NIM Games for Tyler and Noei

William Gasarch-U of MD

7 Mints, 1-2-3 Moves, Game

Tyler and Noei play a game:

7 Mints, 1-2-3 Moves, Game

Tyler and Noei play a game:

1. There are **7 mints** on the table.

7 Mints, 1-2-3 Moves, Game

Tyler and Noei play a game:

1. There are **7 mints** on the table.
2. Tyler removes **1,2,** OR **3** mints.

7 Mints, 1-2-3 Moves, Game

Tyler and Noei play a game:

1. There are **7 mints** on the table.
2. Tyler removes **1,2,** OR **3** mints.
3. Noei removes **1,2,** OR **3** mint.

7 Mints, 1-2-3 Moves, Game

Tyler and Noei play a game:

1. There are **7 mints** on the table.
2. Tyler removes **1,2,** OR **3** mints.
3. Noei removes **1,2,** OR **3** mint.
4. Tyler then removes **1,2,** OR **3** mint.

7 Mints, 1-2-3 Moves, Game

Tyler and Noei play a game:

1. There are **7 mints** on the table.
2. Tyler removes **1,2,** OR **3** mints.
3. Noei removes **1,2,** OR **3** mint.
4. Tyler then removes **1,2,** OR **3** mint.
5. They keep doing this.

7 Mints, 1-2-3 Moves, Game

Tyler and Noei play a game:

1. There are **7 mints** on the table.
2. Tyler removes **1,2,** OR **3** mints.
3. Noei removes **1,2,** OR **3** mint.
4. Tyler then removes **1,2,** OR **3** mint.
5. They keep doing this.
6. The person who removes the **last** mint wins.

7 Mints, 1-2-3 Moves, Game

Tyler and Noei play a game:

1. There are **7 mints** on the table.
2. Tyler removes **1,2,** OR **3** mints.
3. Noei removes **1,2,** OR **3** mint.
4. Tyler then removes **1,2,** OR **3** mint.
5. They keep doing this.
6. The person who removes the **last** mint wins.

Play the game and see if one of the players can **always win**.

Tyler Can Always Win

Tyler Can Always Win

1. Tyler removes **3** mints.

Tyler Can Always Win

1. Tyler removes **3** mints. There are now **4** mints.

Tyler Can Always Win

1. Tyler removes **3** mints. There are now **4** mints.
2. 2.1 If Noei removes **1** mint, Tyler removes **3**.

Tyler Can Always Win

1. Tyler removes **3** mints. There are now **4** mints.
2. 2.1 If Noei removes **1** mint, Tyler removes **3**. Tyler wins.

Tyler Can Always Win

1. Tyler removes **3** mints. There are now **4** mints.
2.
   2.1 If Noei removes **1** mint, Tyler removes **3**. Tyler wins.
   2.2 If Noei removes **2** mint, Tyler removes **2**.

Tyler Can Always Win

1. Tyler removes **3** mints. There are now **4** mints.
2.
   2.1 If Noei removes **1** mint, Tyler removes **3**. Tyler wins.
   2.2 If Noei removes **2** mint, Tyler removes **2**. Tyler wins.

Tyler Can Always Win

1. Tyler removes **3** mints. There are now **4** mints.
2.
   2.1 If Noei removes **1** mint, Tyler removes **3**. Tyler wins.
   2.2 If Noei removes **2** mint, Tyler removes **2**. Tyler wins.
   2.3 If Noei removes **3** mint, Tyler removes **1**.

Tyler Can Always Win

1. Tyler removes **3** mints. There are now **4** mints.
2. 2.1 If Noei removes **1** mint, Tyler removes **3**. Tyler wins.
   2.2 If Noei removes **2** mint, Tyler removes **2**. Tyler wins.
   2.3 If Noei removes **3** mint, Tyler removes **1**. Tyler wins.

What about 5,6,7,8 Mints?

Work on:

1. Can Tyler always win if the game begins with **5** mints?
2. Can Tyler always win if the game begins with **6** mints?
3. Can Tyler always win if the game begins with **7** mints? (Yes)
4. Can Tyler always win if the game begins with **8** mints?

What about 5,6,7,8 Mints? Answers

Work on:

1. Can Tyler always win if the game begins with **5** mints?

What about **5,6,7,8** Mints? Answers

Work on:

1. Can Tyler always win if the game begins with **5** mints?
   Tyler removes 1, there are now 4. Same as before.
2. Can Tyler always win if the game begins with **6** mints?

What about **5,6,7,8** Mints? Answers

Work on:

1. Can Tyler always win if the game begins with **5** mints?
   Tyler removes 1, there are now 4. Same as before.
2. Can Tyler always win if the game begins with **6** mints?
   Tyler removes 2, there are now 4. Same as before.

