Simulation of Low-Depth Quantum Circuits

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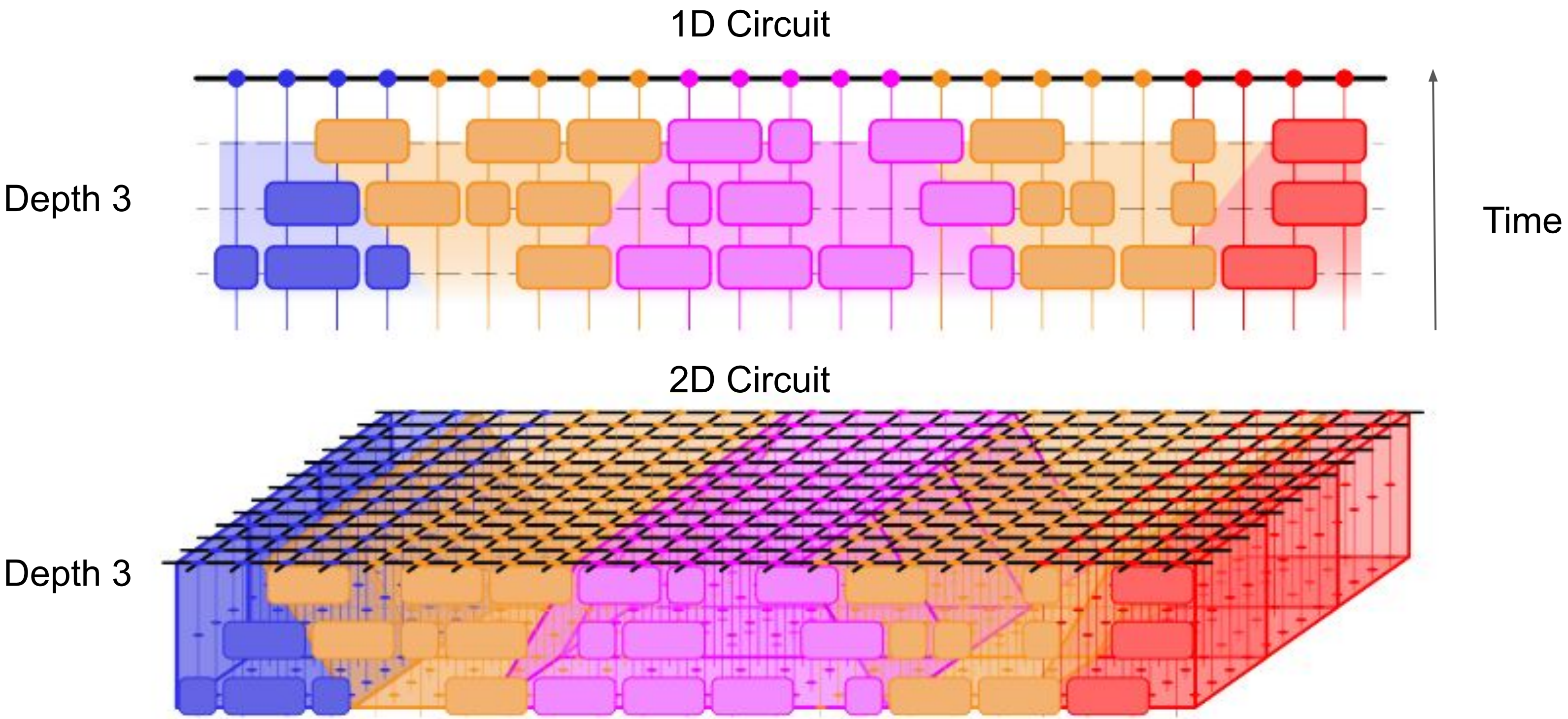
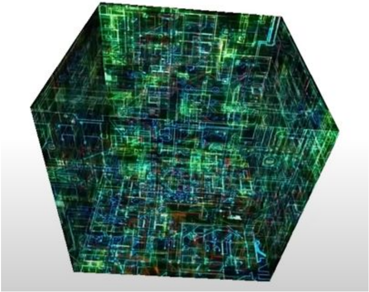
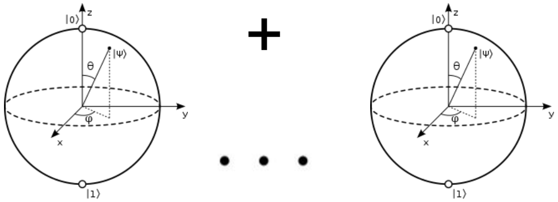
Introduction to Quantum Circuits



