Quantum Bits, Entanglement, and the CHSH Game

Exposition by
William Gasarch and Evan Golub

November 14, 2024

Outline

Outline

1. We describe what qubits are mathematically and how they can be used. We ignore the Physics. Physicists really can create qubits that behave as we describe.

Outline

1. We describe what qubits are mathematically and how they can be used. We ignore the Physics. Physicists really can create qubits that behave as we describe.
2. We describe what entangled qubits are mathematically and how they can be used. We ignore the Physics. Physicists really can create entangled qubits that behave as we describe.

Outline

1. We describe what qubits are mathematically and how they can be used. We ignore the Physics. Physicists really can create qubits that behave as we describe.
2. We describe what entangled qubits are mathematically and how they can be used. We ignore the Physics. Physicists really can create entangled qubits that behave as we describe.
3. We describe the CHSH game.

Outline

1. We describe what qubits are mathematically and how they can be used. We ignore the Physics. Physicists really can create qubits that behave as we describe.
2. We describe what entangled qubits are mathematically and how they can be used. We ignore the Physics. Physicists really can create entangled qubits that behave as we describe.
3. We describe the CHSH game.
4. We give a strategy for the CHSH game where (1) the 2 players are classical, and (2) **the prob of winning is 0.75.** We note that one can prove this is the best two players can do.

Outline

1. We describe what qubits are mathematically and how they can be used. We ignore the Physics. Physicists really can create qubits that behave as we describe.
2. We describe what entangled qubits are mathematically and how they can be used. We ignore the Physics. Physicists really can create entangled qubits that behave as we describe.
3. We describe the CHSH game.
4. We give a strategy for the CHSH game where (1) the 2 players are classical, and (2) **the prob of winning is 0.75.** We note that one can prove this is the best two players can do.
5. We give a strategy for the CHSH game where (1) the 2 players have qubits that are entangled, and (2) **the prob of winning is larger than 0.75.**

Quantum Bits I: Measure Once

Exposition by
William Gasarch and Evan Golub

November 14, 2024

Math Needed

Def

Math Needed

Def

1. Let L be the following function on vectors of complex numbers: $L(\alpha, \beta) = |\alpha|^2 + |\beta|^2$. Note that $L(\alpha, \beta)$ is the square of length of the vector (α, β).

Math Needed

Def

1. Let L be the following function on vectors of complex numbers: $L(\alpha, \beta) = |\alpha|^2 + |\beta|^2$. Note that $L(\alpha, \beta)$ is the square of length of the vector (α, β).

   For the rest of these slides we will assume that L is applied to pairs of reals. We note that the use of complex numbers is very important for quantum mechanics.

Math Needed

Def

1. Let L be the following function on vectors of complex numbers: $L(\alpha, \beta) = |\alpha|^2 + |\beta|^2$. Note that $L(\alpha, \beta)$ is the square of length of the vector (α, β).

   For the rest of these slides we will assume that L is applied to pairs of reals. We note that the use of complex numbers is very important for quantum mechanics.

2. A 2×2 matrix M is **unitary** if when, for all v, $L(Mv) = L(v)$. So M preserves length.

Math Needed

Def

1. Let L be the following function on vectors of complex numbers: $L(\alpha, \beta) = |\alpha|^2 + |\beta|^2$. Note that $L(\alpha, \beta)$ is the square of length of the vector (α, β).

   For the rest of these slides we will assume that L is applied to pairs of reals. We note that the use of complex numbers is very important for quantum mechanics.

2. A 2×2 matrix M is **unitary** if when, for all v, $L(Mv) = L(v)$. So M preserves length.

Example Let $0 \leq \theta \leq 2\pi$. The following matrix is unitary.

Math Needed

Def

1. Let L be the following function on vectors of complex numbers: $L(\alpha, \beta) = |\alpha|^2 + |\beta|^2$. Note that $L(\alpha, \beta)$ is the square of length of the vector (α, β).

   For the rest of these slides we will assume that L is applied to pairs of reals. We note that the use of complex numbers is very important for quantum mechanics.

2. A 2×2 matrix M is **unitary** if when, for all v, $L(Mv) = L(v)$. So M preserves length.

Example Let $0 \le \theta \le 2\pi$. The following matrix is unitary.

$$M_\theta = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}$$

Math Needed

Def

1. Let L be the following function on vectors of complex numbers: $L(\alpha, \beta) = |\alpha|^2 + |\beta|^2$. Note that $L(\alpha, \beta)$ is the square of length of the vector (α, β).

   For the rest of these slides we will assume that L is applied to pairs of reals. We note that the use of complex numbers is very important for quantum mechanics.

2. A 2×2 matrix M is **unitary** if when, for all v, $L(Mv) = L(v)$. So M preserves length.

Example Let $0 \leq \theta \leq 2\pi$. The following matrix is unitary.

$$M_\theta = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}$$

This matrix rotates vectors by θ.

Math Needed

Def

1. Let L be the following function on vectors of complex numbers: $L(\alpha, \beta) = |\alpha|^2 + |\beta|^2$. Note that $L(\alpha, \beta)$ is the square of length of the vector (α, β).

   For the rest of these slides we will assume that L is applied to pairs of reals. We note that the use of complex numbers is very important for quantum mechanics.

2. A 2×2 matrix M is **unitary** if when, for all v, $L(Mv) = L(v)$. So M preserves length.

Example Let $0 \leq \theta \leq 2\pi$. The following matrix is unitary.

$$M_\theta = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}$$

This matrix rotates vectors by θ.
On next slide we show that M_θ is unitary.

Proof that M_θ is Unitary

Let $v = (\alpha, \beta)$ be a vector. We show $N(M_\theta(v)) = N(v)$.

$$\begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \cos(\theta)\alpha - \sin(\theta)\beta \\ \sin(\theta)\alpha + \cos(\theta)\beta \end{pmatrix}$$

Proof that M_θ is Unitary

Let $v = (\alpha, \beta)$ be a vector. We show $N(M_\theta(v)) = N(v)$.

$$\begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \cos(\theta)\alpha - \sin(\theta)\beta \\ \sin(\theta)\alpha + \cos(\theta)\beta \end{pmatrix}$$

$N(\cos(\theta)\alpha - \sin(\theta)\beta, \sin(\theta)\alpha + \cos(\theta)\beta)$

Proof that M_θ is Unitary

Let $v = (\alpha, \beta)$ be a vector. We show $N(M_\theta(v)) = N(v)$.

$$\begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \cos(\theta)\alpha - \sin(\theta)\beta \\ \sin(\theta)\alpha + \cos(\theta)\beta \end{pmatrix}$$

$N(\cos(\theta)\alpha - \sin(\theta)\beta, \sin(\theta)\alpha + \cos(\theta)\beta)$

$= \cos^2(\theta)\alpha^2 + \sin^2(\theta)\beta^2 + \sin^2(\theta)\alpha^2 + \cos^2(\theta)\beta^2$

Proof that M_θ is Unitary

Let $v = (\alpha, \beta)$ be a vector. We show $N(M_\theta(v)) = N(v)$.

$$\begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \cos(\theta)\alpha - \sin(\theta)\beta \\ \sin(\theta)\alpha + \cos(\theta)\beta \end{pmatrix}$$

$N(\cos(\theta)\alpha - \sin(\theta)\beta, \sin(\theta)\alpha + \cos(\theta)\beta)$

$= \cos^2(\theta)\alpha^2 + \sin^2(\theta)\beta^2 + \sin^2(\theta)\alpha^2 + \cos^2(\theta)\beta^2$

$= \cos^2(\theta)\alpha^2 + \sin^2(\theta)\alpha^2 + \sin^2(\theta)\beta^2 + \cos^2(\theta)\beta^2$

Proof that M_θ is Unitary

Let $v = (\alpha, \beta)$ be a vector. We show $N(M_\theta(v)) = N(v)$.

$$\begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \cos(\theta)\alpha - \sin(\theta)\beta \\ \sin(\theta)\alpha + \cos(\theta)\beta \end{pmatrix}$$

$N(\cos(\theta)\alpha - \sin(\theta)\beta, \sin(\theta)\alpha + \cos(\theta)\beta)$

$= \cos^2(\theta)\alpha^2 + \sin^2(\theta)\beta^2 + \sin^2(\theta)\alpha^2 + \cos^2(\theta)\beta^2$

$= \cos^2(\theta)\alpha^2 + \sin^2(\theta)\alpha^2 + \sin^2(\theta)\beta^2 + \cos^2(\theta)\beta^2$

$= (\cos^2(\theta) + \sin^2(\theta))\alpha^2 + (\cos^2(\theta) + \sin^2(\theta))\beta^2$

Proof that M_θ is Unitary

Let $v = (\alpha, \beta)$ be a vector. We show $N(M_\theta(v)) = N(v)$.

$$\begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \cos(\theta)\alpha - \sin(\theta)\beta \\ \sin(\theta)\alpha + \cos(\theta)\beta \end{pmatrix}$$

$$N(\cos(\theta)\alpha - \sin(\theta)\beta, \sin(\theta)\alpha + \cos(\theta)\beta)$$

$$= \cos^2(\theta)\alpha^2 + \sin^2(\theta)\beta^2 + \sin^2(\theta)\alpha^2 + \cos^2(\theta)\beta^2$$

$$= \cos^2(\theta)\alpha^2 + \sin^2(\theta)\alpha^2 + \sin^2(\theta)\beta^2 + \cos^2(\theta)\beta^2$$

$$= (\cos^2(\theta) + \sin^2(\theta))\alpha^2 + (\cos^2(\theta) + \sin^2(\theta))\beta^2$$

$$= \alpha^2 + \beta^2$$

Quantum Bits

Def A **qubit** is something in physics that has a state. The state is an ordered pair (α, β) such that $\alpha^2 + \beta^2 = 1$. If a qubit is in state (α, β) then, when the qubit is measured, the prob that the bit is 0 is α^2 and the prob the bit is 1 is β^2.

Quantum Bits

Def A **qubit** is something in physics that has a state. The state is an ordered pair (α, β) such that $\alpha^2 + \beta^2 = 1$. If a qubit is in state (α, β) then, when the qubit is measured, the prob that the bit is 0 is α^2 and the prob the bit is 1 is β^2.

Caveat A qubit can be measured in many ways:

Quantum Bits

Def A **qubit** is something in physics that has a state. The state is an ordered pair (α, β) such that $\alpha^2 + \beta^2 = 1$. If a qubit is in state (α, β) then, when the qubit is measured, the prob that the bit is 0 is α^2 and the prob the bit is 1 is β^2.

Caveat A qubit can be measured in many ways:
1) **Measuring the qubit in the standard basis**.

Quantum Bits

Def A **qubit** is something in physics that has a state. The state is an ordered pair (α, β) such that $\alpha^2 + \beta^2 = 1$. If a qubit is in state (α, β) then, when the qubit is measured, the prob that the bit is 0 is α^2 and the prob the bit is 1 is β^2.

Caveat A qubit can be measured in many ways:
1) **Measuring the qubit in the standard basis**.
2) **Measure the qubit in a different basis.**

Quantum Bits

Def A **qubit** is something in physics that has a state. The state is an ordered pair (α, β) such that $\alpha^2 + \beta^2 = 1$. If a qubit is in state (α, β) then, when the qubit is measured, the prob that the bit is 0 is α^2 and the prob the bit is 1 is β^2.

Caveat A qubit can be measured in many ways:
1) **Measuring the qubit in the standard basis**.
2) **Measure the qubit in a different basis.**
We multiply the state by a unitary matrix and measure the qubit in this new state.

Quantum Bits

Def A **qubit** is something in physics that has a state. The state is an ordered pair (α, β) such that $\alpha^2 + \beta^2 = 1$. If a qubit is in state (α, β) then, when the qubit is measured, the prob that the bit is 0 is α^2 and the prob the bit is 1 is β^2.

Caveat A qubit can be measured in many ways:
1) **Measuring the qubit in the standard basis**.
2) **Measure the qubit in a different basis.**
We multiply the state by a unitary matrix and measure the qubit in this new state.
So we say

We measure $M_\theta(v)$.

where v was the original state.

Quantum Bits

Def A **qubit** is something in physics that has a state. The state is an ordered pair (α, β) such that $\alpha^2 + \beta^2 = 1$. If a qubit is in state (α, β) then, when the qubit is measured, the prob that the bit is 0 is α^2 and the prob the bit is 1 is β^2.

