

Corrections on Charlie's Rado Exposition

1 Title Page

1. After your name you should have

Mentored by William Gasarch

You would take that out if you submit this anywhere, but for now it goes on the website of REU papers (published or not).

2. Schur's theorem IS a corollary of Rado's theorem- so YES that is a unification.

How do VDW and Rado relate?

VDW is a corollary of Rado for systems of linear equations, but not for the single-equation case.

I think your paper is just on the single-equation case. You can say

His theorem (1) has Schur's theorem as a corollary, and (2) used an extension of van Der Warden's theorem in its proof. As such it tied together different strands of knowledge.

3. You should STATE Rado's theorem early, perhaps right before the paragraph that begins *While Rado's Theorem ...*. Since you will be talking about number-of-colors, your statement of Rado's theorem should include the number-of-colors needed for the case where Rado fails.

Rado's Theorem Let $a_1, \dots, a_n \in \mathbb{Z} - \{0\}$.

- (a) If some subset of $\{a_1, \dots, a_n\}$ sums to 0 then, for all c , there exists $R = R(a_1, \dots, a_n; c)$, such that, for all $COL: [R] \rightarrow [c]$ there exists monochromatic $x_1, \dots, x_n$ such that $\sum_{i=1}^n a_i x_i = 0$.
- (b) If no subset of $\{a_1, \dots, a_n\}$ sums to 0 then there exists c and a coloring $COL: \mathbb{N} \rightarrow [c]$ with no monochromatic $x_1, \dots, x_n$ such that $\sum_{i=1}^n a_i x_i = 0$. (The proof shows c is $\leq$ the least prime that divides all sums of $\{a_1, \dots, a_n\}$.)

Definition $R(a_1, \dots, a_n; c)$ is called the *Rado Number*.

4. On Page 8 and later. I NOW see the point of the paper. You do two very different things later, which is fine, but you should talk about them in the intro.

(a) You study some classes of equations that have $= b$ instead of $= 0$. That is a fine thing to study but you should tell us that right after you state Rado's Theorem.

You should also change the title to

(b) You study some classes that are of the $= 0$ type but have some other restriction. I would consider

$$\sum_{i=1}^n a_i x_i = x_0$$

to be

$$x_0 + \sum_{i=1}^n a_i x_i = x_0$$

and hence to be Rado where at least one of the coefficients is -1.

5. Page 9. Just curious- when they give upper bounds on Rado numbers (or perhaps just prove that there are upper bounds) do they use VDW? Extended VDW? Rado for the $= 0$ case? Or something else. Perhaps they don't use anything already known.

6. Your examples of R_2 are not that interesting since they are of the $= 0$ type. Give some examples when its not $= 0$.

7. WOW, I had forgotten all about the Gasarch-Moriarity-Tumma paper!
No correction here, just surprise.

8. Page 11. I am prob missing something here- the equation

$$ax_1 + \cdots a_m x_m = x_0$$

is just

$$-x_0 + ax_1 + \cdots a_m x_m = 0$$

So its Rado's theorem where one of the variable has coefficient -1 .

Does that make it easier to get the Rado Number?

Apparently so!

No correction here, just surprise.