Problem Setup

Simulated 3D Quantum Circuit





N qubits

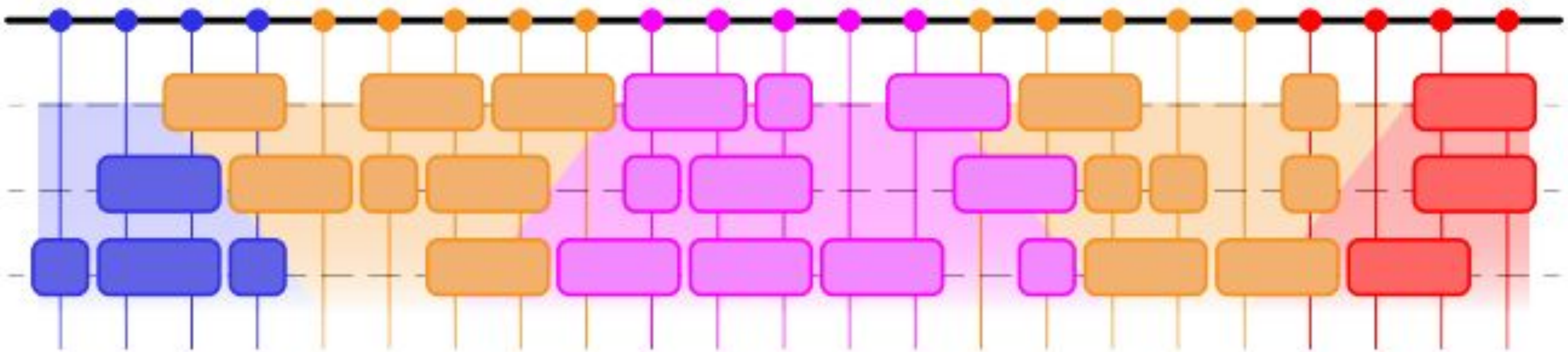
$$\approx P(x \in \{0,1\}^n)$$

Given a 3D geometrically-local, poly-log depth quantum circuit with *n* qubits, what is the probability of outputting some bitstring x, given a series of 0 qubits?

Why is this problem important

One interesting and important question is whether a quantum computer solves problems faster than a classical computer.

Past results (1D Quantum Circuits)

