  is R-free.
3. Consider questions similar to those in the last section for L-free 3-colorings. Come up with a statement like “ $(a, b)$  can be L-free 3-colored iff XXX”.
4. Consider questions similar to those in the last section for R-free 3-colorings. Come up with a statement like “ $(a, b)$  can be R-free 3-colored iff XXX”.
5. Consider the last two questions for 4 colorings.