What about 5,6,7,8 Mints? Answers

Work on:

1. Can Tyler always win if the game begins with **5** mints?
   Tyler removes 1, there are now 4. Same as before.
2. Can Tyler always win if the game begins with **6** mints?
   Tyler removes 2, there are now 4. Same as before.
3. Can Tyler always win if the game begins with **7** mints? (Yes)

What about 5,6,7,8 Mints? Answers

Work on:

1. Can Tyler always win if the game begins with **5** mints?
   Tyler removes 1, there are now 4. Same as before.
2. Can Tyler always win if the game begins with **6** mints?
   Tyler removes 2, there are now 4. Same as before.
3. Can Tyler always win if the game begins with **7** mints? (Yes)
   Tyler removes 3, there are now 4. Same as before.

What about 5,6,7,8 Mints? Answers

Work on:

1. Can Tyler always win if the game begins with **5** mints?
   Tyler removes 1, there are now 4. Same as before.
2. Can Tyler always win if the game begins with **6** mints?
   Tyler removes 2, there are now 4. Same as before.
3. Can Tyler always win if the game begins with **7** mints? (Yes)
   Tyler removes 3, there are now 4. Same as before.
4. What about **8**? Next Side.

What about 8 Mints? Answers

What about 8 Mints? Answers

Noei Wins!

What about 8 Mints? Answers

Noei Wins!

1. If Tyler removes 1 then Noei removes 3.
   Now Tyler is looking at 4 mints and Noei can win.

What about 8 Mints? Answers

Noei Wins!

1. If Tyler removes 1 then Noei removes 3.
   Now Tyler is looking at 4 mints and Noei can win.
2. If Tyler removes 2 then Noei removes 2.
   Now Tyler is looking at 4 mints and Noei can win.

What about 8 Mints? Answers

Noei Wins!

1. If Tyler removes 1 then Noei removes 3.
   Now Tyler is looking at 4 mints and Noei can win.
2. If Tyler removes 2 then Noei removes 2.
   Now Tyler is looking at 4 mints and Noei can win.
3. If Tyler removes 3 then Noei removes 1.
   Now Tyler is looking at 4 mints and Noei can win.

What about 1,2,3,4 Mints?

Work on:

1. Can Tyler always win if the game begins with **0** mints?

What about 1,2,3,4 Mints?

Work on:

1. Can Tyler always win if the game begins with **0** mints?
   No, he can't move.
2. Can Tyler always win if the game begins with **1** mints?

What about 1,2,3,4 Mints?

Work on:

1. Can Tyler always win if the game begins with **0** mints?
   No, he can't move.
2. Can Tyler always win if the game begins with **1** mints?
   Yes-Remove 1.
3. Can Tyler always win if the game begins with **2** mints?

What about 1,2,3,4 Mints?

Work on:

1. Can Tyler always win if the game begins with **0** mints?
   No, he can't move.
2. Can Tyler always win if the game begins with **1** mints?
   Yes-Remove 1.
3. Can Tyler always win if the game begins with **2** mints?
   Yes-Remove 2
4. Can Tyler always win if the game begins with **3** mints?

What about 1,2,3,4 Mints?

Work on:

1. Can Tyler always win if the game begins with **0** mints?
   No, he can't move.
2. Can Tyler always win if the game begins with **1** mints?
   Yes-Remove 1.
3. Can Tyler always win if the game begins with **2** mints?
   Yes-Remove 2
4. Can Tyler always win if the game begins with **3** mints?
   Yes-Remove 3
5. Can Tyler always win if the game begins with **4** mints?

What about 1,2,3,4 Mints?

Work on:

1. Can Tyler always win if the game begins with **0** mints?
   No, he can't move.
2. Can Tyler always win if the game begins with **1** mints?
   Yes-Remove 1.
3. Can Tyler always win if the game begins with **2** mints?
   Yes-Remove 2
4. Can Tyler always win if the game begins with **3** mints?
   Yes-Remove 3
5. Can Tyler always win if the game begins with **4** mints?
   Noei wins-whatever. Tyler does, Noei can remove the rest of the mints.

Table of Who Wins

I means player I. Has been Tyler.
II means player II. Has been Noei.

Table of Who Wins

I means player I. Has been Tyler.
II means player II. Has been Noei.

Numb mints:	0	1	2	3	4	5	6	7	8
Who Wins:	II	I	I	I	II	I	I	I	II

Table of Who Wins

I means player I. Has been Tyler.
II means player II. Has been Noei.

Numb mints:	0	1	2	3	4	5	6	7	8
Who Wins:	II	I	I	I	II	I	I	I	II

What is the pattern?

Table of Who Wins

I means player I. Has been Tyler.
II means player II. Has been Noei.

Numb mints:	0	1	2	3	4	5	6	7	8
Who Wins:	II	I	I	I	II	I	I	I	II

What is the pattern?

Player I wins if the Numb of Mints IS NOT divisible by 4.
Player II wins if the Numb of Mints IS divisible by 4.