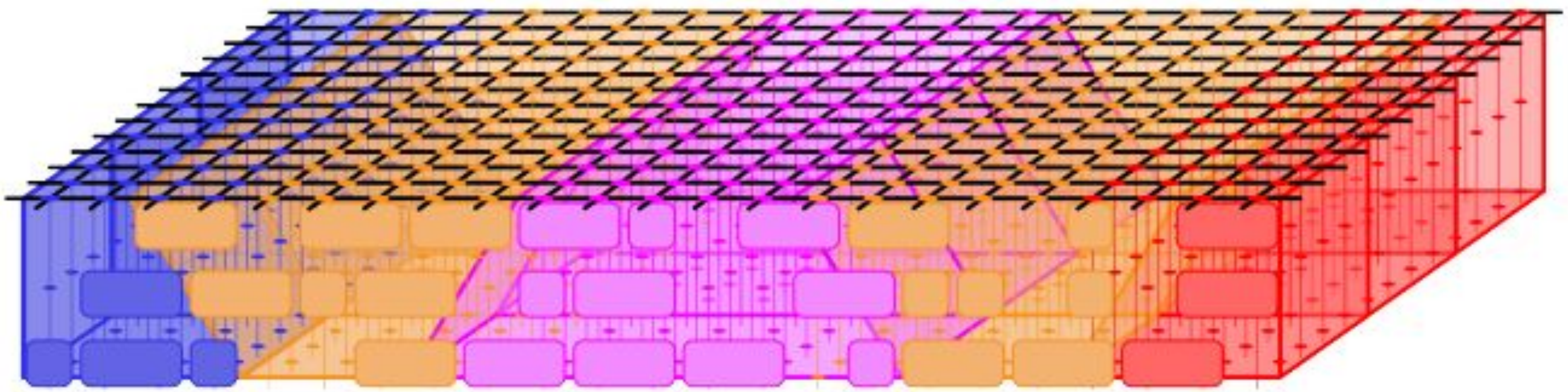


$$n\,poly\left(2^{O(\mathrm{d})}\right)$$

n = no. of qubits

d = depth of circuit

Past results (2D low-depth geometrically-local QC)



*

$n = no. of\ qubits$

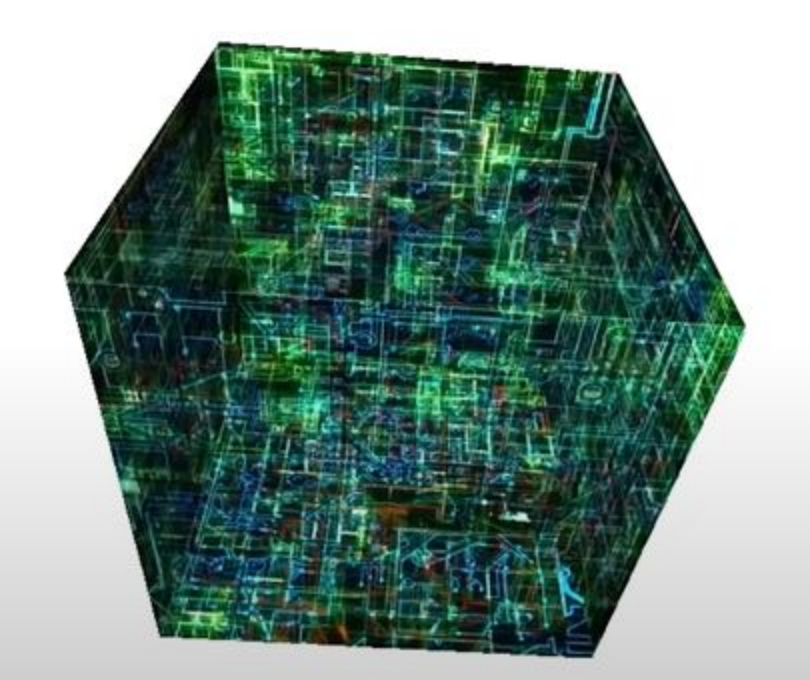
$n\delta^{-2}2^{O(d^2)}$ $d = depth\ of\ circuit$

$\delta = error$

Sergey Bravyi, David Gosset, and Ramis Movassagh, 2020

*N. J. Coble and Coudron, M., "**Quasi-polynomial Time Approximation of Output Probabilities of Constant-depth, Geometrically-local Quantum Circuits**", *Accepted to QIP 2021*, 2020.