Caveat A qubit can be measured in many ways:
1) **Measuring the qubit in the standard basis**.
2) **Measure the qubit in a different basis.**
We multiply the state by a unitary matrix and measure the qubit in this new state.
So we say

We measure $M_\theta(v)$.

where v was the original state.
We will elaborate on this on the next slide.

Measuring a Quantum Bit

If Alice has a qubit in states $v = (\alpha, \beta)$ she could do the following

Measuring a Quantum Bit

If Alice has a qubit in states $v = (\alpha, \beta)$ she could do the following

1) **Measure it in the standard basis.** This means that (1) she will get 0 with prob α^2 and (2) she will get 1 with prob β^2.

Measuring a Quantum Bit

If Alice has a qubit in states $v = (\alpha, \beta)$ she could do the following

1) **Measure it in the standard basis.** This means that (1) she will get 0 with prob α^2 and (2) she will get 1 with prob β^2.

2) **Measure it in basis θ**

Measuring a Quantum Bit

If Alice has a qubit in states $v = (\alpha, \beta)$ she could do the following

1) **Measure it in the standard basis.** This means that (1) she will get 0 with prob α^2 and (2) she will get 1 with prob β^2.

2) **Measure it in basis θ**
First compute $M_\theta(v) = w = (\gamma, \delta)$ where $\gamma^2 + \delta^2 = 1$.

Measuring a Quantum Bit

If Alice has a qubit in states $v = (\alpha, \beta)$ she could do the following

1) **Measure it in the standard basis.** This means that (1) she will get 0 with prob α^2 and (2) she will get 1 with prob β^2.

2) **Measure it in basis θ**
First compute $M_\theta(v) = w = (\gamma, \delta)$ where $\gamma^2 + \delta^2 = 1$.
Second measure w.

Measuring a Quantum Bit

If Alice has a qubit in states $v = (\alpha, \beta)$ she could do the following

1) **Measure it in the standard basis.** This means that (1) she will get 0 with prob α^2 and (2) she will get 1 with prob β^2.

2) **Measure it in basis θ**
First compute $M_\theta(v) = w = (\gamma, \delta)$ where $\gamma^2 + \delta^2 = 1$.
Second measure w.
She will get 0 with prob γ^2 and 1 with prob δ^2

Measuring a Quantum Bit

If Alice has a qubit in states $v = (\alpha, \beta)$ she could do the following

1) **Measure it in the standard basis.** This means that (1) she will get 0 with prob α^2 and (2) she will get 1 with prob β^2.

2) **Measure it in basis θ**
First compute $M_\theta(v) = w = (\gamma, \delta)$ where $\gamma^2 + \delta^2 = 1$.
Second measure w.
She will get 0 with prob γ^2 and 1 with prob δ^2
So she changes the state of the qubit before measuring it.

Measuring a Quantum Bit

If Alice has a qubit in states $v = (\alpha, \beta)$ she could do the following

1) **Measure it in the standard basis.** This means that (1) she will get 0 with prob α^2 and (2) she will get 1 with prob β^2.

2) **Measure it in basis θ**
First compute $M_\theta(v) = w = (\gamma, \delta)$ where $\gamma^2 + \delta^2 = 1$.
Second measure w.
She will get 0 with prob γ^2 and 1 with prob δ^2
So she changes the state of the qubit before measuring it.
This is referred to as **measuring the qubit in a different basis** or in a **different frame.**

Example

Alice has a qubit in state $v = (\alpha, \beta) = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

Example

Alice has a qubit in state $v = (\alpha, \beta) = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

1) She measures the qubit in the standard basis.

Example

Alice has a qubit in state $v = (\alpha, \beta) = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

1) She measures the qubit in the standard basis.
she will get 0 with prob $\frac{1}{2}$, and

Example

Alice has a qubit in state $v = (\alpha, \beta) = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

1) She measures the qubit in the standard basis.
she will get 0 with prob $\frac{1}{2}$, and
she will get 1 with prob $\frac{1}{2}$

Example

Alice has a qubit in state $v = (\alpha, \beta) = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

1) She measures the qubit in the standard basis.
she will get 0 with prob $\frac{1}{2}$, and
she will get 1 with prob $\frac{1}{2}$

2) If instead she measures $M_{\frac{\pi}{6}}(v)$ then we'll see what happens.

Example

Alice has a qubit in state $v = (\alpha, \beta) = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

1) She measures the qubit in the standard basis.
she will get 0 with prob $\frac{1}{2}$, and
she will get 1 with prob $\frac{1}{2}$

2) If instead she measures $M_{\frac{\pi}{6}}(v)$ then we'll see what happens.

Next two slides have the first and second coordinate of $M_{\frac{\pi}{6}}(v)$

Example (cont)

First coordinate of $M_{\frac{\pi}{6}}(v)$ is
$\cos(\theta)\alpha - \sin(\theta)\beta = \cos(\frac{\pi}{6})\frac{1}{\sqrt{2}} - \sin(\frac{\pi}{6})\frac{1}{\sqrt{2}}$

Example (cont)

First coordinate of $M_{\frac{\pi}{6}}(v)$ is
$\cos(\theta)\alpha - \sin(\theta)\beta = \cos(\frac{\pi}{6})\frac{1}{\sqrt{2}} - \sin(\frac{\pi}{6})\frac{1}{\sqrt{2}}$
$= \frac{\sqrt{3}}{2}\frac{1}{\sqrt{2}} - \frac{1}{2}\frac{1}{\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$.

Example (cont)

First coordinate of $M_{\frac{\pi}{6}}(v)$ is
$\cos(\theta)\alpha - \sin(\theta)\beta = \cos(\frac{\pi}{6})\frac{1}{\sqrt{2}} - \sin(\frac{\pi}{6})\frac{1}{\sqrt{2}}$
$= \frac{\sqrt{3}}{2}\frac{1}{\sqrt{2}} - \frac{1}{2}\frac{1}{\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$.

Note $\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^2 = \frac{4-2\sqrt{3}}{8} \sim 0.067$

Example (cont)

First coordinate of $M_{\frac{\pi}{6}}(v)$ is
$\cos(\theta)\alpha - \sin(\theta)\beta = \cos(\frac{\pi}{6})\frac{1}{\sqrt{2}} - \sin(\frac{\pi}{6})\frac{1}{\sqrt{2}}$
$= \frac{\sqrt{3}}{2}\frac{1}{\sqrt{2}} - \frac{1}{2}\frac{1}{\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}.$

Note $\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^2 = \frac{4-2\sqrt{3}}{8} \sim 0.067$

Second coordinate of $M_{\frac{\pi}{6}}(v)$ is
$\sin(\theta)\alpha + \cos(\theta)\beta = \sin(\frac{\pi}{6})\frac{1}{\sqrt{2}} + \cos(\frac{\pi}{6})\frac{1}{\sqrt{2}}$

Example (cont)

First coordinate of $M_{\frac{\pi}{6}}(v)$ is
$\cos(\theta)\alpha - \sin(\theta)\beta = \cos(\frac{\pi}{6})\frac{1}{\sqrt{2}} - \sin(\frac{\pi}{6})\frac{1}{\sqrt{2}}$
$= \frac{\sqrt{3}}{2}\frac{1}{\sqrt{2}} - \frac{1}{2}\frac{1}{\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}.$

Note $\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^2 = \frac{4-2\sqrt{3}}{8} \sim 0.067$

Second coordinate of $M_{\frac{\pi}{6}}(v)$ is
$\sin(\theta)\alpha + \cos(\theta)\beta = \sin(\frac{\pi}{6})\frac{1}{\sqrt{2}} + \cos(\frac{\pi}{6})\frac{1}{\sqrt{2}}$
$= \frac{1}{2}\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\frac{1}{\sqrt{2}} = \frac{1+\sqrt{3}}{2\sqrt{2}}.$

Example (cont)

First coordinate of $M_{\frac{\pi}{6}}(v)$ is
$\cos(\theta)\alpha - \sin(\theta)\beta = \cos(\frac{\pi}{6})\frac{1}{\sqrt{2}} - \sin(\frac{\pi}{6})\frac{1}{\sqrt{2}}$
$= \frac{\sqrt{3}}{2}\frac{1}{\sqrt{2}} - \frac{1}{2}\frac{1}{\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$.

Note $\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^2 = \frac{4-2\sqrt{3}}{8} \sim 0.067$

Second coordinate of $M_{\frac{\pi}{6}}(v)$ is
$\sin(\theta)\alpha + \cos(\theta)\beta = \sin(\frac{\pi}{6})\frac{1}{\sqrt{2}} + \cos(\frac{\pi}{6})\frac{1}{\sqrt{2}}$
$= \frac{1}{2}\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\frac{1}{\sqrt{2}} = \frac{1+\sqrt{3}}{2\sqrt{2}}$.

Note $\left(\frac{1+\sqrt{3}}{2\sqrt{2}}\right)^2 = \frac{4+2\sqrt{3}}{8} \sim 0.933$

Upshot of Example

Alice has qubit in state $v = (\alpha, \beta) = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

Upshot of Example

Alice has qubit in state $v = (\alpha, \beta) = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

1. If she measures the qubit in the standard basis then
   - $\Pr(0) = \frac{1}{2}$
   - $\Pr(1) = \frac{1}{2}$.

Upshot of Example

Alice has qubit in state $v = (\alpha, \beta) = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

1. If she measures the qubit in the standard basis then
   - $\Pr(0) = \frac{1}{2}$
   - $\Pr(1) = \frac{1}{2}$.
2. If instead she measures the qubit in bases $\frac{\pi}{6}$ then she computes $w = M_{\frac{\pi}{6}}(v)$ then
   - $\Pr(0) \sim 0.067$.
   - $\Pr(1) \sim 0.933$.

Upshot of Example

Alice has qubit in state $v = (\alpha, \beta) = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

1. If she measures the qubit in the standard basis then
   - $\Pr(0) = \frac{1}{2}$
   - $\Pr(1) = \frac{1}{2}$.
2. If instead she measures the qubit in bases $\frac{\pi}{6}$ then she computes $w = M_{\frac{\pi}{6}}(v)$ then
   - $\Pr(0) \sim 0.067$.
   - $\Pr(1) \sim 0.933$.

A rotation of 0 gave $\Pr(0) = 0.5$, whereas a rotation of $\frac{\pi}{6}$ made $\Pr(0) = 0.067$ which is much smaller. How does θ affect $\Pr(0)$?

Upshot of Example

Alice has qubit in state $v = (\alpha, \beta) = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

1. If she measures the qubit in the standard basis then
   - $\Pr(0) = \frac{1}{2}$
   - $\Pr(1) = \frac{1}{2}$.
2. If instead she measures the qubit in bases $\frac{\pi}{6}$ then she computes $w = M_{\frac{\pi}{6}}(v)$ then
   - $\Pr(0) \sim 0.067$.
   - $\Pr(1) \sim 0.933$.

A rotation of 0 gave $\Pr(0) = 0.5$, whereas a rotation of $\frac{\pi}{6}$ made $\Pr(0) = 0.067$ which is much smaller. How does θ affect $\Pr(0)$?

as $0 \leq \theta \leq \frac{\pi}{4}$, $\Pr(0)$ goes from $\frac{1}{2}$ to 0.

Upshot of Example

Alice has qubit in state $v = (\alpha, \beta) = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

1. If she measures the qubit in the standard basis then
   - $\Pr(0) = \frac{1}{2}$
   - $\Pr(1) = \frac{1}{2}$.
2. If instead she measures the qubit in bases $\frac{\pi}{6}$ then she computes $w = M_{\frac{\pi}{6}}(v)$ then
   - $\Pr(0) \sim 0.067$.
   - $\Pr(1) \sim 0.933$.

A rotation of 0 gave $\Pr(0) = 0.5$, whereas a rotation of $\frac{\pi}{6}$ made $\Pr(0) = 0.067$ which is much smaller. How does θ affect $\Pr(0)$?

as $0 \leq \theta \leq \frac{\pi}{4}$, $\Pr(0)$ goes from $\frac{1}{2}$ to 0.

The next few slides investigate this issue further.

How Does θ Affect $\Pr(0)$?

Alice has a qubit in state $v = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

How Does θ Affect Pr(0)?