1-2-3-4 Moves, Game

Player I and II play a game:

1-2-3-4 Moves, Game

Player I and II play a game:

1. There are n **mints** on the table.

1-2-3-4 Moves, Game

Player I and II play a game:

1. There are n **mints** on the table.
2. Player I removes **1,2,3** OR **4** mints.

1-2-3-4 Moves, Game

Player I and II play a game:

1. There are n **mints** on the table.
2. Player I removes **1,2,3** OR **4** mints.
3. Player II removes **1,2,3** OR **4** mint.

1-2-3-4 Moves, Game

Player I and II play a game:

1. There are n **mints** on the table.
2. Player I removes **1,2,3** OR **4** mints.
3. Player II removes **1,2,3** OR **4** mint.
4. Player I then removes **1,2,3** OR **4** mint.

1-2-3-4 Moves, Game

Player I and II play a game:

1. There are n **mints** on the table.
2. Player I removes **1,2,3** OR **4** mints.
3. Player II removes **1,2,3** OR **4** mint.
4. Player I then removes **1,2,3** OR **4** mint.
5. They keep doing this.

1-2-3-4 Moves, Game

Player I and II play a game:

1. There are n **mints** on the table.
2. Player I removes **1,2,3** OR **4** mints.
3. Player II removes **1,2,3** OR **4** mint.
4. Player I then removes **1,2,3** OR **4** mint.
5. They keep doing this.
6. The person who removes the **last** mint wins.

1-2-3-4 Moves, Game

Player I and II play a game:

1. There are n **mints** on the table.
2. Player I removes **1,2,3** OR **4** mints.
3. Player II removes **1,2,3** OR **4** mint.
4. Player I then removes **1,2,3** OR **4** mint.
5. They keep doing this.
6. The person who removes the **last** mint wins.

Play the game and see if one of the players can **always win**.

1-2-3-4 Moves, Game

Player I and II play a game:

1. There are n **mints** on the table.
2. Player I removes **1,2,3** OR **4** mints.
3. Player II removes **1,2,3** OR **4** mint.
4. Player I then removes **1,2,3** OR **4** mint.
5. They keep doing this.
6. The person who removes the **last** mint wins.

Play the game and see if one of the players can **always win**.
Try to figure out:

When Player I wins.

When Player II wins.

1-4-5 Game

1-4-5 Game

1. 0 mints:

1-4-5 Game

1. 0 mints: II wins–I can't move.

1-4-5 Game

1. 0 mints: II wins–I can't move.
2. 1 mint:

1-4-5 Game

1. 0 mints: II wins–I can't move.
2. 1 mint: I wins– Remove 1

1-4-5 Game

1. 0 mints: II wins–I can't move.
2. 1 mint: I wins– Remove 1
3. 2 mints:

1-4-5 Game

1. 0 mints: II wins–I can't move.
2. 1 mint: I wins– Remove 1
3. 2 mints: II wins– I cannot get to 0.

1-4-5 Game

1. 0 mints: II wins–I can't move.
2. 1 mint: I wins– Remove 1
3. 2 mints: II wins– I cannot get to 0.
4. 3 mints:

1-4-5 Game

1. 0 mints: II wins–I can't move.
2. 1 mint: I wins– Remove 1
3. 2 mints: II wins– I cannot get to 0.
4. 3 mints: I wins– Remove 1.

1-4-5 Game

1. 0 mints: II wins–I can't move.
2. 1 mint: I wins– Remove 1
3. 2 mints: II wins– I cannot get to 0.
4. 3 mints: I wins– Remove 1.
5. 4 mints:

1-4-5 Game

1. 0 mints: II wins–I can't move.
2. 1 mint: I wins– Remove 1
3. 2 mints: II wins– I cannot get to 0.
4. 3 mints: I wins– Remove 1.
5. 4 mints: I wins– Remove 4.

1-4-5 Game

1. 0 mints: II wins–I can't move.
2. 1 mint: I wins– Remove 1
3. 2 mints: II wins– I cannot get to 0.
4. 3 mints: I wins– Remove 1.
5. 4 mints: I wins– Remove 4.
6. 5 mints:

1-4-5 Game

1. 0 mints: II wins–I can't move.
2. 1 mint: I wins– Remove 1
3. 2 mints: II wins– I cannot get to 0.
4. 3 mints: I wins– Remove 1.
5. 4 mints: I wins– Remove 4.
6. 5 mints: I wins– Remove 5.

1-4-5 Game

1. 0 mints: II wins–I can't move.
2. 1 mint: I wins– Remove 1
3. 2 mints: II wins– I cannot get to 0.
4. 3 mints: I wins– Remove 1.
5. 4 mints: I wins– Remove 4.
6. 5 mints: I wins– Remove 5.
7. 6 mints:

1-4-5 Game

1. 0 mints: II wins–I can't move.
2. 1 mint: I wins– Remove 1
3. 2 mints: II wins– I cannot get to 0.
4. 3 mints: I wins– Remove 1.
5. 4 mints: I wins– Remove 4.
6. 5 mints: I wins– Remove 5.
7. 6 mints: I wins– Remove 4.