Current results (3D low-depth geometrically-local QC)



$$\delta^{-2} 2^{d^3 polylog(n)(1/\delta)^{1/log^2(n)}}$$

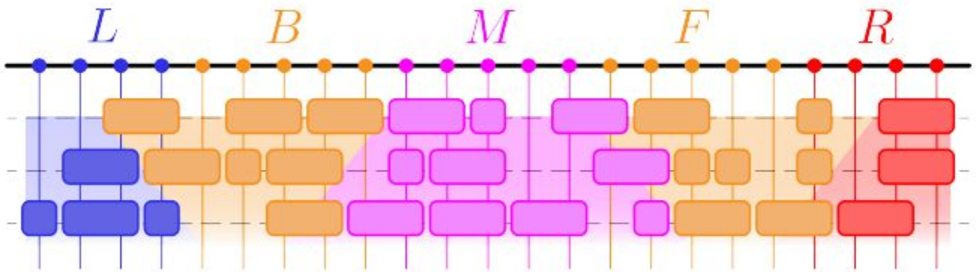
$$n \;=\; no. of\ qubits$$

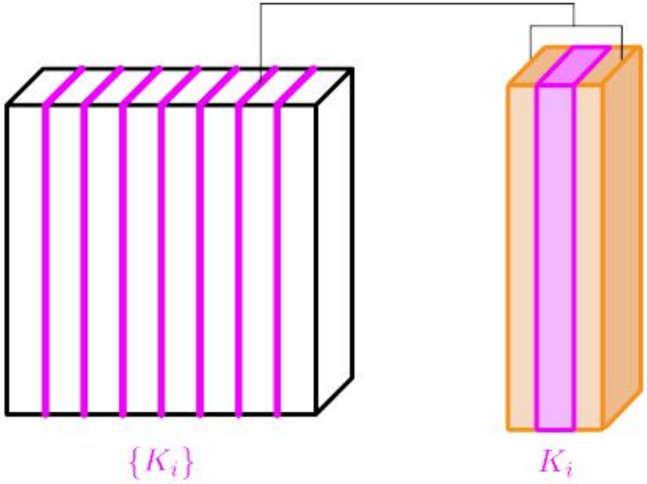
$$d \;=\; depth\ of\ circuit$$

$$\delta = \; error$$

Matthew Coudron and Nolan Coble, 2021

Current approach for 3D QC





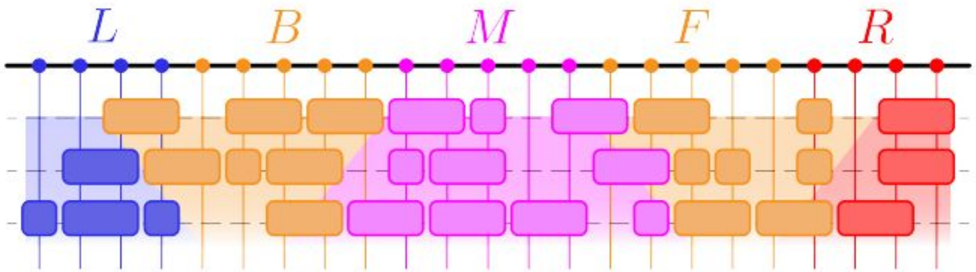
Schmidt Decomposition

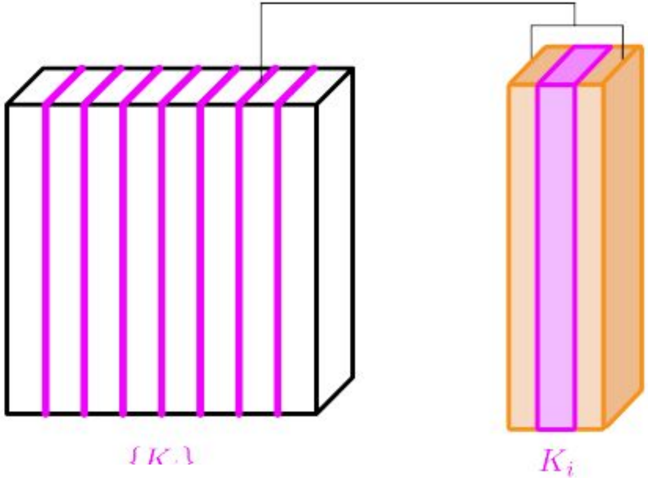
Theorem 2.7: **(Schmidt decomposition)** Suppose $|\psi\rangle$ is a pure state of a composite system, AB. Then there exist orthonormal states $|i_A\rangle$ for system A, and orthonormal states $|i_B\rangle$ of system B such that

