BILL, RECORD LECTURE!!!!

BILL RECORD LECTURE!!!

One Triangle, Two Triangles

William Gasarch

Lets Party Like Its 2019

The following is the first theorem in Ramsey Theory:

Lets Party Like Its 2019

The following is the first theorem in Ramsey Theory:
Thm For all 2-col of the edges of K_6 there is a mono K_3.

Trivial Theorem, Non Trivial Extension

Thm For all 2-cols of edges of K_{12} there are 2 mono K_3's
Question Find n such that

Trivial Theorem, Non Trivial Extension

Thm For all 2-cols of edges of K_{12} there are 2 mono K_3's
Question Find n such that

1. For all 2-col of the edges of K_n there are 2 mono K_3's

Trivial Theorem, Non Trivial Extension

Thm For all 2-cols of edges of K_{12} there are 2 mono K_3's
Question Find n such that

1. For all 2-col of the edges of K_n there are 2 mono K_3's
2. There exists a 2-col of the edges of K_{n-1} that does not have 2 mono K_3's.

Trivial Theorem, Non Trivial Extension

Thm For all 2-cols of edges of K_{12} there are 2 mono K_3's
Question Find n such that

1. For all 2-col of the edges of K_n there are 2 mono K_3's
2. There exists a 2-col of the edges of K_{n-1} that does not have 2 mono K_3's.

VOTE (1) $n = 12$,

Trivial Theorem, Non Trivial Extension

Thm For all 2-cols of edges of K_{12} there are 2 mono K_3's
Question Find n such that

1. For all 2-col of the edges of K_n there are 2 mono K_3's
2. There exists a 2-col of the edges of K_{n-1} that does not have 2 mono K_3's.

VOTE (1) $n = 12$, (2) $9 \leq n \leq 10$,

Trivial Theorem, Non Trivial Extension

Thm For all 2-cols of edges of K_{12} there are 2 mono K_3's
Question Find n such that

1. For all 2-col of the edges of K_n there are 2 mono K_3's
2. There exists a 2-col of the edges of K_{n-1} that does not have 2 mono K_3's.

VOTE (1) $n = 12$, (2) $9 \le n \le 10$, (3) $6 \le n \le 8$.

Trivial Theorem, Non Trivial Extension

Thm For all 2-cols of edges of K_{12} there are 2 mono K_3's
Question Find n such that

1. For all 2-col of the edges of K_n there are 2 mono K_3's
2. There exists a 2-col of the edges of K_{n-1} that does not have 2 mono K_3's.

VOTE (1) $n = 12$, (2) $9 \le n \le 10$, (3) $6 \le n \le 8$.
Answer $n = 6$.

Trivial Theorem, Non Trivial Extension

Thm For all 2-cols of edges of K_{12} there are 2 mono K_3's
Question Find n such that

1. For all 2-col of the edges of K_n there are 2 mono K_3's
2. There exists a 2-col of the edges of K_{n-1} that does not have 2 mono K_3's.

VOTE (1) $n = 12$, (2) $9 \le n \le 10$, (3) $6 \le n \le 8$.
Answer $n = 6$.

1. For all 2-col of the edges of K_6 there are **2** mono K_3's

Trivial Theorem, Non Trivial Extension

Thm For all 2-cols of edges of K_{12} there are 2 mono K_3's
Question Find n such that

1. For all 2-col of the edges of K_n there are 2 mono K_3's
2. There exists a 2-col of the edges of K_{n-1} that does not have 2 mono K_3's.

VOTE (1) $n = 12$, (2) $9 \le n \le 10$, (3) $6 \le n \le 8$.
Answer $n = 6$.

1. For all 2-col of the edges of K_6 there are **2** mono K_3's
2. There exists a 2-col of the edges of K_5 that does not have any mono K_3's.

Proof of K_6 Two Triangles Theorem

Thm For all 2-cols of edges of K_6 there are 2 mono K_3's

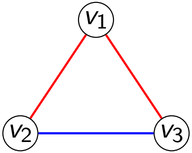
Proof of K_6 Two Triangles Theorem

Thm For all 2-cols of edges of K_6 there are 2 mono K_3's
Proof Let COL be a 2-col of the edges of K_6.
Let R, B, M, be the SET of **RED**, **BLUE**, and **MIXED** triangles.

$$|R| + |B| + |M| = \binom{6}{3} = 20.$$

We show that $|M| \le 18$, so $|R| + |B| \ge 2$.

A Mixed Triangle Has a Vertex Such That



- (v_2, v_1) is red, (v_2, v_3) is blue. View this as $(v_2, \{v_1, v_3\})$.
- (v_3, v_1) is red, (v_3, v_2) is blue. View this as $(v_3, \{v_1, v_2\})$.

Map ZAN to M

Def A **Zan** is an element $(v, \{u, w\}) \in V \times \binom{V}{2}$ such that $v \notin \{u, w\}$ and $COL(v, u) \neq COL(v, w)$. ZAN is the set of Zan's.

Map ZAN to M

Def A **Zan** is an element $(v, \{u, w\}) \in V \times \binom{V}{2}$ such that $v \notin \{u, w\}$ and $COL(v, u) \neq COL(v, w)$. ZAN is the set of Zan's.

Map ZAN to M by mapping $(v, \{u, w\})$ to triangle $\{v, u, w\}$.

Map ZAN to M

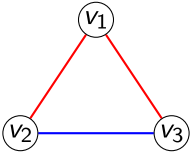
Def A **Zan** is an element $(v, \{u, w\}) \in V \times \binom{V}{2}$ such that $v \notin \{u, w\}$ and $COL(v, u) \neq COL(v, w)$. ZAN is the set of Zan's.

Map ZAN to M by mapping $(v, \{u, w\})$ to triangle $\{v, u, w\}$.
Claim This mapping is exactly 2-to-1.

Map ZAN to M

Def A **Zan** is an element $(v, \{u, w\}) \in V \times \binom{V}{2}$ such that $v \notin \{u, w\}$ and $COL(v, u) \neq COL(v, w)$. ZAN is the set of Zan's.

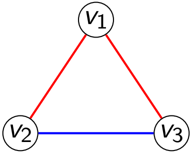
Map ZAN to M by mapping $(v, \{u, w\})$ to triangle $\{v, u, w\}$.
Claim This mapping is exactly 2-to-1.
What Zan's map to the triangle:



Map ZAN to M

Def A **Zan** is an element $(v, \{u, w\}) \in V \times \binom{V}{2}$ such that $v \notin \{u, w\}$ and $COL(v, u) \neq COL(v, w)$. ZAN is the set of Zan's.

Map ZAN to M by mapping $(v, \{u, w\})$ to triangle $\{v, u, w\}$.
Claim This mapping is exactly 2-to-1.
What Zan's map to the triangle:



$(v_2, \{v_1, v_3\})$ and $(v_3, \{v_1, v_2\})$.

Upper Bound on M

There is a 2-to-1 map from ZAN to M. Hence

$$|M| = |ZAN|/2$$

Upper Bound on M

There is a 2-to-1 map from ZAN to M. Hence

$$|M| = |ZAN|/2$$

Now we want to bound $|ZAN|$.

Upper Bound on M

There is a 2-to-1 map from ZAN to M. Hence

$$|M| = |ZAN|/2$$

Now we want to bound $|ZAN|$.
Look at vertex v. How many ZAN's use v as their base point?

Upper Bound on M

There is a 2-to-1 map from ZAN to M. Hence

$$|M| = |ZAN|/2$$

Now we want to bound $|ZAN|$.
Look at vertex v. How many ZAN's use v as their base point?
Depends on $\deg_R(v)$ and $\deg_B(v)$.