1-4-5 Game

1. 0 mints: II wins–I can't move.
2. 1 mint: I wins– Remove 1
3. 2 mints: II wins– I cannot get to 0.
4. 3 mints: I wins– Remove 1.
5. 4 mints: I wins– Remove 4.
6. 5 mints: I wins– Remove 5.
7. 6 mints: I wins– Remove 4.
8. 7 mints:

1-4-5 Game

1. 0 mints: II wins–I can't move.
2. 1 mint: I wins– Remove 1
3. 2 mints: II wins– I cannot get to 0.
4. 3 mints: I wins– Remove 1.
5. 4 mints: I wins– Remove 4.
6. 5 mints: I wins– Remove 5.
7. 6 mints: I wins– Remove 4.
8. 7 mints: I wins– Remove 5.

1-4-5 Game

1. 0 mints: II wins–I can't move.
2. 1 mint: I wins– Remove 1
3. 2 mints: II wins– I cannot get to 0.
4. 3 mints: I wins– Remove 1.
5. 4 mints: I wins– Remove 4.
6. 5 mints: I wins– Remove 5.
7. 6 mints: I wins– Remove 4.
8. 7 mints: I wins– Remove 5.
9. 8 mints:

1-4-5 Game

1. 0 mints: II wins–I can't move.
2. 1 mint: I wins– Remove 1
3. 2 mints: II wins– I cannot get to 0.
4. 3 mints: I wins– Remove 1.
5. 4 mints: I wins– Remove 4.
6. 5 mints: I wins– Remove 5.
7. 6 mints: I wins– Remove 4.
8. 7 mints: I wins– Remove 5.
9. 8 mints: II wins– Cannot get to 0 or 2.

1-4-5 Game

1. 0 mints: II wins–I can't move.
2. 1 mint: I wins– Remove 1
3. 2 mints: II wins– I cannot get to 0.
4. 3 mints: I wins– Remove 1.
5. 4 mints: I wins– Remove 4.
6. 5 mints: I wins– Remove 5.
7. 6 mints: I wins– Remove 4.
8. 7 mints: I wins– Remove 5.
9. 8 mints: II wins– Cannot get to 0 or 2.

Mints:	0	1	2	3	4	5	6	7	8	9	10
Wins:	II	I	II	I	I	I	I	I	II	I	II

1-4-5 Game

1. 0 mints: II wins–I can't move.
2. 1 mint: I wins– Remove 1
3. 2 mints: II wins– I cannot get to 0.
4. 3 mints: I wins– Remove 1.
5. 4 mints: I wins– Remove 4.
6. 5 mints: I wins– Remove 5.
7. 6 mints: I wins– Remove 4.
8. 7 mints: I wins– Remove 5.
9. 8 mints: II wins– Cannot get to 0 or 2.

Mints:	0	1	2	3	4	5	6	7	8	9	10
Wins:	II	I	II	I	I	I	I	I	II	I	II

We want a way to describe this pattern.

Notation

For 1-2-3 Game we said

- Player I wins if n is not divisible by 4
- Player II wins if n is divisible by 4.

Notation

For 1-2-3 Game we said

- Player I wins if n is not divisible by 4
- Player II wins if n is divisible by 4.

We have a notation for this

Notation

For 1-2-3 Game we said

- Player I wins if n is not divisible by 4
- Player II wins if n is divisible by 4.

We have a notation for this

$$n \equiv 0 \pmod 4 \text{ pronounced "} n \text{ is congruent to 0 mod 4"}$$

Means that n is divisible by 4.

Notation

For 1-2-3 Game we said

- Player I wins if n is not divisible by 4
- Player II wins if n is divisible by 4.

We have a notation for this

$$n \equiv 0 \pmod 4 \text{ pronounced "}n\text{ is congruent to 0 mod 4"}$$

Means that n is divisible by 4.

- Player I wins if $n \not\equiv 0 \pmod 4$
- Player II wins if $n \equiv 0 \pmod 4$

Notation

For 1-3-4 Game we said

- Player I wins if when n is divisible by 7 get a remainder that is NOT 0 or 2.
- Player II wins if when divide n by 7 get a remainder of 0 or 2.

Notation

For 1-3-4 Game we said

- Player I wins if when n is divisible by 7 get a remainder that is NOT 0 or 2.
- Player II wins if when divide n by 7 get a remainder of 0 or 2.

We have a notation for this

Notation

For 1-3-4 Game we said

- Player I wins if when n is divisible by 7 get a remainder that is NOT 0 or 2.
- Player II wins if when divide n by 7 get a remainder of 0 or 2.