$$|\psi\rangle = \sum_i \lambda_i |i_A\rangle |i_B\rangle, \tag{2.202}$$

where λ_i are non-negative real numbers satisfying $\sum_i \lambda_i^2 = 1$ known as *Schmidt co-efficients*.

Current approach for 3D QC



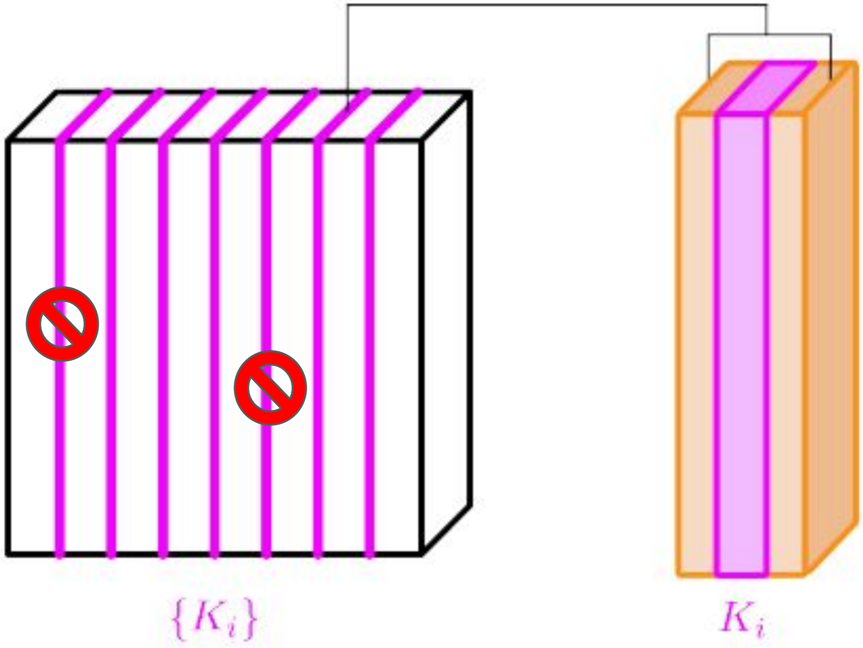


$$|\psi\rangle_{B\cup F} \approx \sum_{i=1}^{p(n)} \lambda_i \, |v_i\rangle_B \otimes |w_i\rangle_F$$

$$\langle 0|_{\mathrm{ALL}}\, C_{L\cup R}\, |0\rangle_{L\cup R} \otimes |\psi\rangle_{B\cup F} \approx \sum_{i=1}^{p(n)} \lambda_i \, \langle 0|_{\mathrm{ALL}}\, C_{L\cup R}\, |0\rangle_{L\cup R} \otimes |v_i\rangle_B \otimes |w_i\rangle_F$$

$$= \sum_{i=1}^{p(n)} \lambda_i \, \langle 0|_{L\cup B}\, C_L\, |0\rangle_L \otimes |v_i\rangle_B \cdot \langle 0|_{F\cup R}\, C_R\, |0\rangle_R \otimes |w_i\rangle_F \, ,$$