Upper Bound on M

There is a 2-to-1 map from ZAN to M. Hence

$$|M| = |ZAN|/2$$

Now we want to bound $|ZAN|$.
Look at vertex v. How many ZAN's use v as their base point?
Depends on $\deg_R(v)$ and $\deg_B(v)$.
Thought experiment If $\deg_R(v) = 3$ and $\deg_B(v) = 2$ then how many ZAN's are of the form

$$\{v, \{x, y\}\}$$

Upper Bound on M

There is a 2-to-1 map from ZAN to M. Hence

$$|M| = |ZAN|/2$$

Now we want to bound $|ZAN|$.
Look at vertex v. How many ZAN's use v as their base point?
Depends on $\deg_R(v)$ and $\deg_B(v)$.
Thought experiment If $\deg_R(v) = 3$ and $\deg_B(v) = 2$ then how many ZAN's are of the form

$$\{v, \{x, y\}\}$$

x: $COL(v, x) =$ **RED**. There are $\deg_R(v)$ of them.

Upper Bound on M

There is a 2-to-1 map from ZAN to M. Hence

$$|M| = |ZAN|/2$$

Now we want to bound $|ZAN|$.
Look at vertex v. How many ZAN's use v as their base point?
Depends on $\deg_R(v)$ and $\deg_B(v)$.
Thought experiment If $\deg_R(v) = 3$ and $\deg_B(v) = 2$ then how many ZAN's are of the form

$$\{v, \{x, y\}\}$$

x: $COL(v, x) =$ **RED**. There are $\deg_R(v)$ of them.
y: $COL(v, y) =$ **BLUE**. There are $\deg_B(v)$ of them.

Upper Bound on M

There is a 2-to-1 map from ZAN to M. Hence

$$|M| = |ZAN|/2$$

Now we want to bound $|ZAN|$.
Look at vertex v. How many ZAN's use v as their base point?
Depends on $\deg_R(v)$ and $\deg_B(v)$.
Thought experiment If $\deg_R(v) = 3$ and $\deg_B(v) = 2$ then how many ZAN's are of the form

$$\{v, \{x, y\}\}$$

x: $COL(v, x) =$ **RED**. There are $\deg_R(v)$ of them.
y: $COL(v, y) =$ **BLUE**. There are $\deg_B(v)$ of them.
So v contributes $\deg_R(v) \times \deg_B(v)$.

Contributions!

Cases

Contributions!

Cases

1. v has $\deg_R(v) = 5$ or $\deg_B(v) = 0$: v contributes 0.

Contributions!

Cases

1. v has $\deg_R(v) = 5$ or $\deg_B(v) = 0$: v contributes 0.
2. v has $\deg_R(v) = 4$ or $\deg_B(v) = 1$: v contributes 4.

Contributions!

Cases

1. v has $\deg_R(v) = 5$ or $\deg_B(v) = 0$: v contributes 0.
2. v has $\deg_R(v) = 4$ or $\deg_B(v) = 1$: v contributes 4.
3. v has $\deg_R(v) = 3$ or $\deg_B(v) = 2$: v contributes 6. Max.

Contributions!

Cases

1. v has $\deg_R(v) = 5$ or $\deg_B(v) = 0$: v contributes 0.
2. v has $\deg_R(v) = 4$ or $\deg_B(v) = 1$: v contributes 4.
3. v has $\deg_R(v) = 3$ or $\deg_B(v) = 2$: v contributes 6. Max.

6 vertices, each contribute ≤ 6,

Contributions!

Cases

1. v has $\deg_R(v) = 5$ or $\deg_B(v) = 0$: v contributes 0.
2. v has $\deg_R(v) = 4$ or $\deg_B(v) = 1$: v contributes 4.
3. v has $\deg_R(v) = 3$ or $\deg_B(v) = 2$: v contributes 6. Max.

6 vertices, each contribute ≤ 6, so

$$|M| = |ZAN|/2 \leq 6 \times 6/2 = 18, \text{ so}$$

Contributions!

Cases

1. v has $\deg_R(v) = 5$ or $\deg_B(v) = 0$: v contributes 0.
2. v has $\deg_R(v) = 4$ or $\deg_B(v) = 1$: v contributes 4.
3. v has $\deg_R(v) = 3$ or $\deg_B(v) = 2$: v contributes 6. Max.

6 vertices, each contribute ≤ 6, so

$$|M| = |ZAN|/2 \leq 6 \times 6/2 = 18, \text{ so}$$

$$|R| + |B| \geq 20 - |M| \geq 2$$

Summary

$$|R| + |B| + |M| = \binom{6}{3} = 20$$

Summary

$$|R| + |B| + |M| = \binom{6}{3} = 20$$

Map ZAN to M. Map is 2-to-1, so $|M| = |ZAN|/2$.

Summary

$$|R| + |B| + |M| = \binom{6}{3} = 20$$

Map ZAN to M. Map is 2-to-1, so $|M| = |ZAN|/2$.

ZAN is max when each vertex: 3 R and 2 B (or 2 R and 3 B).
$|ZAN| \leq 6 \times 6 = 36$.

Summary

$$|R| + |B| + |M| = \binom{6}{3} = 20$$

Map ZAN to M. Map is 2-to-1, so $|M| = |ZAN|/2$.

ZAN is max when each vertex: 3 R and 2 B (or 2 R and 3 B).
$|ZAN| \leq 6 \times 6 = 36$.

$|M| = |ZAN|/2 \leq 18$.

Summary

$$|R| + |B| + |M| = \binom{6}{3} = 20$$

Map ZAN to M. Map is 2-to-1, so $|M| = |ZAN|/2$.

ZAN is max when each vertex: 3 R and 2 B (or 2 R and 3 B).
$|ZAN| \le 6 \times 6 = 36$.

$|M| = |ZAN|/2 \le 18$.

$|R| + |B| \ge 20 - |M| \ge 2$.

Summary

$$|R| + |B| + |M| = \binom{6}{3} = 20$$

Map ZAN to M. Map is 2-to-1, so $|M| = |ZAN|/2$.

ZAN is max when each vertex: 3 R and 2 B (or 2 R and 3 B).
$|ZAN| \le 6 \times 6 = 36$.

$|M| = |ZAN|/2 \le 18$.

$|R| + |B| \ge 20 - |M| \ge 2$.

So there are at least 2 Mono Triangles.

Generalization

If we 2-color the edges of K_n how many mono K_3's do we have?

Generalization

If we 2-color the edges of K_n how many mono K_3's do we have?

VOTE (1) $\sim n^c$ for some $c < 1$, (2) $\sim n$ (3) $\sim n^2$, (4) $\sim n^3$.

Generalization

If we 2-color the edges of K_n how many mono K_3's do we have?

VOTE (1) $\sim n^c$ for some $c < 1$, (2) $\sim n$ (3) $\sim n^2$, (4) $\sim n^3$.

Answer$\sim n^3$. Actually $\frac{n^3}{24} + O(n^2)$.

Generalization

If we 2-color the edges of K_n how many mono K_3's do we have?

VOTE (1) $\sim n^c$ for some $c < 1$, (2) $\sim n$ (3) $\sim n^2$, (4) $\sim n^3$.

Answer$\sim n^3$. Actually $\frac{n^3}{24} + O(n^2)$.

We do one case: $n \equiv 1 \pmod{2}$.

Generalization

If we 2-color the edges of K_n how many mono K_3's do we have?

VOTE (1) $\sim n^c$ for some $c < 1$, (2) $\sim n$ (3) $\sim n^2$, (4) $\sim n^3$.

Answer$\sim n^3$. Actually $\frac{n^3}{24} + O(n^2)$.

We do one case: $n \equiv 1 \pmod{2}$.
Let COL be a coloring of the edges of K_n.

Generalization

If we 2-color the edges of K_n how many mono K_3's do we have?

VOTE (1) $\sim n^c$ for some $c < 1$, (2) $\sim n$ (3) $\sim n^2$, (4) $\sim n^3$.

Answer$\sim n^3$. Actually $\frac{n^3}{24} + O(n^2)$.

We do one case: $n \equiv 1 \pmod{2}$.
Let COL be a coloring of the edges of K_n.
Then degree of each vertex is $n - 1 \equiv 0 \pmod{2}$.

Generalization

If we 2-color the edges of K_n how many mono K_3's do we have?

VOTE (1) $\sim n^c$ for some $c < 1$, (2) $\sim n$ (3) $\sim n^2$, (4) $\sim n^3$.