We have a notation for this

$$n \equiv 0 \pmod{7}$$

Means that n is divisible by 7.

Notation

For 1-3-4 Game we said

- Player I wins if when n is divisible by 7 get a remainder that is NOT 0 or 2.
- Player II wins if when divide n by 7 get a remainder of 0 or 2.

We have a notation for this

$$n \equiv 0 \pmod 7$$

Means that n is divisible by 7.

$$n \equiv 2 \pmod 7$$

Means that when you divide n by 7 the remainder is 2.

Notation

For 1-3-4 Game we said

- Player I wins if when n is divisible by 7 get a remainder that is NOT 0 or 2.
- Player II wins if when divide n by 7 get a remainder of 0 or 2.

We have a notation for this

$$n \equiv 0 \pmod{7}$$

Means that n is divisible by 7.

$$n \equiv 2 \pmod{7}$$

Means that when you divide n by 7 the remainder is 2.

- Player I wins if $n \equiv 1, 3, 5, 6 \pmod{7}$.

Notation

For 1-3-4 Game we said

- Player I wins if when n is divisible by 7 get a remainder that is NOT 0 or 2.
- Player II wins if when divide n by 7 get a remainder of 0 or 2.

We have a notation for this

$$n \equiv 0 \pmod{7}$$

Means that n is divisible by 7.

$$n \equiv 2 \pmod{7}$$

Means that when you divide n by 7 the remainder is 2.

- Player I wins if $n \equiv 1, 3, 5, 6 \pmod{7}$.
- Player II wins if $n \equiv 0, 2 \pmod{7}$

1-4-5

Mints:	0	1	2	3	4	5	6	7	8	9	10
Wins:	II	I	II	I	I	I	I	I	II	I	II

1-4-5

Mints:	0	1	2	3	4	5	6	7	8	9	10
Wins:	II	I	II	I	I	I	I	I	II	I	II

- Player I wins if

1-4-5

Mints:	0	1	2	3	4	5	6	7	8	9	10
Wins:	II	I	II	I	I	I	I	I	II	I	II

- Player I wins if $n \equiv 1, 3, 4, 5, 6, 7 \pmod 8$.

1-4-5

Mints:	0	1	2	3	4	5	6	7	8	9	10
Wins:	II	I	II	I	I	I	I	I	II	I	II

- Player I wins if $n \equiv 1, 3, 4, 5, 6, 7 \pmod 8$.
- Player II wins if

1-4-5

Mints:	0	1	2	3	4	5	6	7	8	9	10
Wins:	II	I	II	I	I	I	I	I	II	I	II

- Player I wins if $n \equiv 1, 3, 4, 5, 6, 7 \pmod 8$.
- Player II wins if $n \equiv 0, 2 \pmod 8$

Patterns of Patterns!

Recall:

Patterns of Patterns!

Recall:

1. 1-2-3 Game:

Patterns of Patterns!

Recall:

1. 1-2-3 Game:
    - Player I wins if

Patterns of Patterns!

Recall:

1. 1-2-3 Game:
   - Player I wins if $n \equiv 1, 2, 3 \pmod{4}$.

Patterns of Patterns!

Recall:

1. 1-2-3 Game:
   - Player I wins if $n \equiv 1, 2, 3 \pmod 4$.
   - Player II wins if

Patterns of Patterns!

Recall:

1. 1-2-3 Game:
   - Player I wins if $n \equiv 1, 2, 3 \pmod 4$.
   - Player II wins if $n \equiv 0 \pmod 4$

Patterns of Patterns!

Recall:

1. 1-2-3 Game:
   - Player I wins if $n \equiv 1, 2, 3 \pmod 4$.
   - Player II wins if $n \equiv 0 \pmod 4$
2. 1-2-3-4 Game:

Patterns of Patterns!

Recall:

1. 1-2-3 Game:
   - Player I wins if $n \equiv 1, 2, 3 \pmod 4$.
   - Player II wins if $n \equiv 0 \pmod 4$
2. 1-2-3-4 Game:
   - Player I wins if

Patterns of Patterns!

Recall:

1. 1-2-3 Game:
   - Player I wins if $n \equiv 1, 2, 3 \pmod 4$.
   - Player II wins if $n \equiv 0 \pmod 4$
2. 1-2-3-4 Game:
   - Player I wins if $n \equiv 1, 2, 3, 4 \pmod 5$.

Patterns of Patterns!

Recall:

1. 1-2-3 Game:
   - Player I wins if $n \equiv 1, 2, 3 \pmod 4$.
   - Player II wins if $n \equiv 0 \pmod 4$
2. 1-2-3-4 Game:
   - Player I wins if $n \equiv 1, 2, 3, 4 \pmod 5$.
   - Player II wins if

Patterns of Patterns!