Reasons why 3D algorithm is quasi-polynomial



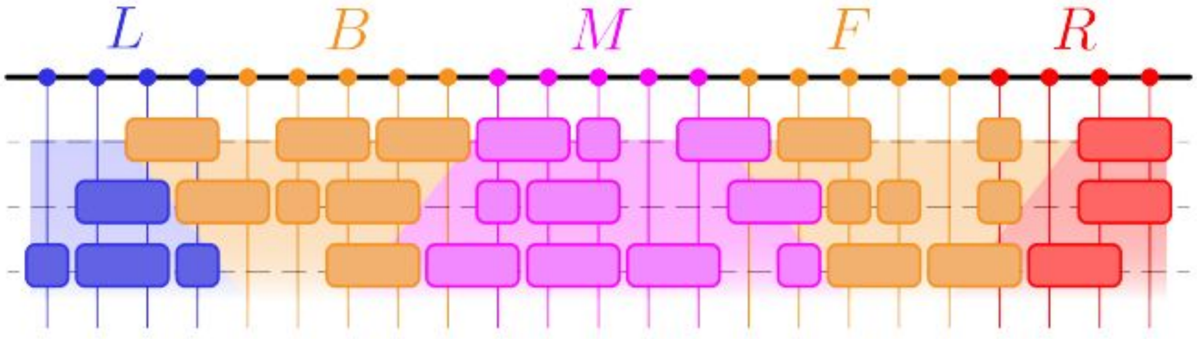
Current work

Two approaches

-Using Chebyshev polynomials to approximate Schmidt vectors for efficient divide and conquer algorithm

-Using a modified divide and conquer algorithm as a base case for induction to higher dimensions

Primary Methodology



By a theorem of Brayvi, any cut needs only top p(n) terms

Old method only used first term, and only when $\lambda_1 \geq 1 - O(e(n))$

$$\langle 0|_{\mathrm{ALL}}\, C_{L\cup R}\, |0\rangle_{L\cup R} \otimes |\psi\rangle_{B\cup F} \approx \sum_{i=1}^{\boxed{p(n)}} \boxed{\lambda_i} \langle 0|_{\mathrm{ALL}}\, C_{L\cup R}\, |0\rangle_{L\cup R} \otimes |v_i\rangle_B \otimes |w_i\rangle_F$$

$$= \sum_{i=1}^{p(n)} \boxed{\lambda_i} \langle 0|_{L\cup B}\, C_L\, |0\rangle_L \otimes \boxed{|v_i\rangle_B} \cdot \langle 0|_{F\cup R}\, C_R\, |0\rangle_R \otimes \boxed{|w_i\rangle_F},$$

Need these for p(n) > i > 1

Chebyshev Series Approximation

$$I_{[a,b]}(x) = \begin{cases} 0 & x \notin [a,b] \\ 1 & x \in [a,b] \end{cases}$$

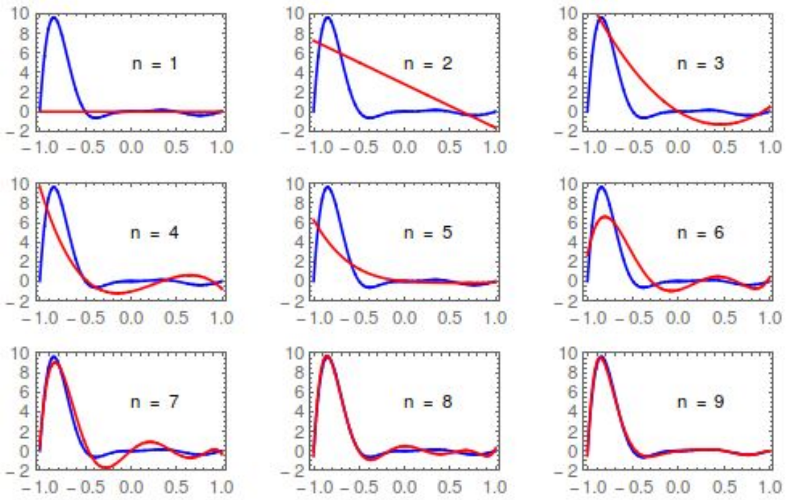
We approximate $I_{[a,b]}(x)$ with a degree K Chebyshev series