Answer$\sim n^3$. Actually $\frac{n^3}{24} + O(n^2)$.

We do one case: $n \equiv 1 \pmod 2$.
Let COL be a coloring of the edges of K_n.
Then degree of each vertex is $n - 1 \equiv 0 \pmod 2$.

We find an upper bound on $|ZAN|$.

Maximize $|ZAN|$

To maximize $|ZAN|$ we would, at each vertex, color half of the edges RED and half BLUE.

Maximize $|ZAN|$

To maximize $|ZAN|$ we would, at each vertex, color half of the edges RED and half BLUE.
Each vertex contributes $(\frac{n-1}{2})^2$. (Recall $n-1 \equiv 0 \pmod 2$.)

Maximize $|ZAN|$

To maximize $|ZAN|$ we would, at each vertex, color half of the edges RED and half BLUE.
Each vertex contributes $(\frac{n-1}{2})^2$. (Recall $n-1 \equiv 0 \pmod 2$.)

$$|ZAN| \leq n\frac{(n-1)^2}{4} = \frac{(n-1)^2 n}{4} \text{ so}$$

Maximize $|ZAN|$

To maximize $|ZAN|$ we would, at each vertex, color half of the edges RED and half BLUE.
Each vertex contributes $(\frac{n-1}{2})^2$. (Recall $n-1 \equiv 0 \pmod 2$.)

$$|ZAN| \le n\frac{(n-1)^2}{4} = \frac{(n-1)^2 n}{4} \text{ so}$$

$$|M| = |ZAN|/2 \le \frac{(n-1)^2 n}{8}$$

Finishing Up The Proof

Recap

$$|M| \leq \frac{(n-1)^2 n}{8}$$

Finishing Up The Proof

Recap

$$|M| \leq \frac{(n-1)^2 n}{8}$$

Recall

$$|R| + |B| + |M| = \binom{n}{3} = \frac{n(n-1)(n-2)}{6} \text{ hence}$$

Finishing Up The Proof

Recap

$$|M| \leq \frac{(n-1)^2 n}{8}$$

Recall

$$|R| + |B| + |M| = \binom{n}{3} = \frac{n(n-1)(n-2)}{6} \text{ hence}$$

$$|R| + |B| = \frac{n(n-1)(n-2)}{6} - |M| \text{ hence}$$

Finishing Up The Proof

Recap

$$|M| \leq \frac{(n-1)^2 n}{8}$$

Recall

$$|R| + |B| + |M| = \binom{n}{3} = \frac{n(n-1)(n-2)}{6} \text{ hence}$$

$$|R| + |B| = \frac{n(n-1)(n-2)}{6} - |M| \text{ hence}$$

$$|R| + |B| \geq \frac{n(n-1)(n-2)}{6} - \frac{(n-1)^2 n}{8}$$

Finishing Up The Proof

Recap

$$|M| \leq \frac{(n-1)^2 n}{8}$$

Recall

$$|R| + |B| + |M| = \binom{n}{3} = \frac{n(n-1)(n-2)}{6} \text{ hence}$$

$$|R| + |B| = \frac{n(n-1)(n-2)}{6} - |M| \text{ hence}$$

$$|R| + |B| \geq \frac{n(n-1)(n-2)}{6} - \frac{(n-1)^2 n}{8}$$

$$= \frac{n^3}{24} - \frac{n^2}{4} + \frac{5n}{24}$$

What About The Other Cases?

We leave the other cases to the reader to both determine the theorem and prove it.

Can This Be Improved?

The bound is known to be tight.

Coloring that Shows the Bound is Tight

Go to this pointer: `tttalk.pdf`

Another Way to Phrase The Results

Thm $\forall$ 2-cols of K_n $\exists \sim \frac{n^3}{24}$ mono K_3.

Another Way to Phrase The Results

Thm $\forall$ 2-cols of K_n $\exists \sim \frac{n^3}{24}$ mono K_3.
In K_n there are $\binom{n}{3}$ triples.

Another Way to Phrase The Results

Thm $\forall$ 2-cols of K_n $\exists \sim \frac{n^3}{24}$ mono K_3.

In K_n there are $\binom{n}{3}$ triples.

We want to know the **fraction** of them that are mono.

Another Way to Phrase The Results

Thm $\forall$ 2-cols of K_n $\exists \sim \frac{n^3}{24}$ mono K_3.

In K_n there are $\binom{n}{3}$ triples.

We want to know the **fraction** of them that are mono.

Thm $\forall$ 2-cols of K_n $\exists \sim \frac{1}{8}\binom{n}{3}$ mono K_3.

Another Way to Phrase The Results

Thm $\forall$ 2-cols of K_n $\exists \sim \frac{n^3}{24}$ mono K_3.

In K_n there are $\binom{n}{3}$ triples.

We want to know the **fraction** of them that are mono.

Thm $\forall$ 2-cols of K_n $\exists \sim \frac{1}{8}\binom{n}{3}$ mono K_3.

The ratio $\frac{1}{8}$ is tight.

One K_4, Two K_4

William Gasarch

Lets Party Like Its 2019

The following is an early theorem in Ramsey Theory:

Lets Party Like Its 2019

The following is an early theorem in Ramsey Theory:
Thm For all 2-col of the edges of K_{18} there is a mono K_4.

Trivial Theorem, Non Trivial Extension

Thm For all 2-cols of edges of K_{36} there are 2 mono K_4's

Trivial Theorem, Non Trivial Extension

Thm For all 2-cols of edges of K_{36} there are 2 mono K_4's
Smallest n such that $\forall$ 2-col of edges of K_n $\exists$ 2 mono K_4's?

Trivial Theorem, Non Trivial Extension

Thm For all 2-cols of edges of K_{36} there are 2 mono K_4's
Smallest n such that $\forall$ 2-col of edges of K_n $\exists$ 2 mono K_4's?
VOTE (1) $n = 36$,

Trivial Theorem, Non Trivial Extension

Thm For all 2-cols of edges of K_{36} there are 2 mono K_4's
Smallest n such that $\forall$ 2-col of edges of K_n $\exists$ 2 mono K_4's?
VOTE (1) $n = 36$, (2) Some n, $19 \le n \le 35$,

Trivial Theorem, Non Trivial Extension

Thm For all 2-cols of edges of K_{36} there are 2 mono K_4's
Smallest n such that $\forall$ 2-col of edges of K_n $\exists$ 2 mono K_4's?
VOTE (1) $n = 36$, (2) Some n, $19 \le n \le 35$, (3) $n = 18$.

Trivial Theorem, Non Trivial Extension

Thm For all 2-cols of edges of K_{36} there are 2 mono K_4's
Smallest n such that $\forall$ 2-col of edges of K_n $\exists$ 2 mono K_4's?
VOTE (1) $n = 36$, (2) Some n, $19 \le n \le 35$, (3) $n = 18$.
Answer This is really two questions.

Trivial Theorem, Non Trivial Extension

Thm For all 2-cols of edges of K_{36} there are 2 mono K_4's
Smallest n such that $\forall$ 2-col of edges of K_n $\exists$ 2 mono K_4's?
VOTE (1) $n = 36$, (2) Some n, $19 \le n \le 35$, (3) $n = 18$.
Answer This is really two questions.

1. As posed the answer is $n = 18$. Piwakoswki and Radziszowski `https://www.cs.rit.edu/~spr/PUBL/paper40.pdf` showed that for every 2-col of K_{18} there are 9 mono K_4's.

Trivial Theorem, Non Trivial Extension

Thm For all 2-cols of edges of K_{36} there are 2 mono K_4's
Smallest n such that $\forall$ 2-col of edges of K_n $\exists$ 2 mono K_4's?
VOTE (1) $n = 36$, (2) Some n, $19 \le n \le 35$, (3) $n = 18$.
Answer This is really two questions.

1. As posed the answer is $n = 18$. Piwakoswki and Radziszowski `https://www.cs.rit.edu/~spr/PUBL/paper40.pdf` showed that for every 2-col of K_{18} there are 9 mono K_4's. The proof used a clever computer search. This is interesting to know but the proof is not math-interesting.