Recall:

1. 1-2-3 Game:
   - Player I wins if $n \equiv 1, 2, 3 \pmod 4$.
   - Player II wins if $n \equiv 0 \pmod 4$
2. 1-2-3-4 Game:
   - Player I wins if $n \equiv 1, 2, 3, 4 \pmod 5$.
   - Player II wins if $n \equiv 0 \pmod 5$

Patterns of Patterns!

Recall:

1. 1-2-3 Game:
   - Player I wins if $n \equiv 1, 2, 3 \pmod 4$.
   - Player II wins if $n \equiv 0 \pmod 4$
2. 1-2-3-4 Game:
   - Player I wins if $n \equiv 1, 2, 3, 4 \pmod 5$.
   - Player II wins if $n \equiv 0 \pmod 5$
3. 1-2-3-4-5 Game:

Patterns of Patterns!

Recall:

1. 1-2-3 Game:
   - Player I wins if $n \equiv 1, 2, 3 \pmod 4$.
   - Player II wins if $n \equiv 0 \pmod 4$
2. 1-2-3-4 Game:
   - Player I wins if $n \equiv 1, 2, 3, 4 \pmod 5$.
   - Player II wins if $n \equiv 0 \pmod 5$
3. 1-2-3-4-5 Game:
   - Player I wins if

Patterns of Patterns!

Recall:

1. 1-2-3 Game:
   - Player I wins if $n \equiv 1, 2, 3 \pmod 4$.
   - Player II wins if $n \equiv 0 \pmod 4$
2. 1-2-3-4 Game:
   - Player I wins if $n \equiv 1, 2, 3, 4 \pmod 5$.
   - Player II wins if $n \equiv 0 \pmod 5$
3. 1-2-3-4-5 Game:
   - Player I wins if $n \equiv 1, 2, 3, 4, 5 \pmod 6$.

Patterns of Patterns!

Recall:

1. 1-2-3 Game:
   - Player I wins if $n \equiv 1, 2, 3 \pmod 4$.
   - Player II wins if $n \equiv 0 \pmod 4$
2. 1-2-3-4 Game:
   - Player I wins if $n \equiv 1, 2, 3, 4 \pmod 5$.
   - Player II wins if $n \equiv 0 \pmod 5$
3. 1-2-3-4-5 Game:
   - Player I wins if $n \equiv 1, 2, 3, 4, 5 \pmod 6$.
   - Player II wins if

Patterns of Patterns!

Recall:

1. 1-2-3 Game:
   - Player I wins if $n \equiv 1, 2, 3 \pmod 4$.
   - Player II wins if $n \equiv 0 \pmod 4$
2. 1-2-3-4 Game:
   - Player I wins if $n \equiv 1, 2, 3, 4 \pmod 5$.
   - Player II wins if $n \equiv 0 \pmod 5$
3. 1-2-3-4-5 Game:
   - Player I wins if $n \equiv 1, 2, 3, 4, 5 \pmod 6$.
   - Player II wins if $n \equiv 0 \pmod 6$
4. 1-2-3-$\cdots$-k Game:

Patterns of Patterns!

Recall:

1. 1-2-3 Game:
   - Player I wins if $n \equiv 1, 2, 3 \pmod 4$.
   - Player II wins if $n \equiv 0 \pmod 4$
2. 1-2-3-4 Game:
   - Player I wins if $n \equiv 1, 2, 3, 4 \pmod 5$.
   - Player II wins if $n \equiv 0 \pmod 5$
3. 1-2-3-4-5 Game:
   - Player I wins if $n \equiv 1, 2, 3, 4, 5 \pmod 6$.
   - Player II wins if $n \equiv 0 \pmod 6$
4. 1-2-3-$\cdots$-k Game:

Patterns of Patterns!

Recall:

1. 1-2-3 Game:
   - Player I wins if $n \equiv 1, 2, 3 \pmod 4$.
   - Player II wins if $n \equiv 0 \pmod 4$
2. 1-2-3-4 Game:
   - Player I wins if $n \equiv 1, 2, 3, 4 \pmod 5$.
   - Player II wins if $n \equiv 0 \pmod 5$
3. 1-2-3-4-5 Game:
   - Player I wins if $n \equiv 1, 2, 3, 4, 5 \pmod 6$.
   - Player II wins if $n \equiv 0 \pmod 6$
4. 1-2-3-$\cdots$-k Game:
   - Player I wins if

Patterns of Patterns!

Recall:

1. 1-2-3 Game:
   - Player I wins if $n \equiv 1, 2, 3 \pmod 4$.
   - Player II wins if $n \equiv 0 \pmod 4$
2. 1-2-3-4 Game:
   - Player I wins if $n \equiv 1, 2, 3, 4 \pmod 5$.
   - Player II wins if $n \equiv 0 \pmod 5$
3. 1-2-3-4-5 Game:
   - Player I wins if $n \equiv 1, 2, 3, 4, 5 \pmod 6$.
   - Player II wins if $n \equiv 0 \pmod 6$
4. 1-2-3-$\cdots$-k Game:
   - Player I wins if $n \equiv 1, 2, 3, , \ldots, k \pmod{k+1}$.

