π Projects for π

Exposition by William Gasarch

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Approx π by Inscribing and Circumscribing n-gons

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n	$P_I(n)$	$P_C(n)$	$A(n)$	$A(n) - \pi$	$A(n) - \frac{22}{7}$
4	3	4	3.5	0.14	0.0001
6	3.1	3.8	3.4	0.10	0.001
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We will later use these numbers to find out how good an approx $A(n)$ is and to prove $\pi \neq \frac{22}{7}$.

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See next slide

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For π use what python says it is.
Write a program that generates the following table (the numbers I have in it are made up, but you'll see the idea) but rather than go out to 3 go out to whatever is reasonable for your computer.

n	$A(n)$	$\pi - A(n)$	$\frac{22}{7} - A(n)$
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How many places of π does $\sqrt{6\sum_{i=1}^{n}\frac{1}{i^2}}$ give?

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Thm $1 + \frac{1}{2^8} + \frac{1}{3^8} + \cdots = \frac{\pi^8}{9450}$.

For each of these find an infinite summation for π and do what you did in the last slide.