1 TO DO on Rado's Theorem

Recall the definition of regular and Rado's Theorem:

Definition 1.1 Let $a_1, \ldots, a_n \in \mathsf{Z}$.

1. $a_1, \ldots, a_n$ is *regular* if the following holds:

   for all $c \ge 1$, there exits $R = R(a_1, \ldots, a_n; c)$ such that

   For all $\mathrm{COL}\colon [R] \to [c]$ there exists a mono solution to $\sum_{i=1}^{n} a_i x_i = 0$.

2. $a_1, \ldots, a_n$ is *distinct regular* if the following holds:

   for all $c \ge 1$, there exits $R = R(a_1, \ldots, a_n; c)$ such that

   For all $\mathrm{COL}\colon [R] \to [c]$ there exists a mono solution to $\sum_{i=1}^{n} a_i x_i = 0$ where all of the a_i's are distinct.

Theorem 1.2 *$a_1, \ldots, a_n$ is regular iff some subset of the $a_1, \ldots, a_n$ sums to 0.*

The following are probably due to me since distinct-regular is probably due to me. More important- I doubt there has been much work on the notion.

Theorem 1.3 *If $a_1, \ldots, a_n$ is such that either $\sum_{i=1}^{n} = 0$ or $\sum_{i=1}^{n-1} = 0$ then $(a_1, \ldots, a_n)$ is distinct regular.*

1. By the proof of Rado's theorem there is a 4-coloring of N with no mono solution to $x+2y-4z = 0$.

   Is there such a 3-coloring of N?

   Is there such a 2-coloring of N?

2. Consider the question above for other equations where no sum of the coefficient is 0.

3. There have been some papers on $\sum_{i=1}^{n} a_i x_i = d$ where d is a constant. I collected them up on a website:

   `https://www.cs.umd.edu/~gasarch/TOPICS/RADO/rado.html`

   they look ugly but you may want to look at them anyway.

4. Consider $x + 2y - 3z + p - 7q$.

   By Rado's Theorem, for any c-coloring has a mono solution.

   However, the proof of Rado's theorem makes $p = q$.

   Is the following true: for any c-coloring has a distinct mono solution.

5. Consider other equations like the one above.