Alice has a qubit in state $v = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.
She is going to measure the qubit in bases θ. Let $w = M_\theta(v)$

How Does θ Affect $\Pr(0)$?

Alice has a qubit in state $v = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.
She is going to measure the qubit in bases θ. Let $w = M_\theta(v)$
$\theta = 0$: $\Pr(0) = \frac{1}{2}$.
$\theta = \pi/60$: $\Pr(0) = 0.448$, close to $\frac{1}{2}$.

How Does θ Affect $\Pr(0)$?

Alice has a qubit in state $v = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.
She is going to measure the qubit in bases θ. Let $w = M_\theta(v)$
$\theta = 0$: $\Pr(0) = \frac{1}{2}$.
$\theta = \pi/60$: $\Pr(0) = 0.448$, close to $\frac{1}{2}$.

As θ gets bigger what happens?

How Does θ Affect Pr(0)?

Alice has a qubit in state $v = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.
She is going to measure the qubit in bases θ. Let $w = M_\theta(v)$
$\theta = 0$: $\Pr(0) = \frac{1}{2}$.
$\theta = \pi/60$: $\Pr(0) = 0.448$, close to $\frac{1}{2}$.

As θ gets bigger what happens?

1. For $0 \le \theta \le \frac{\pi}{4}$, $\Pr(0)$ goes from $\frac{1}{2}$ to 0.

How Does θ Affect $\Pr(0)$?

Alice has a qubit in state $v = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.
She is going to measure the qubit in bases θ. Let $w = M_\theta(v)$
$\theta = 0 : \Pr(0) = \frac{1}{2}$.
$\theta = \pi/60$: $\Pr(0) = 0.448$, close to $\frac{1}{2}$.

As θ gets bigger what happens?

1. For $0 \le \theta \le \frac{\pi}{4}$, $\Pr(0)$ goes from $\frac{1}{2}$ to 0.
2. For $\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$, $\Pr(0)$ goes from 0 to $\frac{1}{2}$.

How Does θ Affect Pr(0)?

Alice has a qubit in state $v = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.
She is going to measure the qubit in bases θ. Let $w = M_\theta(v)$
$\theta = 0$: $\Pr(0) = \frac{1}{2}$.
$\theta = \pi/60$: $\Pr(0) = 0.448$, close to $\frac{1}{2}$.

As θ gets bigger what happens?

1. For $0 \le \theta \le \frac{\pi}{4}$, Pr(0) goes from $\frac{1}{2}$ to 0.
2. For $\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$, Pr(0) goes from 0 to $\frac{1}{2}$.
3. For $\frac{\pi}{2} \le \theta \le \frac{3\pi}{4}$, Pr(0) goes from $\frac{1}{2}$ to 1.

How Does θ Affect $\Pr(0)$?

Alice has a qubit in state $v = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.
She is going to measure the qubit in bases θ. Let $w = M_\theta(v)$
$\theta = 0$: $\Pr(0) = \frac{1}{2}$.
$\theta = \pi/60$: $\Pr(0) = 0.448$, close to $\frac{1}{2}$.

As θ gets bigger what happens?

1. For $0 \le \theta \le \frac{\pi}{4}$, $\Pr(0)$ goes from $\frac{1}{2}$ to 0.
2. For $\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$, $\Pr(0)$ goes from 0 to $\frac{1}{2}$.
3. For $\frac{\pi}{2} \le \theta \le \frac{3\pi}{4}$, $\Pr(0)$ goes from $\frac{1}{2}$ to 1.
4. For $\frac{3\pi}{4} \le \theta \le \pi$, $\Pr(0)$ goes from 1 to $\frac{1}{2}$.

How Does θ Affect $\Pr(0)$?

Alice has a qubit in state $v = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.
She is going to measure the qubit in bases θ. Let $w = M_\theta(v)$
$\theta = 0 : \Pr(0) = \frac{1}{2}$.
$\theta = \pi/60$: $\Pr(0) = 0.448$, close to $\frac{1}{2}$.

As θ gets bigger what happens?

1. For $0 \le \theta \le \frac{\pi}{4}$, $\Pr(0)$ goes from $\frac{1}{2}$ to 0.
2. For $\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$, $\Pr(0)$ goes from 0 to $\frac{1}{2}$.
3. For $\frac{\pi}{2} \le \theta \le \frac{3\pi}{4}$, $\Pr(0)$ goes from $\frac{1}{2}$ to 1.
4. For $\frac{3\pi}{4} \le \theta \le \pi$, $\Pr(0)$ goes from 1 to $\frac{1}{2}$.

The next few slides give actual numbers.

$0 \leq \theta \leq \frac{\pi}{4}$

θ	α	β	$\Pr(0) = \alpha^2$	$\Pr(1) = \beta^2$
0	+0.707	+0.707	0.5	0.5
$\pi/60$	+0.669	+0.743	0.448	0.552
$2\pi/60$	+0.629	+0.777	0.396	0.604
$3\pi/60$	+0.588	+0.809	0.345	0.655
$4\pi/60$	+0.545	+0.839	0.297	0.703
$5\pi/60$	+0.500	+0.866	0.250	0.750
$6\pi/60$	+0.454	+0.891	0.206	0.794
$7\pi/60$	+0.407	+0.914	0.165	0.835
$8\pi/60$	+0.358	+0.934	0.128	0.872
$9\pi/60$	+0.309	+0.951	0.095	0.905
$10\pi/60$	+0.259	+0.966	0.067	0.933
$11\pi/60$	+0.208	+0.978	0.043	0.957
$12\pi/60$	+0.156	+0.988	0.024	0.976
$13\pi/60$	+0.105	+0.995	0.011	0.989
$14\pi/60$	+0.052	+0.999	0.003	0.997
$15\pi/60$	+0.000	+1.000	0.000	1.000

$\frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}$

θ	α	β	$\Pr(0) = \alpha^2$	$\Pr(1) = \beta^2$
$15\pi/60$	$+0.000$	$+1.000$	0.000	1.000
$16\pi/60$	-0.052	$+0.999$	0.003	0.997
$17\pi/60$	-0.105	$+0.995$	0.011	0.989
$18\pi/60$	-0.156	$+0.988$	0.024	0.976
$19\pi/60$	-0.208	$+0.978$	0.043	0.957
$20\pi/60$	-0.259	$+0.966$	0.067	0.933
$21\pi/60$	-0.309	$+0.951$	0.095	0.905
$22\pi/60$	-0.358	$+0.934$	0.128	0.872
$23\pi/60$	-0.407	$+0.914$	0.165	0.835
$24\pi/60$	-0.454	$+0.891$	0.206	0.794
$25\pi/60$	-0.500	$+0.866$	0.250	0.750
$26\pi/60$	-0.545	$+0.839$	0.297	0.703
$27\pi/60$	-0.588	$+0.809$	0.345	0.655
$28\pi/60$	-0.629	$+0.777$	0.396	0.604
$29\pi/60$	-0.669	$+0.743$	0.448	0.552
$30\pi/60$	-0.707	$+0.707$	0.500	0.500

$\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{4}$

θ	α	β	$\Pr(0) = \alpha^2$	$\Pr(1) = \beta^2$
$30\pi/60$	-0.707	$+0.707$	0.500	0.500
$31\pi/60$	-0.743	$+0.669$	0.552	0.448
$32\pi/60$	-0.777	$+0.629$	0.604	0.396
$33\pi/60$	-0.809	$+0.588$	0.655	0.345
$34\pi/60$	-0.839	$+0.545$	0.703	0.297
$35\pi/60$	-0.866	$+0.500$	0.750	0.250
$36\pi/60$	-0.891	$+0.454$	0.794	0.206
$37\pi/60$	-0.914	$+0.407$	0.835	0.165
$38\pi/60$	-0.934	$+0.358$	0.872	0.128
$39\pi/60$	-0.951	$+0.309$	0.905	0.095
$40\pi/60$	-0.966	$+0.259$	0.933	0.067
$41\pi/60$	-0.978	$+0.208$	0.957	0.043
$42\pi/60$	-0.988	$+0.156$	0.976	0.024
$43\pi/60$	-0.995	$+0.105$	0.989	0.011
$44\pi/60$	-0.999	$+0.052$	0.997	0.003
$45\pi/60$	-1.000	$+0.000$	1.000	0.000

$\frac{3\pi}{4} \leq \theta \leq \pi$

θ	α	β	$\Pr(0) = \alpha^2$	$\Pr(1) = \beta^2$
$45\pi/60$	-1.000	$+0.000$	1.000	0.000
$46\pi/60$	-0.999	-0.052	0.997	0.003
$47\pi/60$	-0.995	-0.105	0.989	0.011
$48\pi/60$	-0.988	-0.156	0.976	0.024
$49\pi/60$	-0.978	-0.208	0.957	0.043
$50\pi/60$	-0.966	-0.259	0.933	0.067
$51\pi/60$	-0.951	-0.309	0.905	0.095
$52\pi/60$	-0.934	-0.358	0.872	0.128
$53\pi/60$	-0.914	-0.407	0.835	0.165
$54\pi/60$	-0.891	-0.454	0.794	0.206
$55\pi/60$	-0.866	-0.500	0.750	0.250
$56\pi/60$	-0.839	-0.545	0.703	0.297
$57\pi/60$	-0.809	-0.588	0.655	0.345
$58\pi/60$	-0.777	-0.629	0.604	0.396
$59\pi/60$	-0.743	-0.669	0.552	0.448
$60\pi/60$	-0.707	-0.707	0.500	0.500

Quantum Bits II: Measure Twice

Exposition by
William Gasarch and Evan Golub

November 14, 2024

Measuring a Qubit Twice

EVAN- I HAVE REDONE THIS SLIDE SO PLEASE CHECK.

Alice has a qubit. For this scenario both its state and the basis Alice uses are irrelevant.

Scenario 0:

Measuring a Qubit Twice

EVAN- I HAVE REDONE THIS SLIDE SO PLEASE CHECK.

Alice has a qubit. For this scenario both its state and the basis Alice uses are irrelevant.

Scenario 0:
1) Alice measures qubit. Gets 0. The state is now $(1, 0)$.

Measuring a Qubit Twice

EVAN- I HAVE REDONE THIS SLIDE SO PLEASE CHECK.

Alice has a qubit. For this scenario both its state and the basis Alice uses are irrelevant.

Scenario 0:
1) Alice measures qubit. Gets 0. The state is now $(1, 0)$.
2) Bob measures qubit in basis θ.

Measuring a Qubit Twice

EVAN- I HAVE REDONE THIS SLIDE SO PLEASE CHECK.

Alice has a qubit. For this scenario both its state and the basis Alice uses are irrelevant.

Scenario 0:
1) Alice measures qubit. Gets 0. The state is now $(1, 0)$.
2) Bob measures qubit in basis θ.
State is $M_\theta(1, 0) = (\cos(\theta), \sin(\theta))$.

Measuring a Qubit Twice

EVAN- I HAVE REDONE THIS SLIDE SO PLEASE CHECK.

Alice has a qubit. For this scenario both its state and the basis Alice uses are irrelevant.

Scenario 0:
1) Alice measures qubit. Gets 0. The state is now $(1, 0)$.
2) Bob measures qubit in basis θ.
State is $M_\theta(1, 0) = (\cos(\theta), \sin(\theta))$.
Hence $\Pr(0) = \cos^2(\theta)$.

Measuring a Qubit Twice

EVAN- I HAVE REDONE THIS SLIDE SO PLEASE CHECK.

Alice has a qubit. For this scenario both its state and the basis Alice uses are irrelevant.

Scenario 0:
1) Alice measures qubit. Gets 0. The state is now $(1, 0)$.
2) Bob measures qubit in basis θ.
State is $M_\theta(1, 0) = (\cos(\theta), \sin(\theta))$.
Hence $\Pr(0) = \cos^2(\theta)$.

Scenario 1:

Measuring a Qubit Twice

EVAN- I HAVE REDONE THIS SLIDE SO PLEASE CHECK.

Alice has a qubit. For this scenario both its state and the basis Alice uses are irrelevant.

Scenario 0:
1) Alice measures qubit. Gets 0. The state is now $(1, 0)$.
2) Bob measures qubit in basis θ.
State is $M_\theta(1, 0) = (\cos(\theta), \sin(\theta))$.
Hence $\Pr(0) = \cos^2(\theta)$.