$$\sum_{k=0}^{K} c_k T_k(x), \text{ where}$$

$$c_k = \frac{2}{k\pi}\left(\sin\left(k\cos^{-1}(a)\right) - \sin\left(k\cos^{-1}(b)\right)\right) \text{ and}$$

$$T_k(x) = 2xT_{k-1}(x) - T_{k-2}(x), T_0(x) = 1, T_1(x) = x$$

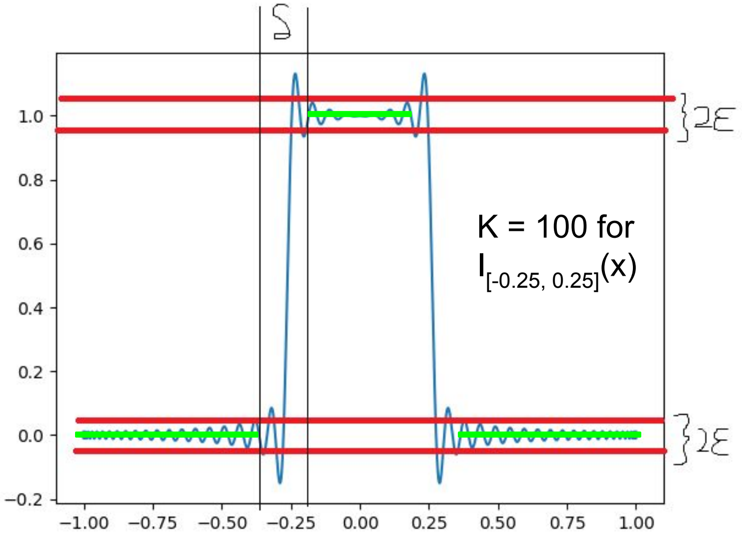
Chebyshev series approximations are near optimal



Chebyshev Series Approximation

Let $\epsilon, \delta > 0$. We define $Cheb^{\delta,\epsilon}_{[a,b]}(x)$ to be a Chebyshev series approximation of $I_{[a,b]}$ such that for $x \in \boxed{[a+\delta/2, b-\delta/2]}$, we get $\left|Cheb^{\delta,\epsilon}_{[a,b]}(x) - 1\right| < \epsilon$, and for $\boxed{x \in [-1, a-\delta/2]}$ $\boxed{\cup\, [b+\delta/2, 1]}$, we get $\left|Cheb^{\delta,\epsilon}_{[a,b]}(x)\right| < \epsilon$.

"usable region"



Conjecture: We can achieve $Cheb^{\delta,\epsilon}_{[a,b]}(x)$ with $K = \Theta(\frac{1}{\epsilon \cdot \delta})$

Chebyshev Series Approach

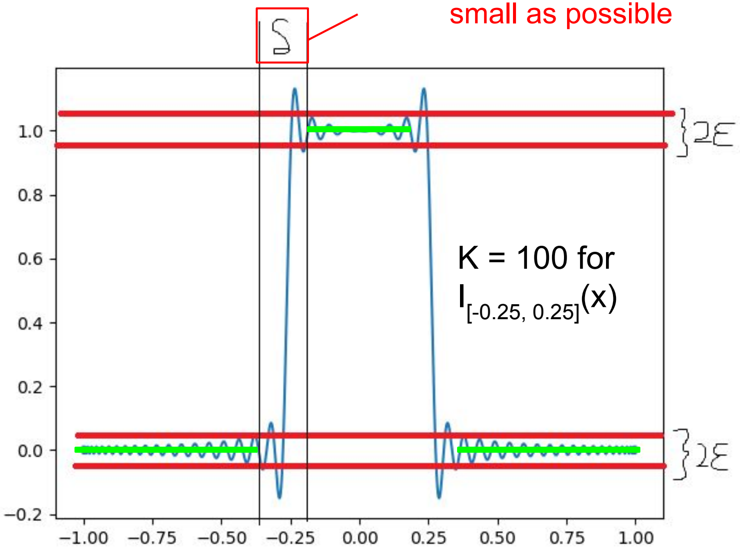
For poly(n) time algorithm to find λ_i, need $K = O(\log^{1/2}(x)) \Rightarrow \delta = \Omega(\log^{-1/2}(x))$

