1 Problems For Leo

1.1 Q and Z

State and prove the following theorems.

Theorem 1.1 *Let E be a finite nonempty subset of $\mathsf{N} - \{0\}$.*

1. *E has property BLAH1 then*

   $(\forall x, y \in \mathsf{Q})$

   $(\bigwedge_{e \in E} x^e + y^e \in \mathsf{Z}) \implies (\forall n \in \mathsf{N})[x^n + y^n \in \mathsf{Z}])$.

2. *E has property BLAH2 then*

   $(\forall x, y \in \mathsf{Q})$

   $(\bigwedge_{e \in E} x^e + y^e \in \mathsf{Z}) \implies (x, y \in \mathsf{Z})]$.

3. *E has property BLAH3 then*

   $(\exists x, y \in \mathsf{Q})$

   $(\bigwedge_{e \in E} x^e + y^e \in \mathsf{Z})$ *and* $(\exists n)[x^n + y^n \notin \mathsf{Z}]$.

NOTE:
You thought BLAH1 was that all of the 2-adic valuations are the same.
You thought BLAH2 was that some pair of 2-adic valuations were different.
I don't think this is true.
Take $E = \{1\}$. This satisfies BLAH1.
Let $x = \frac{1}{2}$ and $y = -\frac{1}{2}$.
$x + y \in \mathsf{Z}$ but $x^2 + y^2 \notin \mathsf{Z}$.
Thats why I included BLAH3.

1.2 R and Z

Theorem 1.2 *Let E be a finite nonempty subset of $\mathsf{N} - \{0\}$.*

1. *E has property BLAH1 then*

   $(\forall x, y \in \mathsf{R})$

   $(\bigwedge_{e \in E} x^e + y^e \in \mathsf{Z}) \implies (\forall n \in \mathsf{N})[x^n + y^n \in \mathsf{Z}])$.

2. *E has property BLAH2 then*

   $(\forall x, y \in \mathsf{R})$

   $(\bigwedge_{e \in E} x^e + y^e \in \mathsf{Z}) \implies (x, y \in \mathsf{Z})]$.

3. *E has property BLAH3 then*

   $(\exists x, y \in \mathsf{R})$

   $(\bigwedge_{e \in E} x^e + y^e \in \mathsf{Z})$ *and* $(\exists n)[x^n + y^n \notin \mathsf{Z}]$.

1.3 C and Z

Theorem 1.3 *Let E be a finite nonempty subset of $\mathsf{N} - \{0\}$.*

1. *E has property BLAH1 then*
   $(\forall x, y \in \mathsf{C})$
   $(\bigwedge_{e \in E} x^e + y^e \in \mathsf{Z}) \implies (\forall n \in \mathsf{N})[x^n + y^n \in \mathsf{Z}])$.
2. *E has property BLAH2 then*
   $(\forall x, y \in \mathsf{C})$
   $(\bigwedge_{e \in E} x^e + y^e \in \mathsf{Z}) \implies (x, y \in \mathsf{Z})]$.
3. *E has property BLAH3 then*
   $(\exists x, y \in \mathsf{C})$
   $(\bigwedge_{e \in E} x^e + y^e \in \mathsf{Z})$ *and* $(\exists n)[x^n + y^n \notin \mathsf{Z}]$.

1.4 R and Q

Theorem 1.4 *Let E be a finite nonempty subset of $\mathsf{N} - \{0\}$.*

1. *E has property BLAH1 then*
   $(\forall x, y \in \mathsf{R})$
   $(\bigwedge_{e \in E} x^e + y^e \in \mathsf{Q}) \implies (\forall n \in \mathsf{N})[x^n + y^n \in \mathsf{Q}])$.
2. *E has property BLAH2 then*
   $(\forall x, y \in \mathsf{R})$
   $(\bigwedge_{e \in E} x^e + y^e \in \mathsf{Q}) \implies (x, y \in \mathsf{Q})]$.
3. *E has property BLAH3 then*
   $(\exists x, y \in \mathsf{R})$
   $(\bigwedge_{e \in E} x^e + y^e \in \mathsf{Q})$ *and* $(\exists n)[x^n + y^n \notin \mathsf{Q}]$.

1.5 C and Q

Theorem 1.5 *Let E be a finite nonempty subset of $\mathsf{N} - \{0\}$.*

1. *E has property BLAH1 then*
   $(\forall x, y \in \mathsf{C})$
   $(\bigwedge_{e \in E} x^e + y^e \in \mathsf{Q}) \implies (\forall n \in \mathsf{N})[x^n + y^n \in \mathsf{Q}])$.
2. *E has property BLAH2 then*
   $(\forall x, y \in \mathsf{C})$
   $(\bigwedge_{e \in E} x^e + y^e \in \mathsf{Q}) \implies (x, y \in \mathsf{Q})]$.
3. *E has property BLAH3 then*
   $(\exists x, y \in \mathsf{C})$
   $(\bigwedge_{e \in E} x^e + y^e \in \mathsf{Q})$ *and* $(\exists n)[x^n + y^n \notin \mathsf{Q}]$.

1.6 C and R

Theorem 1.6 *Let E be a finite nonempty subset of $\mathsf{N} - \{0\}$.*

1. *E has property BLAH1 then*
   $(\forall x, y \in \mathsf{C})$
   $(\bigwedge_{e \in E} x^e + y^e \in \mathsf{R}) \implies (\forall n \in \mathsf{N})[x^n + y^n \in \mathsf{R}])$.
2. *E has property BLAH2 then*
   $(\forall x, y \in \mathsf{C})$
   $(\bigwedge_{e \in E} x^e + y^e \in \mathsf{R}) \implies (x, y \in \mathsf{R})]$.
3. *E has property BLAH3 then*
   $(\exists x, y \in \mathsf{C})$
   $(\bigwedge_{e \in E} x^e + y^e \in \mathsf{R})$ *and* $(\exists n)[x^n + y^n \notin \mathsf{R}]$.