Patterns of Patterns!

Recall:

1. 1-2-3 Game:
   - Player I wins if $n \equiv 1, 2, 3 \pmod 4$.
   - Player II wins if $n \equiv 0 \pmod 4$
2. 1-2-3-4 Game:
   - Player I wins if $n \equiv 1, 2, 3, 4 \pmod 5$.
   - Player II wins if $n \equiv 0 \pmod 5$
3. 1-2-3-4-5 Game:
   - Player I wins if $n \equiv 1, 2, 3, 4, 5 \pmod 6$.
   - Player II wins if $n \equiv 0 \pmod 6$
4. 1-2-3-$\cdots$-k Game:
   - Player I wins if $n \equiv 1, 2, 3, , \ldots, k \pmod{k+1}$.
   - Player II wins if

Patterns of Patterns!

Recall:

1. 1-2-3 Game:
   - Player I wins if $n \equiv 1, 2, 3 \pmod 4$.
   - Player II wins if $n \equiv 0 \pmod 4$
2. 1-2-3-4 Game:
   - Player I wins if $n \equiv 1, 2, 3, 4 \pmod 5$.
   - Player II wins if $n \equiv 0 \pmod 5$
3. 1-2-3-4-5 Game:
   - Player I wins if $n \equiv 1, 2, 3, 4, 5 \pmod 6$.
   - Player II wins if $n \equiv 0 \pmod 6$
4. 1-2-3-$\cdots$-k Game:
   - Player I wins if $n \equiv 1, 2, 3, , \dots, k \pmod{k+1}$.
   - Player II wins if $n \equiv 0 \pmod{k+1}$

More Patterns of Patterns!

Recall:

More Patterns of Patterns!

Recall:

1. 1-3-4 Game:

More Patterns of Patterns!

Recall:

1. 1-3-4 Game:
   - Player I wins if

More Patterns of Patterns!

Recall:

1. 1-3-4 Game:
   - Player I wins if $n \equiv 1, 3, 4, 5, 6 \pmod{7}$.

More Patterns of Patterns!

Recall:

1. 1-3-4 Game:
   - Player I wins if $n \equiv 1, 3, 4, 5, 6 \pmod 7$.
   - Player II wins if

More Patterns of Patterns!

Recall:

1. 1-3-4 Game:
   - Player I wins if $n \equiv 1, 3, 4, 5, 6 \pmod 7$.
   - Player II wins if $n \equiv 0, 2 \pmod 7$

More Patterns of Patterns!

Recall:

1. 1-3-4 Game:
   - Player I wins if $n \equiv 1, 3, 4, 5, 6 \pmod 7$.
   - Player II wins if $n \equiv 0, 2 \pmod 7$
2. 1-4-5 Game:

More Patterns of Patterns!

Recall:

1. 1-3-4 Game:
   - Player I wins if $n \equiv 1, 3, 4, 5, 6 \pmod 7$.
   - Player II wins if $n \equiv 0, 2 \pmod 7$
2. 1-4-5 Game:
   - Player I wins if

More Patterns of Patterns!

Recall:

1. 1-3-4 Game:
   - Player I wins if $n \equiv 1, 3, 4, 5, 6 \pmod 7$.
   - Player II wins if $n \equiv 0, 2 \pmod 7$
2. 1-4-5 Game:
   - Player I wins if $n \equiv 1, 3, 4, 5, 6, 7 \pmod 8$.

More Patterns of Patterns!

Recall:

1. 1-3-4 Game:
   - Player I wins if $n \equiv 1, 3, 4, 5, 6 \pmod 7$.
   - Player II wins if $n \equiv 0, 2 \pmod 7$
2. 1-4-5 Game:
   - Player I wins if $n \equiv 1, 3, 4, 5, 6, 7 \pmod 8$.
   - Player II wins if

More Patterns of Patterns!

Recall:

1. 1-3-4 Game:
   - Player I wins if $n \equiv 1, 3, 4, 5, 6 \pmod 7$.
   - Player II wins if $n \equiv 0, 2 \pmod 7$
2. 1-4-5 Game:
   - Player I wins if $n \equiv 1, 3, 4, 5, 6, 7 \pmod 8$.
   - Player II wins if $n \equiv 0, 2 \pmod 8$)

More Patterns of Patterns!