Scenario 1:
1) Alice measures qubit. Gets bit 1. The state is now $(0, 1)$.

Measuring a Qubit Twice

EVAN- I HAVE REDONE THIS SLIDE SO PLEASE CHECK.

Alice has a qubit. For this scenario both its state and the basis Alice uses are irrelevant.

Scenario 0:
1) Alice measures qubit. Gets 0. The state is now $(1, 0)$.
2) Bob measures qubit in basis θ.
State is $M_\theta(1, 0) = (\cos(\theta), \sin(\theta))$.
Hence $\Pr(0) = \cos^2(\theta)$.

Scenario 1:
1) Alice measures qubit. Gets bit 1. The state is now $(0, 1)$.
2) Bob measures qubit in basis θ.

Measuring a Qubit Twice

EVAN- I HAVE REDONE THIS SLIDE SO PLEASE CHECK.

Alice has a qubit. For this scenario both its state and the basis Alice uses are irrelevant.

Scenario 0:
1) Alice measures qubit. Gets 0. The state is now $(1, 0)$.
2) Bob measures qubit in basis θ.
State is $M_\theta(1, 0) = (\cos(\theta), \sin(\theta))$.
Hence $\Pr(0) = \cos^2(\theta)$.

Scenario 1:
1) Alice measures qubit. Gets bit 1. The state is now $(0, 1)$.
2) Bob measures qubit in basis θ.
State is $M_\theta(0, 1) = (-\sin(\theta), \cos(\theta))$.

Measuring a Qubit Twice

EVAN- I HAVE REDONE THIS SLIDE SO PLEASE CHECK.

Alice has a qubit. For this scenario both its state and the basis Alice uses are irrelevant.

Scenario 0:
1) Alice measures qubit. Gets 0. The state is now $(1,0)$.
2) Bob measures qubit in basis θ.
State is $M_\theta(1,0) = (\cos(\theta), \sin(\theta))$.
Hence $\Pr(0) = \cos^2(\theta)$.

Scenario 1:
1) Alice measures qubit. Gets bit 1. The state is now $(0,1)$.
2) Bob measures qubit in basis θ.
State is $M_\theta(0,1) = (-\sin(\theta), \cos(\theta))$.
Hence $\Pr(1) = \cos^2(\theta)$.

Measuring a Qubit Twice

EVAN- I HAVE REDONE THIS SLIDE SO PLEASE CHECK.

Alice has a qubit. For this scenario both its state and the basis Alice uses are irrelevant.

Scenario 0:
1) Alice measures qubit. Gets 0. The state is now $(1, 0)$.
2) Bob measures qubit in basis θ.
State is $M_\theta(1, 0) = (\cos(\theta), \sin(\theta))$.
Hence $\Pr(0) = \cos^2(\theta)$.

Scenario 1:
1) Alice measures qubit. Gets bit 1. The state is now $(0, 1)$.
2) Bob measures qubit in basis θ.
State is $M_\theta(0, 1) = (-\sin(\theta), \cos(\theta))$.
Hence $\Pr(1) = \cos^2(\theta)$.

Upshot Prob of agreement is $\cos^2(\theta)$.

Measuring a Qubit in Two Different Basis

EVAN- THIS SLIDE NOW SEEMS WRONG
Alice has a qubit in state $v = (\alpha, \beta)$.

Measuring a Qubit in Two Different Basis

EVAN- THIS SLIDE NOW SEEMS WRONG
Alice has a qubit in state $v = (\alpha, \beta)$.

1) Alice measures the qubit in basis θ_1, so in state $w = M_{\theta_1}(v)$, and gets bit b.

Measuring a Qubit in Two Different Basis

EVAN- THIS SLIDE NOW SEEMS WRONG
Alice has a qubit in state $v = (\alpha, \beta)$.

1) Alice measures the qubit in basis θ_1, so in state $w = M_{\theta_1}(v)$, and gets bit b.
2) Bob measures the qubit in basis θ_2, so in state $w' = M_{\theta_2}(w)$.

Measuring a Qubit in Two Different Basis

EVAN- THIS SLIDE NOW SEEMS WRONG
Alice has a qubit in state $v = (\alpha, \beta)$.

1) Alice measures the qubit in basis θ_1, so in state $w = M_{\theta_1}(v)$, and gets bit b.
2) Bob measures the qubit in basis θ_2, so in state $w' = M_{\theta_2}(w)$.
The prob that Bob gets b again is $\cos^2(\theta_1 - \theta_2)$.

Measuring a Qubit in Two Different Basis

EVAN- THIS SLIDE NOW SEEMS WRONG
Alice has a qubit in state $v = (\alpha, \beta)$.

1) Alice measures the qubit in basis θ_1, so in state $w = M_{\theta_1}(v)$, and gets bit b.
2) Bob measures the qubit in basis θ_2, so in state $w' = M_{\theta_2}(w)$.
The prob that Bob gets b again is $\cos^2(\theta_1 - \theta_2)$.
Why $\cos^2(\theta_1 - \theta_2)$?

Measuring a Qubit in Two Different Basis

EVAN- THIS SLIDE NOW SEEMS WRONG
Alice has a qubit in state $v = (\alpha, \beta)$.

1) Alice measures the qubit in basis θ_1, so in state $w = M_{\theta_1}(v)$, and gets bit b.
2) Bob measures the qubit in basis θ_2, so in state $w' = M_{\theta_2}(w)$.
The prob that Bob gets b again is $\cos^2(\theta_1 - \theta_2)$.
Why $\cos^2(\theta_1 - \theta_2)$?
We will explain that later.

Measuring a Qubit in Two Different Basis

EVAN- THIS SLIDE NOW SEEMS WRONG
Alice has a qubit in state $v = (\alpha, \beta)$.

1) Alice measures the qubit in basis θ_1, so in state $w = M_{\theta_1}(v)$, and gets bit b.
2) Bob measures the qubit in basis θ_2, so in state $w' = M_{\theta_2}(w)$.
The prob that Bob gets b again is $\cos^2(\theta_1 - \theta_2)$.
Why $\cos^2(\theta_1 - \theta_2)$?
We will explain that later.
EVAN- WE DO NOT EXPLAIN LATER AND I DO NOT THINK THIS IS TRUE.
ALICE's MEASURE IN θ_1 DOES NOT MATTER SINCE WHEN SHE IS DONE THE STATE IS $(1, 0)$ or $(0, 1)$. SO IS $\cos^2(\theta_1 - \theta_2)$ INCORRECT OR AM I MISSING SOMETHING?

Entanglement

Exposition by
William Gasarch and Evan Golub

November 14, 2024

Alice and Bob Like to Share

We say what Alice and Bob can do if they have qubits that are entangled in a certain way.

Alice and Bob Like to Share

We say what Alice and Bob can do if they have qubits that are entangled in a certain way.

There are several ways Alice and Bob's qubits can be entangled, which intuitively means that measurements made of one of them affects the other even if they are very far apart.

Alice and Bob Like to Share

We say what Alice and Bob can do if they have qubits that are entangled in a certain way.

There are several ways Alice and Bob's qubits can be entangled, which intuitively means that measurements made of one of them affects the other even if they are very far apart.

We will only deal with the case where the pair of qubits are an **EPR pair** (EPR stands for Einstein, Podolsky, Rosen) which is the simplest case of Entanglement. EPR pairs are also called **Bell Pairs.**

Alice and Bob Like to Share

We say what Alice and Bob can do if they have qubits that are entangled in a certain way.

There are several ways Alice and Bob's qubits can be entangled, which intuitively means that measurements made of one of them affects the other even if they are very far apart.

We will only deal with the case where the pair of qubits are an **EPR pair** (EPR stands for Einstein, Podolsky, Rosen) which is the simplest case of Entanglement. EPR pairs are also called **Bell Pairs.**

We will define properties of EPR pairs on the next slide.

EPR Pairs

We will state properties of EPR pairs.

EPR Pairs

We will state properties of EPR pairs.
Alice and Bob **Share an EPR pair** if

EPR Pairs

We will state properties of EPR pairs.
Alice and Bob **Share an EPR pair** if
1) Alice has qubit in state $v_A = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

EPR Pairs

We will state properties of EPR pairs.
Alice and Bob **Share an EPR pair** if
1) Alice has qubit in state $v_A = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.
2) Bob has qubit in state $v_B = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

EPR Pairs

We will state properties of EPR pairs.
Alice and Bob **Share an EPR pair** if
1) Alice has qubit in state $v_A = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.
2) Bob has qubit in state $v_B = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.
See next slide for how the qubits affect each other.

EPR Pairs Affect Each Other

EPR Pairs Affect Each Other

1) Alice measures her qubit in basis θ_A. She will get $\Pr(0) = \Pr(1) = 1/2$.

EPR Pairs Affect Each Other

1) Alice measures her qubit in basis θ_A. She will get
$\Pr(0) = \Pr(1) = 1/2$.
This is different from Alice measuring a bit that is not entangled.
In that case Alice's Prob's depend on $\cos(\theta_A)$.

EPR Pairs Affect Each Other

1) Alice measures her qubit in basis θ_A. She will get $\Pr(0) = \Pr(1) = 1/2$.
This is different from Alice measuring a bit that is not entangled.
In that case Alice's Prob's depend on $\cos(\theta_A)$.
We will see why this is important soon.

EPR Pairs Affect Each Other

1) Alice measures her qubit in basis θ_A. She will get $\Pr(0) = \Pr(1) = 1/2$.
This is different from Alice measuring a bit that is not entangled.
In that case Alice's Prob's depend on $\cos(\theta_A)$.
We will see why this is important soon.
2) Bob then measures his qubit in basis θ_B. The prob his bit will agree with Alice is $\cos^2(\theta_B - \theta_A)$.

EPR Pairs Affect Each Other

1) Alice measures her qubit in basis θ_A. She will get $\Pr(0) = \Pr(1) = 1/2$.
This is different from Alice measuring a bit that is not entangled.
In that case Alice's Prob's depend on $\cos(\theta_A)$.
We will see why this is important soon.
2) Bob then measures his qubit in basis θ_B. The prob his bit will agree with Alice is $\cos^2(\theta_B - \theta_A)$.
Example $\theta_A = \theta_B = 0$. Then Bob will get exactly what Alice got.

EPR Pairs Affect Each Other

1) Alice measures her qubit in basis θ_A. She will get $\Pr(0) = \Pr(1) = 1/2$.
This is different from Alice measuring a bit that is not entangled.
In that case Alice's Prob's depend on $\cos(\theta_A)$.
We will see why this is important soon.
2) Bob then measures his qubit in basis θ_B. The prob his bit will agree with Alice is $\cos^2(\theta_B - \theta_A)$.
Example $\theta_A = \theta_B = 0$. Then Bob will get exactly what Alice got.
Question Does this violate relativity? Alice and Bob are over a light year apart and Alice knows what Bob's Bit is.

EPR Pairs Affect Each Other

1) Alice measures her qubit in basis θ_A. She will get $\Pr(0) = \Pr(1) = 1/2$.
This is different from Alice measuring a bit that is not entangled.
In that case Alice's Prob's depend on $\cos(\theta_A)$.
We will see why this is important soon.
2) Bob then measures his qubit in basis θ_B. The prob his bit will agree with Alice is $\cos^2(\theta_B - \theta_A)$.
Example $\theta_A = \theta_B = 0$. Then Bob will get exactly what Alice got.
Question Does this violate relativity? Alice and Bob are over a light year apart and Alice knows what Bob's Bit is.
Answer No. Alice has no control over what the bit is. If she had control over the bit, or even control over the $\Pr(0)$ and $\Pr(1)$ (e.g., if measuring in a basis changed that) then this would violate Relativity.

EPR Pairs Affect Each Other

1) Alice measures her qubit in basis θ_A. She will get $\Pr(0) = \Pr(1) = 1/2$.
This is different from Alice measuring a bit that is not entangled.
In that case Alice's Prob's depend on $\cos(\theta_A)$.
We will see why this is important soon.
2) Bob then measures his qubit in basis θ_B. The prob his bit will agree with Alice is $\cos^2(\theta_B - \theta_A)$.
Example $\theta_A = \theta_B = 0$. Then Bob will get exactly what Alice got.
Question Does this violate relativity? Alice and Bob are over a light year apart and Alice knows what Bob's Bit is.
Answer No. Alice has no control over what the bit is. If she had control over the bit, or even control over the Pr(0) and Pr(1) (e.g., if measuring in a basis changed that) then this would violate Relativity.
EVAN- I THINK THE ABOVE Q AND A WAS YOUR CONCERN. WAS IT? IS MY EXPLANATION CORRECT?

