

## Ramsey Multiplicity TO DO

1. The version of the paper on the website now has the graph colorings that give the exact lower bounds. READ IT and UNDERSTAND IT.

2. Prove a theorem of the following form:

For all 3-colorings of the edges of  $K_n$  there will be at least BLAH mono  $K_3$ .

Make BLAH as exact as you can.

Try to prove the result is optimal or almost-optimal.

3. Prove a theorem of the following form:

For all  $c$ -colorings of the edges of  $K_n$  there will be at least BLAH mono  $K_3$ .

Make BLAH as exact as you can.

Try to prove the result is optimal or almost-optimal.

4. YOU proved:

For all 2-colorings of the edges of  $K_{20}$  there are  $\geq 2$  mono  $K_4$ 's.

Write it up carefully.

Try to extend it so that you have a table (might need a program for this) like the one below (my numbers are surely wrong) where  $m$  is the desired number of  $K_4$ 's you want, and  $n$  is such that for all 2-colorings of  $K_n$  there are  $m$  mono  $K_m$ 's.

$m$	$n$
2	20
3	22
4	24

(Do more numbers than this.)

Write a program to get the same chart from my proof. See if you get lower  $n$ 's than I got.

5. I COULD ask you to look at 3-colorings and  $K_4$ 's. this might be hard to really do much with since

$$128 \leq R(4, 4, 4) \leq 229.$$

6. Look into how many mono  $K_5$ 's you get, assuming  $R(5) = 43$ .

7. I put three papers on Ramsey Multiplicity (thats the name of this field) on the project website. Giraud's 1979 paper. Its not in English. I think its in French. If one of you is fluent in French (or whatever language its in) we can look into it. I suspect not. Yasmine IS fluent in French so I may at least ask her what the Theorems in it are.

Susanne Neib's 2012 Paper on Ramsey Multiplicity. Read this and do a good write up of with examples and intuitions.

Rob Brady's 2022 Masters Thesis on Ramsey Multiplicity I supervised this theorem so hopefully its already well written. Read it. I have the latex so if you find a typo or a better way to say something I can fix it.

8. Programming:

RANDOMLY 2-coloring  $K_n$  and see what fraction of the  $\binom{n}{3}$  triangles are mono.

RANDOMLY 2-coloring  $K_n$  and see what fraction of the  $\binom{n}{4}$   $K_4$ 's are mono.

RANDOMLY 3-coloring  $K_n$  and see what fraction of the  $\binom{n}{3}$  triangles are mono.

RANDOMLY 3-coloring  $K_n$  and see what fraction of the  $\binom{n}{4}$   $K_4$ 's are mono.