Trivial Theorem, Non Trivial Extension

Thm For all 2-cols of edges of K_{36} there are 2 mono K_4's
Smallest n such that $\forall$ 2-col of edges of K_n $\exists$ 2 mono K_4's?
VOTE (1) $n = 36$, (2) Some n, $19 \le n \le 35$, (3) $n = 18$.
Answer This is really two questions.

1. As posed the answer is $n = 18$. Piwakoswki and Radziszowski `https://www.cs.rit.edu/~spr/PUBL/paper40.pdf` showed that for every 2-col of K_{18} there are 9 mono K_4's. The proof used a clever computer search. This is interesting to know but the proof is not math-interesting.
2. I will present a math-interesting proof of the following: *For all 2-cols of K_{19} there are TWO mono K_4's.*

Proof of K_{19} Two K_4 Theorem

Thm For all 2-cols of edges of K_{19} there are 2 mono K_4's

Proof of K_{19} Two K_4 Theorem

Thm For all 2-cols of edges of K_{19} there are 2 mono K_4's
Assume you have a 2-col of the edges of K_{19}.

Proof of K_{19} Two K_4 Theorem

Thm For all 2-cols of edges of K_{19} there are 2 mono K_4's
Assume you have a 2-col of the edges of K_{19}.

List out all subsets of $V = \{1, \ldots, 19\}$ of size $R(4) = 18$.

Proof of K_{19} Two K_4 Theorem

Thm For all 2-cols of edges of K_{19} there are 2 mono K_4's
Assume you have a 2-col of the edges of K_{19}.

List out all subsets of $V = \{1, \ldots, 19\}$ of size $R(4) = 18$.

$$A_1, A_2, \ldots, A_{\binom{19}{18}}.$$

Proof of K_{19} Two K_4 Theorem

Thm For all 2-cols of edges of K_{19} there are 2 mono K_4's
Assume you have a 2-col of the edges of K_{19}.

List out all subsets of $V = \{1, \ldots, 19\}$ of size $R(4) = 18$.

$$A_1, A_2, \ldots, A_{\binom{19}{18}}.$$

Since $|A_i| = R(4)$, each A_i has a mono K_4.

Proof of K_{19} Two K_4 Theorem

Thm For all 2-cols of edges of K_{19} there are 2 mono K_4's
Assume you have a 2-col of the edges of K_{19}.

List out all subsets of $V = \{1, \ldots, 19\}$ of size $R(4) = 18$.

$$A_1, A_2, \ldots, A_{\binom{19}{18}}.$$

Since $|A_i| = R(4)$, each A_i has a mono K_4.

So we get $\binom{19}{18}$ mono K_4's. So we are almost there.

Proof of K_{19} Two K_4 Theorem

Thm For all 2-cols of edges of K_{19} there are 2 mono K_4's
Assume you have a 2-col of the edges of K_{19}.

List out all subsets of $V = \{1, \ldots, 19\}$ of size $R(4) = 18$.

$$A_1, A_2, \ldots, A_{\binom{19}{18}}.$$

Since $|A_i| = R(4)$, each A_i has a mono K_4.

So we get $\binom{19}{18}$ mono K_4's. So we are almost there.

YOU"VE BEEN PUNKED. It is quite possible that the mono K_4 from A_3 and the mono K_4 from A_9 are the same.

Proof of K_{19} Two K_4 Theorem

Thm For all 2-cols of edges of K_{19} there are 2 mono K_4's
Assume you have a 2-col of the edges of K_{19}.

List out all subsets of $V = \{1, \ldots, 19\}$ of size $R(4) = 18$.

$$A_1, A_2, \ldots, A_{\binom{19}{18}}.$$

Since $|A_i| = R(4)$, each A_i has a mono K_4.

So we get $\binom{19}{18}$ mono K_4's. So we are almost there.

YOU"VE BEEN PUNKED. It is quite possible that the mono K_4 from A_3 and the mono K_4 from A_9 are the same.

For the real proof, see next slide.

Proof of K_{19} Two K_4 Theorem (cont)

List out all subsets of $V = \{1, \ldots, 19\}$ of size $R(4) = 18$.

Proof of K_{19} Two K_4 Theorem (cont)

List out all subsets of $V = \{1, \ldots, 19\}$ of size $R(4) = 18$.

$$A_1, A_2, \ldots, A_{\binom{19}{18}}.$$

Proof of K_{19} Two K_4 Theorem (cont)

List out all subsets of $V = \{1, \ldots, 19\}$ of size $R(4) = 18$.

$$A_1, A_2, \ldots, A_{\binom{19}{18}}.$$

(There are just 19 of these, $A_i = \{1, \ldots, 19\} - \{i\}$.)

Proof of K_{19} Two K_4 Theorem (cont)

List out all subsets of $V = \{1, \ldots, 19\}$ of size $R(4) = 18$.

$$A_1, A_2, \ldots, A_{\binom{19}{18}}.$$

(There are just 19 of these, $A_i = \{1, \ldots, 19\} - \{i\}$.)

1) Find a mono K_4 in A_1. Say its $\{16, 17, 18, 19\}$.

Proof of K_{19} Two K_4 Theorem (cont)

List out all subsets of $V = \{1, \ldots, 19\}$ of size $R(4) = 18$.

$$A_1, A_2, \ldots, A_{\binom{19}{18}}.$$

(There are just 19 of these, $A_i = \{1, \ldots, 19\} - \{i\}$.)

1) Find a mono K_4 in A_1. Say its $\{16, 17, 18, 19\}$.

2) REMOVE all A_i's that have all of $\{16, 17, 18, 19\}$.

Proof of K_{19} Two K_4 Theorem (cont)

List out all subsets of $V = \{1, \ldots, 19\}$ of size $R(4) = 18$.

$$A_1, A_2, \ldots, A_{\binom{19}{18}}.$$

(There are just 19 of these, $A_i = \{1, \ldots, 19\} - \{i\}$.)

1) Find a mono K_4 in A_1. Say its $\{16, 17, 18, 19\}$.

2) REMOVE all A_i's that have all of $\{16, 17, 18, 19\}$.
There are $\binom{19-4}{18-4} = \binom{15}{14} = 15$ of these.

Proof of K_{19} Two K_4 Theorem (cont)

List out all subsets of $V = \{1, \ldots, 19\}$ of size $R(4) = 18$.

$$A_1, A_2, \ldots, A_{\binom{19}{18}}.$$

(There are just 19 of these, $A_i = \{1, \ldots, 19\} - \{i\}$.)

1) Find a mono K_4 in A_1. Say its $\{16, 17, 18, 19\}$.

2) REMOVE all A_i's that have all of $\{16, 17, 18, 19\}$.
There are $\binom{19-4}{18-4} = \binom{15}{14} = 15$ of these.
There are $\binom{19}{18} - \binom{15}{14} = 19 - 15 = 4$ left. Call then B_1, B_2, B_3, B_4.

Proof of K_{19} Two K_4 Theorem (cont)

List out all subsets of $V = \{1, \ldots, 19\}$ of size $R(4) = 18$.

$$A_1, A_2, \ldots, A_{\binom{19}{18}}.$$

(There are just 19 of these, $A_i = \{1, \ldots, 19\} - \{i\}$.)

1) Find a mono K_4 in A_1. Say its $\{16, 17, 18, 19\}$.

2) REMOVE all A_i's that have all of $\{16, 17, 18, 19\}$.
There are $\binom{19-4}{18-4} = \binom{15}{14} = 15$ of these.
There are $\binom{19}{18} - \binom{15}{14} = 19 - 15 = 4$ left. Call then B_1, B_2, B_3, B_4.

3) Since B_1 has $18 = R(4)$ vertices, there is a mono K_4

Proof of K_{19} Two K_4 Theorem (cont)

List out all subsets of $V = \{1, \ldots, 19\}$ of size $R(4) = 18$.

$$A_1, A_2, \ldots, A_{\binom{19}{18}}.$$

(There are just 19 of these, $A_i = \{1, \ldots, 19\} - \{i\}$.)