Contrast Independent Pairs and EPR pairs

Alice has a qubit $v_A = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

Contrast Independent Pairs and EPR pairs

Alice has a qubit $v_A = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

Bob has a qubit $v_B = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

Contrast Independent Pairs and EPR pairs

Alice has a qubit $v_A = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

Bob has a qubit $v_B = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

1) Alice measures her qubit. St. basis. Gets 0.

Contrast Independent Pairs and EPR pairs

Alice has a qubit $v_A = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

Bob has a qubit $v_B = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

1) Alice measures her qubit. St. basis. Gets 0.

2) Bob measures his qubit. Basis $\frac{\pi}{6}$. state $M_{\frac{\pi}{6}}(v_B) = (0.259, 0.996)$.

Contrast Independent Pairs and EPR pairs

Alice has a qubit $v_A = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

Bob has a qubit $v_B = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

1) Alice measures her qubit. St. basis. Gets 0.

2) Bob measures his qubit. Basis $\frac{\pi}{6}$. state $M_{\frac{\pi}{6}}(v_B) = (0.259, 0.996)$.

2a) If v_A and v_B are independent of each other

$$\Pr(\text{Bob gets } 0) = 0.5 \qquad \Pr(\text{Bob gets } 1) = (0.259)^2 = 0.067.$$

Contrast Independent Pairs and EPR pairs

Alice has a qubit $v_A = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

Bob has a qubit $v_B = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

1) Alice measures her qubit. St. basis. Gets 0.

2) Bob measures his qubit. Basis $\frac{\pi}{6}$. state $M_{\frac{\pi}{6}}(v_B) = (0.259, 0.996)$.

2a) If v_A and v_B are independent of each other

$$\Pr(\text{Bob gets } 0) = 0.5 \qquad \Pr(\text{Bob gets } 1) = (0.259)^2 = 0.067.$$

2b) If v_A and v_B are an EPR pair then

Contrast Independent Pairs and EPR pairs

Alice has a qubit $v_A = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

Bob has a qubit $v_B = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

1) Alice measures her qubit. St. basis. Gets 0.

2) Bob measures his qubit. Basis $\frac{\pi}{6}$. state
$M_{\frac{\pi}{6}}(v_B) = (0.259, 0.996)$.

2a) If v_A and v_B are independent of each other

$$\Pr(\text{Bob gets } 0) = 0.5 \qquad \Pr(\text{Bob gets } 1) = (0.259)^2 = 0.067.$$

2b) If v_A and v_B are an EPR pair then

$$\Pr(\text{Bob gets } 0) = \Pr(\text{Alice \& Bob agree}) = \cos^2\left(\frac{\pi}{6} - 0\right) = 0.75.$$

Contrast Independent Pairs and EPR pairs

Alice has a qubit $v_A = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

Bob has a qubit $v_B = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

1) Alice measures her qubit. St. basis. Gets 0.

2) Bob measures his qubit. Basis $\frac{\pi}{6}$. state $M_{\frac{\pi}{6}}(v_B) = (0.259, 0.996)$.

2a) If v_A and v_B are independent of each other

$$\Pr(\text{Bob gets } 0) = 0.5 \qquad \Pr(\text{Bob gets } 1) = (0.259)^2 = 0.067.$$

2b) If v_A and v_B are an EPR pair then

$$\Pr(\text{Bob gets } 0) = \Pr(\text{Alice \& Bob agree}) = \cos^2\left(\frac{\pi}{6} - 0\right) = 0.75.$$

$$\Pr(\text{Bob gets } 1) = \Pr(\text{Alice \& Bob disagree}) = 1 - \cos^2\left(\frac{\pi}{6} - 0\right) = 0.25.$$

The CHSH Game

Exposition by
William Gasarch and Evan Golub

November 14, 2024

The CHSH Game

(CHSH stands for the authors of the paper this appeared in: John Clauser, Michael Horne, Abner Shimony, Richard Holt.)

The CHSH Game

(CHSH stands for the authors of the paper this appeared in:
John Clauser, Michael Horne, Abner Shimony, Richard Holt.)

1. Charles sends Alice a bit x and Bob a bit y. Both x and y were chosen uniformly at random.

The CHSH Game

(CHSH stands for the authors of the paper this appeared in: John Clauser, Michael Horne, Abner Shimony, Richard Holt.)

1. Charles sends Alice a bit x and Bob a bit y. Both x and y were chosen uniformly at random.
2. Alice sends Charles a bit a. Bob sends Charles a bit b.

The CHSH Game

(CHSH stands for the authors of the paper this appeared in: John Clauser, Michael Horne, Abner Shimony, Richard Holt.)

1. Charles sends Alice a bit x and Bob a bit y. Both x and y were chosen uniformly at random.
2. Alice sends Charles a bit a. Bob sends Charles a bit b.
3. If $x \wedge y = a \oplus b$ then Alice and Bob win. Else they lose.

Classic Strategies

On the next few slides we discuss strategies with an eye towards asking how often they win.

All 0 Strategy

Since $x \wedge y$ is mostly 0, make $a \oplus b$ always 0, so a strong strategy is for Alice and Bob to both send 0.

All 0 Strategy

Since $x \wedge y$ is mostly 0, make $a \oplus b$ always 0, so a strong strategy is for Alice and Bob to both send 0.

x	y	a	b	$x \wedge y$	$a \oplus b$	Wins?
0	0	0	0	0	0	Y
0	1	0	0	0	0	Y
1	0	0	0	0	0	Y
1	1	0	0	1	0	N

All 0 Strategy

Since $x \wedge y$ is mostly 0, make $a \oplus b$ always 0, so a strong strategy is for Alice and Bob to both send 0.

x	y	a	b	$x \wedge y$	$a \oplus b$	Wins?
0	0	0	0	0	0	Y
0	1	0	0	0	0	Y
1	0	0	0	0	0	Y
1	1	0	0	1	0	N

Alice and Bob win with probability 0.75.

Mostly 0 Strategy

Since $x \wedge y$ is mostly 0 but not **all** the time we want Alice to sometimes send a 1.

Mostly 0 Strategy

Since $x \wedge y$ is mostly 0 but not **all** the time we want Alice to sometimes send a 1.

If Alice sees a 1 then with prob p (to be determined) she sends a 1. Bob still always sends a 0

Mostly 0 Strategy

Since $x \wedge y$ is mostly 0 but not **all** the time we want Alice to sometimes send a 1.

If Alice sees a 1 then with prob p (to be determined) she sends a 1. Bob still always sends a 0

Alice will flip a **coin** with sides 0 and 1, prob p of getting a 1.

Mostly 0 Strategy

Since $x \wedge y$ is mostly 0 but not **all** the time we want Alice to sometimes send a 1.

If Alice sees a 1 then with prob p (to be determined) she sends a 1. Bob still always sends a 0

Alice will flip a **coin** with sides 0 and 1, prob p of getting a 1.

Next slide analyzes the prob that they win.

Analyzing the Mostly 0 Strategy

x	y	coin	a	b	$x \wedge y$	$a \oplus b$	Wins?
0	0	0	0	0	0	0	Y
0	0	1	0	0	0	0	Y
0	1	0	0	0	0	0	Y
0	1	1	0	0	0	0	Y
1	0	0	0	0	0	0	Y
1	0	1	1	0	0	1	N
1	1	0	0	0	1	0	N
1	1	1	1	0	1	1	Y

Analyzing the Mostly 0 Strategy

x	y	coin	a	b	$x \wedge y$	$a \oplus b$	Wins?
0	0	0	0	0	0	0	Y
0	0	1	0	0	0	0	Y
0	1	0	0	0	0	0	Y
0	1	1	0	0	0	0	Y
1	0	0	0	0	0	0	Y
1	0	1	1	0	0	1	N
1	1	0	0	0	1	0	N
1	1	1	1	0	1	1	Y

In the first four rows the coin flip is irrelevant.

Analyzing the Mostly 0 Strategy

x	y	coin	a	b	$x \wedge y$	$a \oplus b$	Wins?
0	0	0	0	0	0	0	Y
0	0	1	0	0	0	0	Y
0	1	0	0	0	0	0	Y
0	1	1	0	0	0	0	Y
1	0	0	0	0	0	0	Y
1	0	1	1	0	0	1	N
1	1	0	0	0	1	0	N
1	1	1	1	0	1	1	Y

In the first four rows the coin flip is irrelevant.
If $(x, y) = (1, 0)$ then they win if the coin is 0, so prob $1 - p$.

Analyzing the Mostly 0 Strategy

x	y	coin	a	b	$x \wedge y$	$a \oplus b$	Wins?
0	0	0	0	0	0	0	Y
0	0	1	0	0	0	0	Y
0	1	0	0	0	0	0	Y
0	1	1	0	0	0	0	Y
1	0	0	0	0	0	0	Y
1	0	1	1	0	0	1	N
1	1	0	0	0	1	0	N
1	1	1	1	0	1	1	Y

In the first four rows the coin flip is irrelevant.
If $(x, y) = (1, 0)$ then they win if the coin is 0, so prob $1 - p$.
If $(x, y) = (1, 1)$ then they win if the coin is 1, so prob p.

Analyzing the Mostly 0 Strategy

x	y	coin	a	b	$x \wedge y$	$a \oplus b$	Wins?
0	0	0	0	0	0	0	Y
0	0	1	0	0	0	0	Y
0	1	0	0	0	0	0	Y
0	1	1	0	0	0	0	Y
1	0	0	0	0	0	0	Y
1	0	1	1	0	0	1	N
1	1	0	0	0	1	0	N
1	1	1	1	0	1	1	Y

In the first four rows the coin flip is irrelevant.
If $(x, y) = (1, 0)$ then they win if the coin is 0, so prob $1 - p$.
If $(x, y) = (1, 1)$ then they win if the coin is 1, so prob p.

Hence they win when any of the following happen:

Analyzing the Mostly 0 Strategy

x	y	coin	a	b	$x \wedge y$	$a \oplus b$	Wins?
0	0	0	0	0	0	0	Y
0	0	1	0	0	0	0	Y
0	1	0	0	0	0	0	Y
0	1	1	0	0	0	0	Y
1	0	0	0	0	0	0	Y
1	0	1	1	0	0	1	N
1	1	0	0	0	1	0	N
1	1	1	1	0	1	1	Y

In the first four rows the coin flip is irrelevant.
If $(x, y) = (1, 0)$ then they win if the coin is 0, so prob $1 - p$.
If $(x, y) = (1, 1)$ then they win if the coin is 1, so prob p.

Hence they win when any of the following happen:

1) $(x, y) \in \{(0, 0), (0, 1)\}$. Thats prob $\frac{1}{2}$.

Analyzing the Mostly 0 Strategy

x	y	coin	a	b	$x \wedge y$	$a \oplus b$	Wins?
0	0	0	0	0	0	0	Y
0	0	1	0	0	0	0	Y
0	1	0	0	0	0	0	Y
0	1	1	0	0	0	0	Y
1	0	0	0	0	0	0	Y
1	0	1	1	0	0	1	N
1	1	0	0	0	1	0	N
1	1	1	1	0	1	1	Y

In the first four rows the coin flip is irrelevant.
If $(x, y) = (1, 0)$ then they win if the coin is 0, so prob $1 - p$.
If $(x, y) = (1, 1)$ then they win if the coin is 1, so prob p.

Hence they win when any of the following happen:

1) $(x, y) \in \{(0, 0), (0, 1)\}$. Thats prob $\frac{1}{2}$.

2) $(x, y) = (1, 0)$ and the coin is 0. Thats prob $\frac{1}{4} \times (1 - p)$.

Analyzing the Mostly 0 Strategy

x	y	coin	a	b	$x \wedge y$	$a \oplus b$	Wins?
0	0	0	0	0	0	0	Y
0	0	1	0	0	0	0	Y
0	1	0	0	0	0	0	Y
0	1	1	0	0	0	0	Y
1	0	0	0	0	0	0	Y
1	0	1	1	0	0	1	N
1	1	0	0	0	1	0	N
1	1	1	1	0	1	1	Y

In the first four rows the coin flip is irrelevant.
If $(x, y) = (1, 0)$ then they win if the coin is 0, so prob $1 - p$.
If $(x, y) = (1, 1)$ then they win if the coin is 1, so prob p.

