Results

Chid's Result

Thm Let $\mathbb{D}_1, \mathbb{D}_2$ be integral domains that are all subsets of $\mathbb{C}$. Assume $\mathbb{D}_2$ is a field. Let $x, y \in \mathbb{D}_1$. If $x + y \in \mathbb{D}_2$ and $x^2 + y^2 \in \mathbb{D}_2$ then $(\forall k \in \mathbb{N})[x^k + y^k \in \mathbb{D}_2]$.

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Proof $x + y \in \mathbb{D}_2$ so $(x + y)^2 \in \mathbb{D}_2$. Since $x^2 + y^2 \in \mathbb{D}_2$ we have $(x+y)^2 - (x+y) \in \mathbb{D}_2$, so $2xy \in \mathbb{D}_2$. Since $\mathbb{D}_2$ is a field, $xy \in \mathbb{D}_2$.

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$x^k + y^k = (x + y)(x^{k-1} + y^{k-1}) - xy(x^{k-2} + y^{k-2}) \in \mathbb{D}_2$.