1) Find a mono K_4 in A_1. Say its $\{16, 17, 18, 19\}$.

2) REMOVE all A_i's that have all of $\{16, 17, 18, 19\}$.
There are $\binom{19-4}{18-4} = \binom{15}{14} = 15$ of these.
There are $\binom{19}{18} - \binom{15}{14} = 19 - 15 = 4$ left. Call then B_1, B_2, B_3, B_4.

3) Since B_1 has $18 = R(4)$ vertices, there is a mono K_4

4) The mono K_4 from A_1, and the mono K_4 from B are different.

Proof of K_{19} Two K_4 Theorem (cont)

List out all subsets of $V = \{1, \ldots, 19\}$ of size $R(4) = 18$.

$$A_1, A_2, \ldots, A_{\binom{19}{18}}.$$

(There are just 19 of these, $A_i = \{1, \ldots, 19\} - \{i\}$.)

1) Find a mono K_4 in A_1. Say its $\{16, 17, 18, 19\}$.

2) REMOVE all A_i's that have all of $\{16, 17, 18, 19\}$.
There are $\binom{19-4}{18-4} = \binom{15}{14} = 15$ of these.
There are $\binom{19}{18} - \binom{15}{14} = 19 - 15 = 4$ left. Call then B_1, B_2, B_3, B_4.

3) Since B_1 has $18 = R(4)$ vertices, there is a mono K_4

4) The mono K_4 from A_1, and the mono K_4 from B are different.
Those are our 2 mono K_4's.

Can the Proof Give 3 mono K_4's?

We show that the technique to get 2 mono K_4's cannot be extended to give 3 mono K_4's.

Can the Proof Give 3 mono K_4's?

We show that the technique to get 2 mono K_4's cannot be extended to give 3 mono K_4's.

Assume that the first K_4 is $\{16, 17, 18, 19\}$

Can the Proof Give 3 mono K_4's?

We show that the technique to get 2 mono K_4's cannot be extended to give 3 mono K_4's.

Assume that the first K_4 is $\{16, 17, 18, 19\}$

Assume that the second K_4 is $\{12, 13, 14, 15\}$

Can the Proof Give 3 mono K_4's?

We show that the technique to get 2 mono K_4's cannot be extended to give 3 mono K_4's.

Assume that the first K_4 is $\{16, 17, 18, 19\}$

Assume that the second K_4 is $\{12, 13, 14, 15\}$

So we have removed all $A \subseteq \{1, \ldots, 19\}$ of size 18 where

Can the Proof Give 3 mono K_4's?

We show that the technique to get 2 mono K_4's cannot be extended to give 3 mono K_4's.

Assume that the first K_4 is $\{16, 17, 18, 19\}$

Assume that the second K_4 is $\{12, 13, 14, 15\}$

So we have removed all $A \subseteq \{1, \ldots, 19\}$ of size 18 where

A has all of $\{16, 17, 18, 19\}$, or

Can the Proof Give 3 mono K_4's?

We show that the technique to get 2 mono K_4's cannot be extended to give 3 mono K_4's.

Assume that the first K_4 is $\{16, 17, 18, 19\}$

Assume that the second K_4 is $\{12, 13, 14, 15\}$

So we have removed all $A \subseteq \{1, \ldots, 19\}$ of size 18 where

A has all of $\{16, 17, 18, 19\}$, or

A has all of $\{12, 13, 14, 15\}$.

Can the Proof Give 3 mono K_4's?

We show that the technique to get 2 mono K_4's cannot be extended to give 3 mono K_4's.

Assume that the first K_4 is $\{16, 17, 18, 19\}$

Assume that the second K_4 is $\{12, 13, 14, 15\}$

So we have removed all $A \subseteq \{1, \ldots, 19\}$ of size 18 where

A has all of $\{16, 17, 18, 19\}$, or

A has all of $\{12, 13, 14, 15\}$.

Whats left?

Can the Proof Give 3 mono K_4's?

We show that the technique to get 2 mono K_4's cannot be extended to give 3 mono K_4's.

Assume that the first K_4 is $\{16, 17, 18, 19\}$

Assume that the second K_4 is $\{12, 13, 14, 15\}$

So we have removed all $A \subseteq \{1, \ldots, 19\}$ of size 18 where

A has all of $\{16, 17, 18, 19\}$, or

A has all of $\{12, 13, 14, 15\}$.

Whats left?All $A \subseteq \{1, \ldots, 19\}$ of size 18 that are missing at least one of $\{16, 17, 18, 19\}$ and at least one of $\{12, 13, 14, 15\}$.

Can the Proof Give 3 mono K_4's?

We show that the technique to get 2 mono K_4's cannot be extended to give 3 mono K_4's.

Assume that the first K_4 is $\{16, 17, 18, 19\}$

Assume that the second K_4 is $\{12, 13, 14, 15\}$

So we have removed all $A \subseteq \{1, \ldots, 19\}$ of size 18 where

A has all of $\{16, 17, 18, 19\}$, or

A has all of $\{12, 13, 14, 15\}$.

Whats left?All $A \subseteq \{1, \ldots, 19\}$ of size 18 that are missing at least one of $\{16, 17, 18, 19\}$ and at least one of $\{12, 13, 14, 15\}$.

If $A \subseteq \{1, \ldots, 19\}$ is missing two elements it is of size 17.

Can the Proof Give 3 mono K_4's?

We show that the technique to get 2 mono K_4's cannot be extended to give 3 mono K_4's.

Assume that the first K_4 is $\{16, 17, 18, 19\}$

Assume that the second K_4 is $\{12, 13, 14, 15\}$

So we have removed all $A \subseteq \{1, \ldots, 19\}$ of size 18 where

A has all of $\{16, 17, 18, 19\}$, or

A has all of $\{12, 13, 14, 15\}$.

Whats left?All $A \subseteq \{1, \ldots, 19\}$ of size 18 that are missing at least one of $\{16, 17, 18, 19\}$ and at least one of $\{12, 13, 14, 15\}$.

If $A \subseteq \{1, \ldots, 19\}$ is missing two elements it is of size 17.

Hence there might be no mono K_4's.

Can the Proof Give 3 mono K_4's?

We show that the technique to get 2 mono K_4's cannot be extended to give 3 mono K_4's.

Assume that the first K_4 is $\{16, 17, 18, 19\}$

Assume that the second K_4 is $\{12, 13, 14, 15\}$

So we have removed all $A \subseteq \{1, \ldots, 19\}$ of size 18 where

A has all of $\{16, 17, 18, 19\}$, or

A has all of $\{12, 13, 14, 15\}$.

Whats left?All $A \subseteq \{1, \ldots, 19\}$ of size 18 that are missing at least one of $\{16, 17, 18, 19\}$ and at least one of $\{12, 13, 14, 15\}$.

If $A \subseteq \{1, \ldots, 19\}$ is missing two elements it is of size 17.

Hence there might be no mono K_4's.

Note We only showed that the **proof** cannot be extended. As noted above any 2-col of K_{18} has 9 mono K_4's.

Want n such that $\forall$ 2-col $\exists$3 Mono K_4's

Want n such that $\forall$ 2-col $\exists$3 Mono K_4's

$\forall$ *2-col of* K_n $\exists$ *3 mono* K_4*'s.*

Want n such that $\forall$ 2-col $\exists 3$ Mono K_4's

$\forall$ 2-col of K_n $\exists$ 3 mono K_4's.

List out all subsets of $V = \{1, \ldots, n\}$ of size $R(4) = 18$.

Want n such that $\forall$ 2-col $\exists 3$ Mono K_4's

$\forall$ 2-col of K_n $\exists$ 3 mono K_4's.

List out all subsets of $V = \{1, \ldots, n\}$ of size $R(4) = 18$.

$$A_1, A_2, \ldots, A_{\binom{n}{18}}.$$

Want n such that $\forall$ 2-col $\exists$3 Mono K_4's

$\forall$ 2-col of K_n $\exists$ 3 mono K_4's.