Conjecture: We can achieve $Cheb^{\delta,\epsilon}_{[a,b]}(x)$ with $\boxed{K} = \Theta(\frac{1}{\epsilon \cdot \delta})$

Let $\epsilon, \delta > 0$. We define $Cheb^{\delta,\epsilon}_{[a,b]}(x)$ to be a Chebyshev series approximation of $I_{[a,b]}$ such that for $x \in \boxed{[a + \delta/2, b - \delta/2]}$, we get $\left|Cheb^{\delta,\epsilon}_{[a,b]}(x) - 1\right| < \epsilon$, and for $\boxed{x \in [-1, a - \delta/2]}$ $\boxed{\cup [b + \delta/2, 1]}$, we get $\left|Cheb^{\delta,\epsilon}_{[a,b]}(x)\right| < \epsilon$.

"usable region"

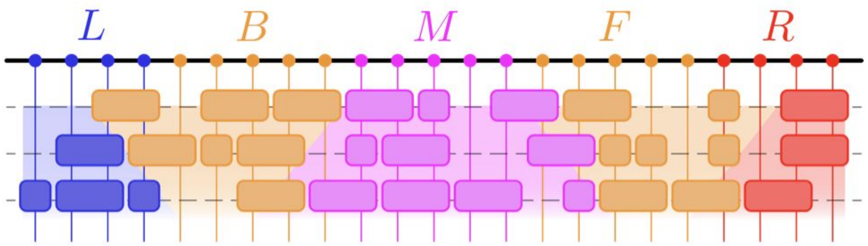
But for good approximation, need δ as small as possible



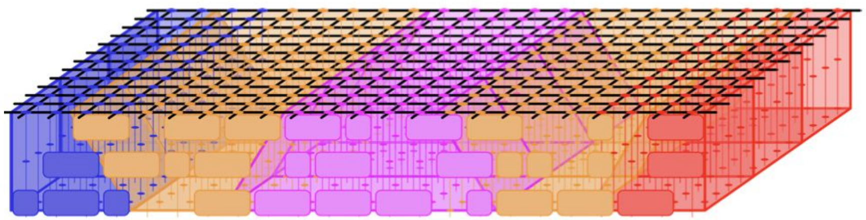
Induction to higher dimensions

Current algorithm: divide and conquer approach to split 3D circuit into 2D slices, using BGM as a base case for 2D

Goal: Simplify algorithm so that it can be generalized for any dimension circuit



1D circuit*



2D circuit*

*N. J. Coble and Coudron, M., "**Quasi-polynomial Time Approximation of Output Probabilities of Constant-depth, Geometrically-local Quantum Circuits**", *Accepted to QIP 2021*, 2020.

Method

-Use the current algorithm to break 3D into 2D with a constant width in the 3rd dimension

-Use this method again for the 2D case

-Use the 1D algorithm to solve this case

Current progress

-Generalizing language so it can apply to other dimensions

-Starting with 2D case

- -Modifying 3D algorithm to run on a 2D circuit
- -Making sure lemmas and proofs hold in other dimensions

-Next:

- -Analyze runtime of new algorithm
- -Apply to higher dimensions

Analysis of this approach

-Likely not a faster runtime than using BGM

-Only one algorithm

-Can incorporate other improvements to the algorithm

-Applications to real circuits

Summary

-Chebyshev approach

-Induction approach

Acknowledgements

Matthew Coudron Gorjan Alagic William Gasarch Stephen Smith Nolan Coble

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