Avoiding Short Progressions in Ramsey Theory

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L3 Result

$$E^n \not\to (\ell_3, \ell_{20})$$

We already know what L_n means

There exists a coloring such that no 3 collinear red points and no 20 collinear blue points exist

The Coloring

We choose a set $S=\{0, 1, 2, 3, 4, 5, 6\}$

Color point x red if $[7|x|^2] \bmod 29$ is in S

If not color it blue

Color depends on the norm, and every point on same sphere has same color

L3 Property

Let x, y, z form L3.

Then by pappus theorem (or Apollonius) $|x|^2+|z|^2=2(|y|^2+1)$, $|x|^2-2|y|^2+|z|^2=2$

Multiplying both side by 7: $7|x|^2-2*7|y|^2+7|z|^2=14$

Since floor can cause error of maximum 1, $[7|x|^2]-2*[7|y|^2]+[7|z|^2] = \{13,14,15\}$

Check with S={0,1,2,3,4,5,6}

Out of 7^3=343 combinations, no combination work to result in {13,14,15}

Blue L_{20} goes similarly

Let $x_0 \sim x_{19}$ form L_{20}

Let $X_k = |x_k|^2$

Then $X_{k+2} = 2X_{k+1} - X_k + 2$

Solve the recurrence: $X_k = k^2 + (X_1 - X_0 - 1)k + X_0$

So for all X_0 and X_1 mod 29 in {7, 8, …, 28},

check k=0~19 if all X_k mod 29 are in {7, 8, …, 28}

None of them works

Thus in this coloring no red L_3 and no blue L_{20} exists

Why these numbers?

Why $S=\{0,1,2,3,4,5,6\}$, mod 29, and $7[|x|^2]$?

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Why S={0,1,2,3,4,5,6}, mod p=29, and d=7[|x|^2]?

UNKNOWN TO SCIENCE

More specifically, they did computational search over 3≤p≤45, 1≤d≤p/2, |S|≤10 and searched for: no red L_3 and no blue L_N for smallest N

So bound of N can still improve!

Some reasoning behind these numbers

They wanted S to be as large as possible

- Larger red density helps destroy long blue progressions

Why consecutive elements in S?

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Apparently it just happened to be so.

L_4,L_{14} and L_5,L_8 goes similar

For L_4, they used S={0,1,2,3,4,5,6,7,8}, p=29, and d=10

For L_5, they used S={0,1}, p=5, and d=2