List out all subsets of $V = \{1, \ldots, n\}$ of size $R(4) = 18$.

$$A_1, A_2, \ldots, A_{\binom{n}{18}}.$$

1) Find a mono K_4 in A_1. Say its $\{x_1, x_2, x_3, x_4\}$.

Want n such that $\forall$ 2-col $\exists$3 Mono K_4's

$\forall$ 2-col of K_n $\exists$ 3 mono K_4's.

List out all subsets of $V = \{1, \ldots, n\}$ of size $R(4) = 18$.

$$A_1, A_2, \ldots, A_{\binom{n}{18}}.$$

1) Find a mono K_4 in A_1. Say its $\{x_1, x_2, x_3, x_4\}$.

2) REMOVE all A_i's that have all of $\{x_1, x_2, x_3, x_4\}$.

Want n such that $\forall$ 2-col $\exists$3 Mono K_4's

$\forall$ 2-col of K_n $\exists$ 3 mono K_4's.

List out all subsets of $V = \{1, \ldots, n\}$ of size $R(4) = 18$.

$$A_1, A_2, \ldots, A_{\binom{n}{18}}.$$

1) Find a mono K_4 in A_1. Say its $\{x_1, x_2, x_3, x_4\}$.

2) REMOVE all A_i's that have all of $\{x_1, x_2, x_3, x_4\}$.
$\binom{n-4}{18-4} = \binom{n-4}{14}$ of these. There are $\binom{n}{18} - \binom{n-4}{14}$ left.

Want n such that $\forall$ 2-col $\exists$3 Mono K_4's

$\forall$ 2-col of K_n $\exists$ 3 mono K_4's.

List out all subsets of $V = \{1, \ldots, n\}$ of size $R(4) = 18$.

$$A_1, A_2, \ldots, A_{\binom{n}{18}}.$$

1) Find a mono K_4 in A_1. Say its $\{x_1, x_2, x_3, x_4\}$.

2) REMOVE all A_i's that have all of $\{x_1, x_2, x_3, x_4\}$.
$\binom{n-4}{18-4} = \binom{n-4}{14}$ of these. There are $\binom{n}{18} - \binom{n-4}{14}$ left.

3) Find a mono K_4 in one of the sets left. Say its $\{y_1, y_2, y_3, y_4\}$.

Want n such that $\forall$ 2-col $\exists$3 Mono K_4's

$\forall$ 2-col of K_n $\exists$ 3 mono K_4's.

List out all subsets of $V = \{1, \ldots, n\}$ of size $R(4) = 18$.

$$A_1, A_2, \ldots, A_{\binom{n}{18}}.$$

1) Find a mono K_4 in A_1. Say its $\{x_1, x_2, x_3, x_4\}$.

2) REMOVE all A_i's that have all of $\{x_1, x_2, x_3, x_4\}$.
$\binom{n-4}{18-4} = \binom{n-4}{14}$ of these. There are $\binom{n}{18} - \binom{n-4}{14}$ left.

3) Find a mono K_4 in one of the sets left. Say its $\{y_1, y_2, y_3, y_4\}$.

4) REMOVE all A_i's that have all of $\{y_1, y_2, y_3, y_4\}$.

Want n such that $\forall$ 2-col $\exists$3 Mono K_4's

$\forall$ 2-col of K_n $\exists$ 3 mono K_4's.

List out all subsets of $V = \{1, \ldots, n\}$ of size $R(4) = 18$.

$$A_1, A_2, \ldots, A_{\binom{n}{18}}.$$

1) Find a mono K_4 in A_1. Say its $\{x_1, x_2, x_3, x_4\}$.

2) REMOVE all A_i's that have all of $\{x_1, x_2, x_3, x_4\}$.
$\binom{n-4}{18-4} = \binom{n-4}{14}$ of these. There are $\binom{n}{18} - \binom{n-4}{14}$ left.

3) Find a mono K_4 in one of the sets left. Say its $\{y_1, y_2, y_3, y_4\}$.

4) REMOVE all A_i's that have all of $\{y_1, y_2, y_3, y_4\}$.
$\binom{n-4}{18-4} = \binom{n-4}{14}$ of these. There are $\binom{n}{18} - 2\binom{n-4}{14}$ left.

Want n such that $\forall$ 2-col $\exists$3 Mono K_4's

$\forall$ 2-col of K_n $\exists$ 3 mono K_4's.

List out all subsets of $V = \{1, \ldots, n\}$ of size $R(4) = 18$.

$$A_1, A_2, \ldots, A_{\binom{n}{18}}.$$

1) Find a mono K_4 in A_1. Say its $\{x_1, x_2, x_3, x_4\}$.

2) REMOVE all A_i's that have all of $\{x_1, x_2, x_3, x_4\}$.
$\binom{n-4}{18-4} = \binom{n-4}{14}$ of these. There are $\binom{n}{18} - \binom{n-4}{14}$ left.

3) Find a mono K_4 in one of the sets left. Say its $\{y_1, y_2, y_3, y_4\}$.

4) REMOVE all A_i's that have all of $\{y_1, y_2, y_3, y_4\}$.
$\binom{n-4}{18-4} = \binom{n-4}{14}$ of these. There are $\binom{n}{18} - 2\binom{n-4}{14}$ left.

5) Find a mono K_4 in one of the sets left. Now have 3. But. . ..

Want 3 Mono K_4's (cont)

Need

Want 3 Mono K_4's (cont)

Need

$$\binom{n}{18} - 2\binom{n-4}{14} \geq 1$$

Want 3 Mono K_4's (cont)

Need

$$\binom{n}{18} - 2\binom{n-4}{14} \geq 1$$

$$\binom{n}{18} - 2\binom{n-4}{14} > 0$$

Want 3 Mono K_4's (cont)

Need

$$\binom{n}{18} - 2\binom{n-4}{14} \geq 1$$

$$\binom{n}{18} - 2\binom{n-4}{14} > 0$$

$$\frac{n!}{18!(n-18)!} > 2\frac{(n-4)!}{14!(n-18)!}$$

Want 3 Mono K_4's (cont)

Need

$$\binom{n}{18} - 2\binom{n-4}{14} \geq 1$$

$$\binom{n}{18} - 2\binom{n-4}{14} > 0$$

$$\frac{n!}{18!(n-18)!} > 2\frac{(n-4)!}{14!(n-18)!}$$

$$\frac{n!}{18 \times 17 \times 16 \times 15} > 2(n-4)!$$

Want 3 Mono K_4's (cont)

Need

$$\binom{n}{18} - 2\binom{n-4}{14} \geq 1$$

$$\binom{n}{18} - 2\binom{n-4}{14} > 0$$

$$\frac{n!}{18!(n-18)!} > 2\frac{(n-4)!}{14!(n-18)!}$$

$$\frac{n!}{18 \times 17 \times 16 \times 15} > 2(n-4)!$$

$$n(n-1)(n-2)(n-3) > 2 \times 18 \times 17 \times 16 \times 15$$

Want 3 Mono K_4's (cont)

$$n(n-1)(n-2)(n-3) > 2 \times 18 \times 17 \times 16 \times 15 = 146889.$$

Want 3 Mono K_4's (cont)

$$n(n-1)(n-2)(n-3) > 2 \times 18 \times 17 \times 16 \times 15 = 146889.$$

n	$n(n-1)(n-2)(n-3)$
19	93024
20	116280
21	143640
22	175560

Want 3 Mono K_4's (cont)

$$n(n-1)(n-2)(n-3) > 2 \times 18 \times 17 \times 16 \times 15 = 146889.$$

n	$n(n-1)(n-2)(n-3)$
19	93024
20	116280
21	143640
22	175560

Thm $\forall$ 2-cols of the edges of K_{22} $\exists$ 3 mono K_4's.

Want m Mono K_4's

The key to the prior proof is how many A_i's do you remove.

Want m Mono K_4's

The key to the prior proof is how many A_i's do you remove.

We removed $\binom{n-4}{18-4} = \binom{n-4}{14}$ in each iteration.

