HAT GAME

The Set Up: n people, working together as a team, must stand in a line. Each person can see the heads of everyone in front of her, but not her own head, or the heads of those in back of her. BEFORE hats are placed (the next step) they can discuss strategy; however, the adversary listens in on that conversation.

The Adversary's Move: The Adversary places either a red hat or a blue hat on top of each contestant's head. The contestants cannot communicate at all except as specified in the next step.

The Contestants Move: After the hats have been placed, each contestant, in turn starting from the back of the line and proceeding one by one to the front of the line, will call out one of the two colors, red or blue. Their goal is to get as many people as possible to correctly call out their own hat color.

Note: The n people may discuss strategy ahead of time. However, the adversary will be listening.

1. Is there a strategy that is guaranteed to get at least $n/2$ hats correct? (YES)
2. Is there a strategy that is guaranteed to get MORE THAN $n/2$ hats correct?
3. What is the best they can do?
4. Ask Bill to tell you about when this problem was a project in another summer program.

Extensions:

1. What if there are 3 colors of hats? c colors of hats?
2. What if there are just 2 colors of hats BUT there are a COUNTABLE number of people? Can you make sure that all but a finite number guess there correct hat color?