Hence they win when any of the following happen:

1) $(x, y) \in \{(0, 0), (0, 1)\}$. Thats prob $\frac{1}{2}$.

2) $(x, y) = (1, 0)$ and the coin is 0. Thats prob $\frac{1}{4} \times (1 - p)$.

3) $(x, y) = (1, 1)$ and the coin is 1. Thats prob $\frac{1}{4} \times p$.

Analyzing the Mostly 0 Strategy

x	y	coin	a	b	$x \wedge y$	$a \oplus b$	Wins?
0	0	0	0	0	0	0	Y
0	0	1	0	0	0	0	Y
0	1	0	0	0	0	0	Y
0	1	1	0	0	0	0	Y
1	0	0	0	0	0	0	Y
1	0	1	1	0	0	1	N
1	1	0	0	0	1	0	N
1	1	1	1	0	1	1	Y

In the first four rows the coin flip is irrelevant.
If $(x, y) = (1, 0)$ then they win if the coin is 0, so prob $1 - p$.
If $(x, y) = (1, 1)$ then they win if the coin is 1, so prob p.

Hence they win when any of the following happen:

1) $(x, y) \in \{(0, 0), (0, 1)\}$. Thats prob $\frac{1}{2}$.

2) $(x, y) = (1, 0)$ and the coin is 0. Thats prob $\frac{1}{4} \times (1 - p)$.

3) $(x, y) = (1, 1)$ and the coin is 1. Thats prob $\frac{1}{4} \times p$.

So the prob of winning is $\frac{1}{2} + \frac{1-p}{4} + \frac{p}{4} = \frac{3}{4} = 0.75$.

Analyzing the Mostly 0 Strategy

x	y	coin	a	b	$x \wedge y$	$a \oplus b$	Wins?
0	0	0	0	0	0	0	Y
0	0	1	0	0	0	0	Y
0	1	0	0	0	0	0	Y
0	1	1	0	0	0	0	Y
1	0	0	0	0	0	0	Y
1	0	1	1	0	0	1	N
1	1	0	0	0	1	0	N
1	1	1	1	0	1	1	Y

In the first four rows the coin flip is irrelevant.
If $(x, y) = (1, 0)$ then they win if the coin is 0, so prob $1 - p$.
If $(x, y) = (1, 1)$ then they win if the coin is 1, so prob p.

Hence they win when any of the following happen:

1) $(x, y) \in \{(0, 0), (0, 1)\}$. Thats prob $\frac{1}{2}$.

2) $(x, y) = (1, 0)$ and the coin is 0. Thats prob $\frac{1}{4} \times (1 - p)$.

3) $(x, y) = (1, 1)$ and the coin is 1. Thats prob $\frac{1}{4} \times p$.

So the prob of winning is $\frac{1}{2} + \frac{1-p}{4} + \frac{p}{4} = \frac{3}{4} = 0.75$. No better.

Is There a Better Strategy?

The following are known:

Is There a Better Strategy?

The following are known:

1. There is no deterministic strategy that can win with probability more than 0.75.

Is There a Better Strategy?

The following are known:

1. There is no deterministic strategy that can win with probability more than 0.75.
2. There is no randomized strategy that can win with probability more than 0.75.

If Alice and Bob Share an EPR Pair . . .

We will show on the next two slides that if Alice and Bob share an EPR pair,

If Alice and Bob Share an EPR Pair . . .

We will show on the next two slides that if Alice and Bob share an EPR pair,

then Alice and Bob have a strategy that wins the CHSH game with probability $\frac{13}{16} = 0.8125 > 0.75$.

If Alice and Bob Share an EPR Pair ...

Alice and Bob share an EPR pair. Alice's (Bob's) qubit is in state v_A (v_B).

If Alice and Bob Share an EPR Pair . . .

Alice and Bob share an EPR pair. Alice's (Bob's) qubit is in state v_A (v_B).
Alice gets x, Bob gets y.

If Alice and Bob Share an EPR Pair ...

Alice and Bob share an EPR pair. Alice's (Bob's) qubit is in state v_A (v_B).
Alice gets x, Bob gets y.

1. $x = 0$: Alice measures $M_{\frac{\pi}{3}}(v_A)$. a is result.

If Alice and Bob Share an EPR Pair . . .

Alice and Bob share an EPR pair. Alice's (Bob's) qubit is in state v_A (v_B).
Alice gets x, Bob gets y.

1. $x = 0$: Alice measures $M_{\frac{\pi}{3}}(v_A)$. a is result.
2. $x = 1$: Alice measures v_A in the standard basis. a is result.

If Alice and Bob Share an EPR Pair ...

Alice and Bob share an EPR pair. Alice's (Bob's) qubit is in state v_A (v_B).
Alice gets x, Bob gets y.

1. $x = 0$: Alice measures $M_{\frac{\pi}{3}}(v_A)$. a is result.
2. $x = 1$: Alice measures v_A in the standard basis. a is result.
3. $y = 0$: Bob measures $M_{\frac{\pi}{6}}(v_B)$. b is result.

If Alice and Bob Share an EPR Pair ...

Alice and Bob share an EPR pair. Alice's (Bob's) qubit is in state v_A (v_B).
Alice gets x, Bob gets y.

1. $x = 0$: Alice measures $M_{\frac{\pi}{3}}(v_A)$. a is result.
2. $x = 1$: Alice measures v_A in the standard basis. a is result.
3. $y = 0$: Bob measures $M_{\frac{\pi}{6}}(v_B)$. b is result.
4. $y = 1$: Bob measures $M_{\frac{\pi}{2}}(v_B)$. b is result.

If Alice and Bob Share an EPR Pair ...

Alice and Bob share an EPR pair. Alice's (Bob's) qubit is in state v_A (v_B).
Alice gets x, Bob gets y.

1. $x = 0$: Alice measures $M_{\frac{\pi}{3}}(v_A)$. a is result.
2. $x = 1$: Alice measures v_A in the standard basis. a is result.
3. $y = 0$: Bob measures $M_{\frac{\pi}{6}}(v_B)$. b is result.
4. $y = 1$: Bob measures $M_{\frac{\pi}{2}}(v_B)$. b is result.

We analyze all four cases $(x, y) \in \{(0,0),(0,1),(1,0),(1,1)\}$ on the next slides.

Each Scenario

Each Scenario

1. $(x, y) = (0, 0)$. Alice: $\frac{\pi}{3}$. Bob: $\frac{\pi}{6}$. Prob they agree: $\cos^2(\frac{\pi}{3} - \frac{\pi}{6}) = \cos^2(\frac{\pi}{6}) = (\frac{\sqrt{3}}{2})^2 = \frac{3}{4}$.

Each Scenario

1. $(x, y) = (0, 0)$. Alice: $\frac{\pi}{3}$. Bob: $\frac{\pi}{6}$. Prob they agree: $\cos^2(\frac{\pi}{3} - \frac{\pi}{6}) = \cos^2(\frac{\pi}{6}) = (\frac{\sqrt{3}}{2})^2 = \frac{3}{4}$.
2. $(x, y) = (0, 1)$. Alice: $\frac{\pi}{3}$. Bob: $\frac{\pi}{2}$. Prob they agree: $\cos^2(\frac{\pi}{3} - \frac{\pi}{2}) = \cos^2(-\frac{\pi}{6}) = (\frac{\sqrt{3}}{2})^2 = \frac{3}{4}$.

Each Scenario

1. $(x, y) = (0, 0)$. Alice: $\frac{\pi}{3}$. Bob: $\frac{\pi}{6}$. Prob they agree: $\cos^2(\frac{\pi}{3} - \frac{\pi}{6}) = \cos^2(\frac{\pi}{6}) = (\frac{\sqrt{3}}{2})^2 = \frac{3}{4}$.
2. $(x, y) = (0, 1)$. Alice: $\frac{\pi}{3}$. Bob: $\frac{\pi}{2}$. Prob they agree: $\cos^2(\frac{\pi}{3} - \frac{\pi}{2}) = \cos^2(-\frac{\pi}{6}) = (\frac{\sqrt{3}}{2})^2 = \frac{3}{4}$.
3. $(x, y) = (1, 0)$. Alice: 0. Bob: $\frac{\pi}{6}$. Prob they agree: $\cos^2(\frac{\pi}{6} - 0) = \cos^2(\frac{\pi}{6}) = \frac{3}{4}$.

Each Scenario

1. $(x, y) = (0, 0)$. Alice: $\frac{\pi}{3}$. Bob: $\frac{\pi}{6}$. Prob they agree: $\cos^2(\frac{\pi}{3} - \frac{\pi}{6}) = \cos^2(\frac{\pi}{6}) = (\frac{\sqrt{3}}{2})^2 = \frac{3}{4}$.
2. $(x, y) = (0, 1)$. Alice: $\frac{\pi}{3}$. Bob: $\frac{\pi}{2}$. Prob they agree: $\cos^2(\frac{\pi}{3} - \frac{\pi}{2}) = \cos^2(-\frac{\pi}{6}) = (\frac{\sqrt{3}}{2})^2 = \frac{3}{4}$.
3. $(x, y) = (1, 0)$. Alice: 0. Bob: $\frac{\pi}{6}$. Prob they agree: $\cos^2(\frac{\pi}{6} - 0) = \cos^2(\frac{\pi}{6}) = \frac{3}{4}$.
4. $(x, y) = (1, 1)$. Alice: 0. Bob: $\frac{\pi}{2}$. Prob they agree: $\cos^2(\frac{\pi}{2} - 0) = \cos^2(\frac{\pi}{2}) = 0$.

Each Scenario

1. $(x, y) = (0, 0)$. Alice: $\frac{\pi}{3}$. Bob: $\frac{\pi}{6}$. Prob they agree: $\cos^2(\frac{\pi}{3} - \frac{\pi}{6}) = \cos^2(\frac{\pi}{6}) = (\frac{\sqrt{3}}{2})^2 = \frac{3}{4}$.
2. $(x, y) = (0, 1)$. Alice: $\frac{\pi}{3}$. Bob: $\frac{\pi}{2}$. Prob they agree: $\cos^2(\frac{\pi}{3} - \frac{\pi}{2}) = \cos^2(-\frac{\pi}{6}) = (\frac{\sqrt{3}}{2})^2 = \frac{3}{4}$.
3. $(x, y) = (1, 0)$. Alice: 0. Bob: $\frac{\pi}{6}$. Prob they agree: $\cos^2(\frac{\pi}{6} - 0) = \cos^2(\frac{\pi}{6}) = \frac{3}{4}$.
4. $(x, y) = (1, 1)$. Alice: 0. Bob: $\frac{\pi}{2}$. Prob they agree: $\cos^2(\frac{\pi}{2} - 0) = \cos^2(\frac{\pi}{2}) = 0$.
   So prob they do not agree is 1.

Each Scenario

1. $(x, y) = (0, 0)$. Alice: $\frac{\pi}{3}$. Bob: $\frac{\pi}{6}$. Prob they agree: $\cos^2(\frac{\pi}{3} - \frac{\pi}{6}) = \cos^2(\frac{\pi}{6}) = (\frac{\sqrt{3}}{2})^2 = \frac{3}{4}$.
2. $(x, y) = (0, 1)$. Alice: $\frac{\pi}{3}$. Bob: $\frac{\pi}{2}$. Prob they agree: $\cos^2(\frac{\pi}{3} - \frac{\pi}{2}) = \cos^2(-\frac{\pi}{6}) = (\frac{\sqrt{3}}{2})^2 = \frac{3}{4}$.
3. $(x, y) = (1, 0)$. Alice: 0. Bob: $\frac{\pi}{6}$. Prob they agree: $\cos^2(\frac{\pi}{6} - 0) = \cos^2(\frac{\pi}{6}) = \frac{3}{4}$.
4. $(x, y) = (1, 1)$. Alice: 0. Bob: $\frac{\pi}{2}$. Prob they agree: $\cos^2(\frac{\pi}{2} - 0) = \cos^2(\frac{\pi}{2}) = 0$.
   So prob they do not agree is 1.