Want m Mono K_4's

The key to the prior proof is how many A_i's do you remove.

We removed $\binom{n-4}{18-4} = \binom{n-4}{14}$ in each iteration.

Thm Let $m, n \ge \mathbb{N}$. Assume $\binom{n}{18} - m\binom{n-4}{14} \ge 1$. For any 2-col of K_n there exists $m + 1$ mono K_4's.

Want m Mono K_4's

The key to the prior proof is how many A_i's do you remove.

We removed $\binom{n-4}{18-4} = \binom{n-4}{14}$ in each iteration.

Thm Let $m, n \ge \mathbb{N}$. Assume $\binom{n}{18} - m\binom{n-4}{14} \ge 1$. For any 2-col of K_n there exists $m+1$ mono K_4's.

Subsets of $V = \{1, \ldots, n\}$, size $R(4) = 18$: $A_1, A_2, \ldots, A_{\binom{n}{18}}$.

Want m Mono K_4's

The key to the prior proof is how many A_i's do you remove.

We removed $\binom{n-4}{18-4} = \binom{n-4}{14}$ in each iteration.

Thm Let $m, n \ge \mathbb{N}$. Assume $\binom{n}{18} - m\binom{n-4}{14} \ge 1$. For any 2-col of K_n there exists $m+1$ mono K_4's.

Subsets of $V = \{1, \ldots, n\}$, size $R(4) = 18$: $A_1, A_2, \ldots, A_{\binom{n}{18}}$.

1) $\mathrm{SETA} = \{A_1, A_2, \ldots, A_{\binom{n}{18}}\}$. $\mathrm{SETK4} = \emptyset$.

Want m Mono K_4's

The key to the prior proof is how many A_i's do you remove.

We removed $\binom{n-4}{18-4} = \binom{n-4}{14}$ in each iteration.

Thm Let $m, n \ge \mathbb{N}$. Assume $\binom{n}{18} - m\binom{n-4}{14} \ge 1$. For any 2-col of K_n there exists $m+1$ mono K_4's.

Subsets of $V = \{1, \ldots, n\}$, size $R(4) = 18$: $A_1, A_2, \ldots, A_{\binom{n}{18}}$.

1) $\mathrm{SETA} = \{A_1, A_2, \ldots, A_{\binom{n}{18}}\}$. $\mathrm{SETK4} = \emptyset$.

2) Take arb $A \in \mathrm{SETA}$. $\exists$ mono K_4 in A, $K = \{x_1, x_2, x_3, x_4\}$.

Want m Mono K_4's

The key to the prior proof is how many A_i's do you remove.

We removed $\binom{n-4}{18-4} = \binom{n-4}{14}$ in each iteration.

Thm Let $m, n \ge \mathbb{N}$. Assume $\binom{n}{18} - m\binom{n-4}{14} \ge 1$. For any 2-col of K_n there exists $m+1$ mono K_4's.

Subsets of $V = \{1, \ldots, n\}$, size $R(4) = 18$: $A_1, A_2, \ldots, A_{\binom{n}{18}}$.

1) $\mathrm{SETA} = \{A_1, A_2, \ldots, A_{\binom{n}{18}}\}$. $\mathrm{SETK4} = \emptyset$.

2) Take arb $A \in \mathrm{SETA}$. $\exists$ mono K_4 in A, $K = \{x_1, x_2, x_3, x_4\}$.

- $\mathrm{SETK4} = \mathrm{SETK4} \cup \{K\}$.

Want m Mono K_4's

The key to the prior proof is how many A_i's do you remove.

We removed $\binom{n-4}{18-4} = \binom{n-4}{14}$ in each iteration.

Thm Let $m, n \ge \mathbb{N}$. Assume $\binom{n}{18} - m\binom{n-4}{14} \ge 1$. For any 2-col of K_n there exists $m+1$ mono K_4's.

Subsets of $V = \{1, \ldots, n\}$, size $R(4) = 18$: $A_1, A_2, \ldots, A_{\binom{n}{18}}$.

1) $\mathrm{SETA} = \{A_1, A_2, \ldots, A_{\binom{n}{18}}\}$. $\mathrm{SETK4} = \emptyset$.

2) Take arb $A \in \mathrm{SETA}$. $\exists$ mono K_4 in A, $K = \{x_1, x_2, x_3, x_4\}$.

- $\mathrm{SETK4} = \mathrm{SETK4} \cup \{K\}$.
- $\mathrm{SETA} = \mathrm{SETA} - \{A \in \mathrm{SETA} : x_1, x_2, x_3, x_4 \in A\}$.

Want m Mono K_4's

The key to the prior proof is how many A_i's do you remove.

We removed $\binom{n-4}{18-4} = \binom{n-4}{14}$ in each iteration.

Thm Let $m, n \geq \mathbb{N}$. Assume $\binom{n}{18} - m\binom{n-4}{14} \geq 1$. For any 2-col of K_n there exists $m+1$ mono K_4's.

Subsets of $V = \{1, \ldots, n\}$, size $R(4) = 18$: $A_1, A_2, \ldots, A_{\binom{n}{18}}$.

1) $\mathrm{SETA} = \{A_1, A_2, \ldots, A_{\binom{n}{18}}\}$. $\mathrm{SETK4} = \emptyset$.

2) Take arb $A \in \mathrm{SETA}$. $\exists$ mono K_4 in A, $K = \{x_1, x_2, x_3, x_4\}$.

- $\mathrm{SETK4} = \mathrm{SETK4} \cup \{K\}$.
- $\mathrm{SETA} = \mathrm{SETA} - \{A \in \mathrm{SETA} : x_1, x_2, x_3, x_4 \in A\}$.

3) If $\mathrm{SETA} \neq \emptyset$ then go to step 2. Else STOP.

Want m Mono K_4's

The key to the prior proof is how many A_i's do you remove.

We removed $\binom{n-4}{18-4} = \binom{n-4}{14}$ in each iteration.

Thm Let $m, n \ge \mathbb{N}$. Assume $\binom{n}{18} - m\binom{n-4}{14} \ge 1$. For any 2-col of K_n there exists $m+1$ mono K_4's.

Subsets of $V = \{1, \ldots, n\}$, size $R(4) = 18$: $A_1, A_2, \ldots, A_{\binom{n}{18}}$.

1) $\mathrm{SETA} = \{A_1, A_2, \ldots, A_{\binom{n}{18}}\}$. $\mathrm{SETK4} = \emptyset$.

2) Take arb $A \in \mathrm{SETA}$. $\exists$ mono K_4 in A, $K = \{x_1, x_2, x_3, x_4\}$.

- $\mathrm{SETK4} = \mathrm{SETK4} \cup \{K\}$.
- $\mathrm{SETA} = \mathrm{SETA} - \{A \in \mathrm{SETA} : x_1, x_2, x_3, x_4 \in A\}$.

3) If $\mathrm{SETA} \ne \emptyset$ then go to step 2. Else STOP.

Since $\binom{n}{18} - m\binom{n-4}{14} \ge 1$ this process can go for $\ge m+1$ iterations and produce $\ge m+1$ mono K_4's.

Want m Mono K_4's (cont)

We just proved that for all $n, m \in \mathbb{N}$:
Thm If $\binom{n}{18} - m\binom{n-4}{14} \geq 1$ then $\forall$ 2-col of K_n $\exists$ $m+1$ mono K_4's.

Want m Mono K_4's (cont)

We just proved that for all $n, m \in \mathbb{N}$:
Thm If $\binom{n}{18} - m\binom{n-4}{14} \geq 1$ then $\forall$ 2-col of K_n $\exists$ $m+1$ mono K_4's.

We want m as a function of n.

Want m Mono K_4's (cont)

We just proved that for all $n, m \in \mathbb{N}$:
Thm If $\binom{n}{18} - m\binom{n-4}{14} \geq 1$ then $\forall$ 2-col of K_n $\exists$ $m+1$ mono K_4's.

We want m as a function of n.

$$\binom{n}{18} - m\binom{n-4}{14} \geq 0$$

Want m Mono K_4's (cont)

We just proved that for all $n, m \in \mathbb{N}$:
Thm If $\binom{n}{18} - m\binom{n-4}{14} \ge 1$ then $\forall$ 2-col of K_n $\exists$ $m+1$ mono K_4's.

We want m as a function of n.

$$\binom{n}{18} - m\binom{n-4}{14} \ge 0$$

$$m \le \frac{\binom{n}{18}}{\binom{n-4}{14}} = \frac{n!}{18!(n-18)!}\frac{14!(n-18)!}{(n-4)!} = \frac{n(n-1)(n-2)(n-3)}{18 \times 17 \times 16 \times 15}$$

Want m Mono K_4's (cont)

We just proved that for all $n, m \in \mathbb{N}$:
Thm If $\binom{n}{18} - m\binom{n-4}{14} \ge 1$ then $\forall$ 2-col of K_n $\exists$ $m+1$ mono K_4's.
We want m as a function of n.

$$\binom{n}{18} - m\binom{n-4}{14} \ge 0$$

$$m \le \frac{\binom{n}{18}}{\binom{n-4}{14}} = \frac{n!}{18!(n-18)!}\frac{14!(n-18)!}{(n-4)!} = \frac{n(n-1)(n-2)(n-3)}{18\times 17\times 16\times 15}$$

We state a theorem which expresses this in several ways, on the next slide.

More Versions

More Versions

Thm Let $n \geq 19$. $\forall$ 2-col of K_n the following happens.

More Versions

Thm Let $n \geq 19$. $\forall$ 2-col of K_n the following happens.

1) There are $\left\lfloor \frac{\binom{n}{18}}{\binom{n-4}{14}} \right\rfloor + 1$ mono K_4's.

More Versions

Thm Let $n \geq 19$. $\forall$ 2-col of K_n the following happens.

1) There are $\left\lfloor \frac{\binom{n}{18}}{\binom{n-4}{14}} \right\rfloor + 1$ mono K_4's.

2) There are $\frac{n(n-1)(n-2)(n-3)}{18\times17\times16\times15} + 1$ mono K_4's.

More Versions

Thm Let $n \ge 19$. $\forall$ 2-col of K_n the following happens.

1) There are $\left\lfloor \frac{\binom{n}{18}}{\binom{n-4}{14}} \right\rfloor + 1$ mono K_4's.

2) There are $\frac{n(n-1)(n-2)(n-3)}{18\times 17\times 16\times 15} + 1$ mono K_4's.

3) There are $\frac{n(n-1)(n-2)(n-3)}{73440} + 1$ mono K_4's.

More Versions

Thm Let $n \geq 19$. $\forall$ 2-col of K_n the following happens.

1) There are $\left\lfloor \frac{\binom{n}{18}}{\binom{n-4}{14}} \right\rfloor + 1$ mono K_4's.

2) There are $\frac{n(n-1)(n-2)(n-3)}{18\times 17\times 16\times 15} + 1$ mono K_4's.

3) There are $\frac{n(n-1)(n-2)(n-3)}{73440} + 1$ mono K_4's.

4) There are $\frac{n^4}{73440} - \frac{n^3}{12240} + \Omega(n^2)$ mono K_4's.

Another Way to Phrase The Results

Another Way to Phrase The Results

There are $\sim \frac{n^4}{73440}$ mono K_4's.

Another Way to Phrase The Results

There are $\sim \frac{n^4}{73440}$ mono K_4's.

We rephrase this as what fraction of the $\binom{n}{4}$ K_4's are mono.

Another Way to Phrase The Results

There are $\sim \frac{n^4}{73440}$ mono K_4's.

We rephrase this as what fraction of the $\binom{n}{4}$ K_4's are mono.

There are $\frac{1}{3060}\binom{n}{4}$ mono K_4's.

Another Way to Phrase The Results

There are $\sim \frac{n^4}{73440}$ mono K_4's.

We rephrase this as what fraction of the $\binom{n}{4}$ K_4's are mono.

There are $\frac{1}{3060}\binom{n}{4}$ mono K_4's.

$\frac{1}{3060} \sim 0.00032$.

Another Way to Phrase The Results

There are $\sim \frac{n^4}{73440}$ mono K_4's.

We rephrase this as what fraction of the $\binom{n}{4}$ K_4's are mono.

There are $\frac{1}{3060}\binom{n}{4}$ mono K_4's.
$\frac{1}{3060} \sim 0.00032$.
Is better known? Speculate. Answer on next page.

Want: If 2-Color Edges of K_n Then How many K_L's?

Want: If 2-Color Edges of K_n Then How many K_L's?

Thm For all $\mathrm{COL}\colon \binom{[n]}{2} \rightarrow [2]$, the fraction of 4-sets that are mono K_4's is $\ge \frac{1}{3060} \sim 0.00032$.

Want: If 2-Color Edges of K_n Then How many K_L's?

Thm For all $\mathrm{COL}\colon \binom{[n]}{2} \rightarrow [2]$, the fraction of 4-sets that are mono K_4's is $\ge \frac{1}{3060} \sim 0.00032$.

Better is known.

Want: If 2-Color Edges of K_n Then How many K_L's?

Thm For all $\mathrm{COL}\colon \binom{[n]}{2} \rightarrow [2]$, the fraction of 4-sets that are mono K_4's is $\ge \frac{1}{3060} \sim 0.00032$.

Better is known.

In 1979 Girad showed that fraction is $\ge \frac{1}{46} \sim 0.0217$.

Want: If 2-Color Edges of K_n Then How many K_L's?

Thm For all $\mathrm{COL}\colon \binom{[n]}{2} \rightarrow [2]$, the fraction of 4-sets that are mono K_4's is $\ge \frac{1}{3060} \sim 0.00032$.

Better is known.

In 1979 Girad showed that fraction is $\ge \frac{1}{46} \sim 0.0217$.

In 2012 Susanne NieB showed

Want: If 2-Color Edges of K_n Then How many K_L's?

Thm For all $\mathrm{COL}\colon \binom{[n]}{2} \rightarrow [2]$, the fraction of 4-sets that are mono K_4's is $\ge \frac{1}{3060} \sim 0.00032$.

Better is known.

In 1979 Girad showed that fraction is $\ge \frac{1}{46} \sim 0.0217$.

In 2012 Susanne NieB showed
the fraction is $\ge \frac{204603019}{7112448000} \sim 0.0287$.

Want: If 2-Color Edges of K_n Then How many K_L's?

Thm For all $\mathrm{COL}\colon \binom{[n]}{2} \rightarrow [2]$, the fraction of 4-sets that are mono K_4's is $\ge \frac{1}{3060} \sim 0.00032$.

Better is known.

In 1979 Girad showed that fraction is $\ge \frac{1}{46} \sim 0.0217$.

In 2012 Susanne NieB showed
the fraction is $\ge \frac{204603019}{7112448000} \sim 0.0287$.
the fraction is $\le \frac{1}{33} \sim 0.0303$.
see `https://arxiv.org/pdf/1207.4714`

Want: If 2-Color Edges of K_n Then How many K_L's?

Thm For all $\mathrm{COL}\colon \binom{[n]}{2} \rightarrow [2]$, the fraction of 4-sets that are mono K_4's is $\ge \frac{1}{3060} \sim 0.00032$.

Better is known.

In 1979 Girad showed that fraction is $\ge \frac{1}{46} \sim 0.0217$.

In 2012 Susanne NieB showed
the fraction is $\ge \frac{204603019}{7112448000} \sim 0.0287$.
the fraction is $\le \frac{1}{33} \sim 0.0303$.
see `https://arxiv.org/pdf/1207.4714`

Want: If 2-Color Edges of K_n Then How many K_L's?

This will be a HW.

Generalize

Left to the reader

Generalize

Left to the reader

1. Generalize to mono K_m.

Generalize

Left to the reader

1. Generalize to mono K_m.
2. Generalize to c colors.

Generalize

Left to the reader

1. Generalize to mono K_m.
2. Generalize to c colors.
3. Generalize to c colors and mono K_m.