Hence the prob of a win is
$\frac{1}{4} \times \frac{3}{4} + \frac{1}{4} \times \frac{3}{4} + \frac{1}{4} \times \frac{3}{4} + \frac{1}{4} \times 1 = \frac{13}{16} = 0.8125$.

What Does This Mean?

What Does This Mean?

1. Physicists have actually done this in the lab.

What Does This Mean?

1. Physicists have actually done this in the lab.
2. This is evidence that quantum mechanics is correct.

What Does This Mean?

1. Physicists have actually done this in the lab.
2. This is evidence that quantum mechanics is correct.
3. There are things we can do **better** in the quantum world than in the classical world.

Can We Do Better?

We have shown the following:

If Alice and Bob share an EPR pair then Alice and Bob have a strategy that wins the CHSH game with Prob 0.8125

Can We Do Better?

We have shown the following:

If Alice and Bob share an EPR pair then Alice and Bob have a strategy that wins the CHSH game with Prob 0.8125

Assume Alice and Bob share an EPR pair.

Can We Do Better?

We have shown the following:

If Alice and Bob share an EPR pair then Alice and Bob have a strategy that wins the CHSH game with Prob 0.8125

Assume Alice and Bob share an EPR pair.

Vote Which of the following is true:

Can We Do Better?

We have shown the following:

If Alice and Bob share an EPR pair then Alice and Bob have a strategy that wins the CHSH game with Prob 0.8125

Assume Alice and Bob share an EPR pair.

Vote Which of the following is true:

1. Alice and Bob have a strategy that wins the CHSH game with Prob $p > 0.8125$ and this is **known**.

Can We Do Better?

We have shown the following:

If Alice and Bob share an EPR pair then Alice and Bob have a strategy that wins the CHSH game with Prob 0.8125

Assume Alice and Bob share an EPR pair.

Vote Which of the following is true:

1. Alice and Bob have a strategy that wins the CHSH game with Prob $p > 0.8125$ and this is **known**.
2. The best Alice and ... can do is 0.8125 and this is **known**.

Can We Do Better?

We have shown the following:

If Alice and Bob share an EPR pair then Alice and Bob have a strategy that wins the CHSH game with Prob 0.8125

Assume Alice and Bob share an EPR pair.

Vote Which of the following is true:

1. Alice and Bob have a strategy that wins the CHSH game with Prob $p > 0.8125$ and this is **known**.
2. The best Alice and … can do is 0.8125 and this is **known**.
3. The question of if Alice and Bob can do better than 0.8125 is **Unknown to Science**.

Can We Do Better?

We have shown the following:
If Alice and Bob share an EPR pair then Alice and Bob have a strategy that wins the CHSH game with Prob 0.8125
Assume Alice and Bob share an EPR pair.

Vote Which of the following is true:

1. Alice and Bob have a strategy that wins the CHSH game with Prob $p > 0.8125$ and this is **known**.
2. The best Alice and ... can do is 0.8125 and this is **known**.
3. The question of if Alice and Bob can do better than 0.8125 is **Unknown to Science**.

Answer on the next slide.

Alice and Bob Can Do Better than 0.8125

Alice and Bob share an EPR pair.

Alice and Bob Can Do Better than 0.8125

Alice and Bob share an EPR pair.
Alice gets x, Bob gets y.

Alice and Bob Can Do Better than 0.8125

Alice and Bob share an EPR pair.
Alice gets x, Bob gets y.

1. $x = 0$: Alice measures $M_{\frac{\pi}{4}}(v_A)$. a is result.

Alice and Bob Can Do Better than 0.8125

Alice and Bob share an EPR pair.
Alice gets x, Bob gets y.

1. $x = 0$: Alice measures $M_{\frac{\pi}{4}}(v_A)$. a is result.
2. $x = 1$: Alice measures v_A in the standard basis. a is result.

Alice and Bob Can Do Better than 0.8125

Alice and Bob share an EPR pair.
Alice gets x, Bob gets y.

1. $x = 0$: Alice measures $M_{\frac{\pi}{4}}(v_A)$. a is result.
2. $x = 1$: Alice measures v_A in the standard basis. a is result.
3. $y = 0$: Bob measures $M_{\frac{\pi}{8}}(v_B)$. b is result.

Alice and Bob Can Do Better than 0.8125

Alice and Bob share an EPR pair.
Alice gets x, Bob gets y.

1. $x = 0$: Alice measures $M_{\frac{\pi}{4}}(v_A)$. a is result.
2. $x = 1$: Alice measures v_A in the standard basis. a is result.
3. $y = 0$: Bob measures $M_{\frac{\pi}{8}}(v_B)$. b is result.
4. $y = 1$: Bob measures $M_{\frac{3\pi}{8}}(v_B)$. b is result.

Alice and Bob Can Do Better than 0.8125

Alice and Bob share an EPR pair.
Alice gets x, Bob gets y.

1. $x = 0$: Alice measures $M_{\frac{\pi}{4}}(v_A)$. a is result.
2. $x = 1$: Alice measures v_A in the standard basis. a is result.
3. $y = 0$: Bob measures $M_{\frac{\pi}{8}}(v_B)$. b is result.
4. $y = 1$: Bob measures $M_{\frac{3\pi}{8}}(v_B)$. b is result.

We analyze all four cases $(x, y) \in \{(0,0),(0,1),(1,0),(1,1)\}$ on the next slides.

Each Scenario

Alice and Bob share an EPR pair.

Each Scenario

Alice and Bob share an EPR pair.

1. $(x, y) = (0, 0)$. Alice: $\frac{\pi}{4}$. Bob: $\frac{\pi}{8}$. Prob they agree: $\cos^2(\frac{\pi}{4} - \frac{\pi}{8}) = \cos^2(\frac{\pi}{8}) \sim 0.853$.

Each Scenario

Alice and Bob share an EPR pair.

1. $(x, y) = (0, 0)$. Alice: $\frac{\pi}{4}$. Bob: $\frac{\pi}{8}$. Prob they agree: $\cos^2(\frac{\pi}{4} - \frac{\pi}{8}) = \cos^2(\frac{\pi}{8}) \sim 0.853$.
2. $(x, y) = (0, 1)$. Alice: $\frac{\pi}{4}$. Bob: $\frac{3\pi}{8}$. Prob they agree: $\cos^2(\frac{3\pi}{8} - \frac{\pi}{4}) = \cos^2(\frac{\pi}{8}) \sim 0.853$.

Each Scenario

Alice and Bob share an EPR pair.

1. $(x, y) = (0, 0)$. Alice: $\frac{\pi}{4}$. Bob: $\frac{\pi}{8}$. Prob they agree: $\cos^2(\frac{\pi}{4} - \frac{\pi}{8}) = \cos^2(\frac{\pi}{8}) \sim 0.853$.
2. $(x, y) = (0, 1)$. Alice: $\frac{\pi}{4}$. Bob: $\frac{3\pi}{8}$. Prob they agree: $\cos^2(\frac{3\pi}{8} - \frac{\pi}{4}) = \cos^2(\frac{\pi}{8}) \sim 0.853$.
3. $(x, y) = (1, 0)$. Alice: 0. Bob: $\frac{\pi}{8}$. Prob they agree: $\cos^2(\frac{\pi}{8} - 0) = \cos^2(\frac{\pi}{8}) \sim 0.853$.

Each Scenario

Alice and Bob share an EPR pair.

1. $(x,y) = (0,0)$. Alice: $\frac{\pi}{4}$. Bob: $\frac{\pi}{8}$. Prob they agree: $\cos^2(\frac{\pi}{4} - \frac{\pi}{8}) = \cos^2(\frac{\pi}{8}) \sim 0.853$.
2. $(x,y) = (0,1)$. Alice: $\frac{\pi}{4}$. Bob: $\frac{3\pi}{8}$. Prob they agree: $\cos^2(\frac{3\pi}{8} - \frac{\pi}{4}) = \cos^2(\frac{\pi}{8}) \sim 0.853$.
3. $(x,y) = (1,0)$. Alice: 0. Bob: $\frac{\pi}{8}$. Prob they agree: $\cos^2(\frac{\pi}{8} - 0) = \cos^2(\frac{\pi}{8}) \sim 0.853$.
4. $(x,y) = (1,1)$. Alice: 0. Bob: $3\pi/8$. Prob they agree: $\cos^2(\frac{3\pi}{8} - 0) = \cos^2(\frac{3\pi}{8}) = 1 - \cos^2(\frac{\pi}{8})$.

Each Scenario

Alice and Bob share an EPR pair.

1. $(x, y) = (0, 0)$. Alice: $\frac{\pi}{4}$. Bob: $\frac{\pi}{8}$. Prob they agree: $\cos^2(\frac{\pi}{4} - \frac{\pi}{8}) = \cos^2(\frac{\pi}{8}) \sim 0.853$.
2. $(x, y) = (0, 1)$. Alice: $\frac{\pi}{4}$. Bob: $\frac{3\pi}{8}$. Prob they agree: $\cos^2(\frac{3\pi}{8} - \frac{\pi}{4}) = \cos^2(\frac{\pi}{8}) \sim 0.853$.
3. $(x, y) = (1, 0)$. Alice: 0. Bob: $\frac{\pi}{8}$. Prob they agree: $\cos^2(\frac{\pi}{8} - 0) = \cos^2(\frac{\pi}{8}) \sim 0.853$.
4. $(x, y) = (1, 1)$. Alice: 0. Bob: $3\pi/8$. Prob they agree: $\cos^2(\frac{3\pi}{8} - 0) = \cos^2(\frac{3\pi}{8}) = 1 - \cos^2(\frac{\pi}{8})$.
   So prob they do not agree is
   $1 - (1 - \cos^2(\frac{\pi}{8})) = \cos^2(\frac{\pi}{8}) \sim 0.853$.

Each Scenario

Alice and Bob share an EPR pair.

1. $(x, y) = (0, 0)$. Alice: $\frac{\pi}{4}$. Bob: $\frac{\pi}{8}$. Prob they agree: $\cos^2(\frac{\pi}{4} - \frac{\pi}{8}) = \cos^2(\frac{\pi}{8}) \sim 0.853$.
2. $(x, y) = (0, 1)$. Alice: $\frac{\pi}{4}$. Bob: $\frac{3\pi}{8}$. Prob they agree: $\cos^2(\frac{3\pi}{8} - \frac{\pi}{4}) = \cos^2(\frac{\pi}{8}) \sim 0.853$.
3. $(x, y) = (1, 0)$. Alice: 0. Bob: $\frac{\pi}{8}$. Prob they agree: $\cos^2(\frac{\pi}{8} - 0) = \cos^2(\frac{\pi}{8}) \sim 0.853$.
4. $(x, y) = (1, 1)$. Alice: 0. Bob: $3\pi/8$. Prob they agree: $\cos^2(\frac{3\pi}{8} - 0) = \cos^2(\frac{3\pi}{8}) = 1 - \cos^2(\frac{\pi}{8})$.
   So prob they do not agree is
   $1 - (1 - \cos^2(\frac{\pi}{8})) = \cos^2(\frac{\pi}{8}) \sim 0.853$.

$(x, y) \in \{(0, 0), (0, 1), (1, 0)\} \implies \Pr(WIN) = \Pr(a = b) \sim 0.853$.

Each Scenario

Alice and Bob share an EPR pair.

1. $(x, y) = (0, 0)$. Alice: $\frac{\pi}{4}$. Bob: $\frac{\pi}{8}$. Prob they agree: $\cos^2(\frac{\pi}{4} - \frac{\pi}{8}) = \cos^2(\frac{\pi}{8}) \sim 0.853$.
2. $(x, y) = (0, 1)$. Alice: $\frac{\pi}{4}$. Bob: $\frac{3\pi}{8}$. Prob they agree: $\cos^2(\frac{3\pi}{8} - \frac{\pi}{4}) = \cos^2(\frac{\pi}{8}) \sim 0.853$.
3. $(x, y) = (1, 0)$. Alice: 0. Bob: $\frac{\pi}{8}$. Prob they agree: $\cos^2(\frac{\pi}{8} - 0) = \cos^2(\frac{\pi}{8}) \sim 0.853$.
4. $(x, y) = (1, 1)$. Alice: 0. Bob: $3\pi/8$. Prob they agree: $\cos^2(\frac{3\pi}{8} - 0) = \cos^2(\frac{3\pi}{8}) = 1 - \cos^2(\frac{\pi}{8})$.
   So prob they do not agree is
   $1 - (1 - \cos^2(\frac{\pi}{8})) = \cos^2(\frac{\pi}{8}) \sim 0.853$.

$(x, y) \in \{(0, 0), (0, 1), (1, 0)\} \implies \Pr(WIN) = \Pr(a = b) \sim 0.853$.
$(x, y) = (1, 1) \implies \Pr(WIN) = \Pr(a = b) \sim 0.853$.

Each Scenario

Alice and Bob share an EPR pair.

1. $(x, y) = (0, 0)$. Alice: $\frac{\pi}{4}$. Bob: $\frac{\pi}{8}$. Prob they agree: $\cos^2(\frac{\pi}{4} - \frac{\pi}{8}) = \cos^2(\frac{\pi}{8}) \sim 0.853$.
2. $(x, y) = (0, 1)$. Alice: $\frac{\pi}{4}$. Bob: $\frac{3\pi}{8}$. Prob they agree: $\cos^2(\frac{3\pi}{8} - \frac{\pi}{4}) = \cos^2(\frac{\pi}{8}) \sim 0.853$.
3. $(x, y) = (1, 0)$. Alice: 0. Bob: $\frac{\pi}{8}$. Prob they agree: $\cos^2(\frac{\pi}{8} - 0) = \cos^2(\frac{\pi}{8}) \sim 0.853$.
4. $(x, y) = (1, 1)$. Alice: 0. Bob: $3\pi/8$. Prob they agree: $\cos^2(\frac{3\pi}{8} - 0) = \cos^2(\frac{3\pi}{8}) = 1 - \cos^2(\frac{\pi}{8})$.
   So prob they do not agree is
   $1 - (1 - \cos^2(\frac{\pi}{8})) = \cos^2(\frac{\pi}{8}) \sim 0.853$.

$(x, y) \in \{(0, 0), (0, 1), (1, 0)\} \implies \Pr(WIN) = \Pr(a = b) \sim 0.853$.
$(x, y) = (1, 1) \implies \Pr(WIN) = \Pr(a = b) \sim 0.853$.
So
$\Pr(WIN) \sim \frac{3}{4}(0.853) + \frac{1}{4}(0.853) = 0.853 > 0.8125$.

Each Scenario

Alice and Bob share an EPR pair.

1. $(x, y) = (0, 0)$. Alice: $\frac{\pi}{4}$. Bob: $\frac{\pi}{8}$. Prob they agree: $\cos^2(\frac{\pi}{4} - \frac{\pi}{8}) = \cos^2(\frac{\pi}{8}) \sim 0.853$.
2. $(x, y) = (0, 1)$. Alice: $\frac{\pi}{4}$. Bob: $\frac{3\pi}{8}$. Prob they agree: $\cos^2(\frac{3\pi}{8} - \frac{\pi}{4}) = \cos^2(\frac{\pi}{8}) \sim 0.853$.
3. $(x, y) = (1, 0)$. Alice: 0. Bob: $\frac{\pi}{8}$. Prob they agree: $\cos^2(\frac{\pi}{8} - 0) = \cos^2(\frac{\pi}{8}) \sim 0.853$.
4. $(x, y) = (1, 1)$. Alice: 0. Bob: $3\pi/8$. Prob they agree: $\cos^2(\frac{3\pi}{8} - 0) = \cos^2(\frac{3\pi}{8}) = 1 - \cos^2(\frac{\pi}{8})$.
   So prob they do not agree is
   $1 - (1 - \cos^2(\frac{\pi}{8})) = \cos^2(\frac{\pi}{8}) \sim 0.853$.

$(x, y) \in \{(0, 0), (0, 1), (1, 0)\} \implies \Pr(WIN) = \Pr(a = b) \sim 0.853$.
$(x, y) = (1, 1) \implies \Pr(WIN) = \Pr(a = b) \sim 0.853$.
So
$\Pr(WIN) \sim \frac{3}{4}(0.853) + \frac{1}{4}(0.853) = 0.853 > 0.8125$.
The exact prob of winning is $\cos^2(\frac{\pi}{8})$.

Can Alice and Bob Do Better?

Assume Alice and Bob share an EPR pair.

Can Alice and Bob Do Better?

Assume Alice and Bob share an EPR pair.

Vote Which of the following is true:

Can Alice and Bob Do Better?

Assume Alice and Bob share an EPR pair.

Vote Which of the following is true:

1. Alice and Bob have a strategy that wins the CHSH game with Prob $p > \cos^2(\frac{\pi}{8})$ and this is **known**.

Can Alice and Bob Do Better?

Assume Alice and Bob share an EPR pair.

Vote Which of the following is true:

1. Alice and Bob have a strategy that wins the CHSH game with Prob $p > \cos^2(\frac{\pi}{8})$ and this is **known**.
2. The best prob of winning that Alice and Bob can achieve is $\cos^2(\frac{\pi}{8})$ and this is **known**.

Can Alice and Bob Do Better?

Assume Alice and Bob share an EPR pair.

Vote Which of the following is true:

1. Alice and Bob have a strategy that wins the CHSH game with Prob $p > \cos^2(\frac{\pi}{8})$ and this is **known**.
2. The best prob of winning that Alice and Bob can achieve is $\cos^2(\frac{\pi}{8})$ and this is **known**.
3. The question of if Alice and Bob can do better than $\cos^2(\frac{\pi}{8}$ is **Unknown to Science**.

Can Alice and Bob Do Better?

Assume Alice and Bob share an EPR pair.

Vote Which of the following is true:

1. Alice and Bob have a strategy that wins the CHSH game with Prob $p > \cos^2(\frac{\pi}{8})$ and this is **known**.
2. The best prob of winning that Alice and Bob can achieve is $\cos^2(\frac{\pi}{8})$ and this is **known**.
3. The question of if Alice and Bob can do better than $\cos^2(\frac{\pi}{8}$ is **Unknown to Science**.

Answer to a particular part of this problem on the Next Page

Can Alice and Bob Do Better With a Diff Choice of Angles?

Recall

Can Alice and Bob Do Better With a Diff Choice of Angles?

Recall

1. First Strategy:
   Alice used $\frac{\pi}{3}$ and 0,

Can Alice and Bob Do Better With a Diff Choice of Angles?

Recall

1. First Strategy:
   Alice used $\frac{\pi}{3}$ and 0,
   Bob used $\frac{\pi}{6}$ and $\frac{\pi}{2}$,

Can Alice and Bob Do Better With a Diff Choice of Angles?

Recall

1. First Strategy:
   Alice used $\frac{\pi}{3}$ and 0,
   Bob used $\frac{\pi}{6}$ and $\frac{\pi}{2}$,
   and got prob of winning 0.8125.

Can Alice and Bob Do Better With a Diff Choice of Angles?

Recall

1. First Strategy:
   Alice used $\frac{\pi}{3}$ and 0,
   Bob used $\frac{\pi}{6}$ and $\frac{\pi}{2}$,
   and got prob of winning 0.8125.
2. Second Strategy:
   Alice used $\frac{\pi}{4}$ and 0,

Can Alice and Bob Do Better With a Diff Choice of Angles?

Recall

1. First Strategy:
   Alice used $\frac{\pi}{3}$ and 0,
   Bob used $\frac{\pi}{6}$ and $\frac{\pi}{2}$,
   and got prob of winning 0.8125.
2. Second Strategy:
   Alice used $\frac{\pi}{4}$ and 0,
   Bob used $\frac{\pi}{8}$ and $\frac{3\pi}{8}$,

Can Alice and Bob Do Better With a Diff Choice of Angles?

Recall

1. First Strategy:
   Alice used $\frac{\pi}{3}$ and 0,
   Bob used $\frac{\pi}{6}$ and $\frac{\pi}{2}$,
   and got prob of winning 0.8125.
2. Second Strategy:
   Alice used $\frac{\pi}{4}$ and 0,
   Bob used $\frac{\pi}{8}$ and $\frac{3\pi}{8}$,
   and got prob of winning $\cos^2(\frac{\pi}{8}) \sim 0.853$.

Can Alice and Bob Do Better With a Diff Choice of Angles?

Recall

1. First Strategy:
   Alice used $\frac{\pi}{3}$ and 0,
   Bob used $\frac{\pi}{6}$ and $\frac{\pi}{2}$,
   and got prob of winning 0.8125.
2. Second Strategy:
   Alice used $\frac{\pi}{4}$ and 0,
   Bob used $\frac{\pi}{8}$ and $\frac{3\pi}{8}$,
   and got prob of winning $\cos^2(\frac{\pi}{8}) \sim 0.853$.

Can Alice and Bob obtain a higher prob of winning with a different choice of angles?

Can Alice and Bob Do Better With a Diff Choice of Angles?

Recall

1. First Strategy:
   Alice used $\frac{\pi}{3}$ and 0,
   Bob used $\frac{\pi}{6}$ and $\frac{\pi}{2}$,
   and got prob of winning 0.8125.
2. Second Strategy:
   Alice used $\frac{\pi}{4}$ and 0,
   Bob used $\frac{\pi}{8}$ and $\frac{3\pi}{8}$,
   and got prob of winning $\cos^2(\frac{\pi}{8}) \sim 0.853$.

Can Alice and Bob obtain a higher prob of winning with a different choice of angles?
Answer on the Next Page.

Can Alice and Bob Do Better With a Diff Choice of Angles?

No.

Can Alice and Bob Do Better With a Diff Choice of Angles?

No.
This can be proven solving maximizing

$$\cos^2(x_0 - y_0) + \cos^2(x_0 - y_1) + \cos^2(x_1 - y_0) + \cos^2(x_1 - y_1).$$

EVAN AND BILL- BILL ESP- CHECK ON THIS- VERIFY THIS IS WHAT YOU NEED TO MAXIMIZE AND FIND THE MAX.

Can Alice and Bob Do Better With a Diff Approach? More EPR-Pairs?

Can Alice and Bob Do Better With a Diff Approach? More EPR-Pairs?

In 1980 Tsirelson proved the following:

Can Alice and Bob Do Better With a Diff Approach? More EPR-Pairs?

In 1980 Tsirelson proved the following:

Even allowing Alice and Bob to share many EPR pairs, there is no strategy that gives a prob of winning $> \cos^2(\frac{\pi}{8})$.

Final Thoughts

Final Thoughts

1. Classically: There is a strategy for CHSH that has prob of winning 0.75 and it is known you cannot do better than that.

Final Thoughts

1. Classically: There is a strategy for CHSH that has prob of winning 0.75 and it is known you cannot do better than that.
2. If Alice and Bob share an EPR pair then there is a strategy that has prob of winning $\cos^2(\frac{\pi}{8}) \sim 0.853$.

Final Thoughts

1. Classically: There is a strategy for CHSH that has prob of winning 0.75 and it is known you cannot do better than that.
2. If Alice and Bob share an EPR pair then there is a strategy that has prob of winning $\cos^2(\frac{\pi}{8}) \sim 0.853$.
3. I am amazed that with a shared EPR pair Alice and Bob can do better.

Final Thoughts

1. Classically: There is a strategy for CHSH that has prob of winning 0.75 and it is known you cannot do better than that.
2. If Alice and Bob share an EPR pair then there is a strategy that has prob of winning $\cos^2(\frac{\pi}{8}) \sim 0.853$.
3. I am amazed that with a shared EPR pair Alice and Bob can do better.
4. I am amazed that with a shared EPR pair Alice and Bob can do **so much better**. I would have have thought something like $0.75 + \epsilon$.

Final Thoughts

1. Classically: There is a strategy for CHSH that has prob of winning 0.75 and it is known you cannot do better than that.
2. If Alice and Bob share an EPR pair then there is a strategy that has prob of winning $\cos^2(\frac{\pi}{8}) \sim 0.853$.
3. I am amazed that with a shared EPR pair Alice and Bob can do better.
4. I am amazed that with a shared EPR pair Alice and Bob can do **so much better**. I would have have thought something like $0.75 + \epsilon$.
5. Even with many EPR pairs and any kind of strategy Alice and Bob cannot do better than $\cos^2(\frac{\pi}{8})$. I am not amazed this is true, but I am amazed its been proven. (Proof is hard.)