Recall:

1. 1-3-4 Game:
   - Player I wins if $n \equiv 1, 3, 4, 5, 6 \pmod 7$.
   - Player II wins if $n \equiv 0, 2 \pmod 7$
2. 1-4-5 Game:
   - Player I wins if $n \equiv 1, 3, 4, 5, 6, 7 \pmod 8$.
   - Player II wins if $n \equiv 0, 2 \pmod 8$
3. 1-5-6 Game: I leave this to you

More Patterns of Patterns!

Recall:

1. 1-3-4 Game:
   - Player I wins if $n \equiv 1, 3, 4, 5, 6 \pmod 7$.
   - Player II wins if $n \equiv 0, 2 \pmod 7$
2. 1-4-5 Game:
   - Player I wins if $n \equiv 1, 3, 4, 5, 6, 7 \pmod 8$.
   - Player II wins if $n \equiv 0, 2 \pmod 8$
3. 1-5-6 Game: I leave this to you
4. General Pattern: I leave this to you.

1-5-6

Mints:	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Wins:	II	I	II	I	II	I	I	I	I	I	I	II	I	II

1-5-6

Mints:	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Wins:	II	I	II	I	II	I	I	I	I	I	I	II	I	II

- Player I wins if

1-5-6

Mints:	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Wins:	II	I	II	I	II	I	I	I	I	I	I	II	I	II

- Player I wins if $n \equiv 1, 3, 5, 6, 7, 8, 9, 10 \pmod{11}$.

1-5-6

Mints:	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Wins:	II	I	II	I	II	I	I	I	I	I	I	II	I	II

- Player I wins if $n \equiv 1, 3, 5, 6, 7, 8, 9, 10 \pmod{11}$.
- Player II wins if

1-5-6

Mints:	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Wins:	II	I	II	I	II	I	I	I	I	I	I	II	I	II

- Player I wins if $n \equiv 1, 3, 5, 6, 7, 8, 9, 10 \pmod{11}$.
- Player II wins if $n \equiv 0, 2, 4 \pmod{11}$

1-6-7

Mints:	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Wins:	II	I	II	I	II	I	I	I	I	I	I	I	II	I	II

1-6-7

Mints:	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Wins:	II	I	II	I	II	I	I	I	I	I	I	I	II	I	II

- Player I wins if

1-6-7

Mints:	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Wins:	II	I	II	I	II	I	I	I	I	I	I	I	II	I	II

- Player I wins if $n \equiv 1, 3, 5, 6, 7, 8, 9, 10, 11 \pmod{11}$.

1-6-7

Mints:	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Wins:	II	I	II	I	II	I	I	I	I	I	I	I	II	I	II

- Player I wins if $n \equiv 1, 3, 5, 6, 7, 8, 9, 10, 11 \pmod{11}$.
- Player II wins if

1-6-7

Mints:	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Wins:	II	I	II	I	II	I	I	I	I	I	I	I	II	I	II

- Player I wins if $n \equiv 1, 3, 5, 6, 7, 8, 9, 10, 11 \pmod{11}$.
- Player II wins if $n \equiv 0, 2, 4 \pmod{12}$

Patterns for 1-a-$a+1$, a odd

1-3-4

Patterns for 1-a-$a+1$, a odd

1-3-4

- Player I wins if $n \not\equiv 0, 2 \pmod 7$.

Patterns for 1-a-$a+1$, a odd

1-3-4

- Player I wins if $n \not\equiv 0, 2 \pmod 7$.
- Player II wins if $n \equiv 0, 2 \pmod 7$

Patterns for 1-a-$a+1$, a odd

1-3-4

- Player I wins if $n \not\equiv 0, 2 \pmod 7$.
- Player II wins if $n \equiv 0, 2 \pmod 7$

1-5-6

Patterns for 1-a-$a+1$, a odd

1-3-4

- Player I wins if $n \not\equiv 0, 2 \pmod 7$.
- Player II wins if $n \equiv 0, 2 \pmod 7$

1-5-6

- Player I wins if

Patterns for 1-a-$a+1$, a odd

1-3-4

- Player I wins if $n \not\equiv 0, 2 \pmod 7$.
- Player II wins if $n \equiv 0, 2 \pmod 7$

1-5-6

- Player I wins if $n \not\equiv 0, 2, 4 \pmod{11}$.

Patterns for 1-a-$a+1$, a odd

1-3-4

- Player I wins if $n \not\equiv 0, 2 \pmod 7$.
- Player II wins if $n \equiv 0, 2 \pmod 7$

1-5-6

- Player I wins if $n \not\equiv 0, 2, 4 \pmod{11}$.
- Player II wins if

Patterns for 1-a-$a+1$, a odd

1-3-4

- Player I wins if $n \not\equiv 0, 2 \pmod 7$.
- Player II wins if $n \equiv 0, 2 \pmod 7$

1-5-6

- Player I wins if $n \not\equiv 0, 2, 4 \pmod{11}$.
- Player II wins if $n \equiv 0, 2, 4 \pmod{11}$

Patterns for 1-a-$a+1$, a odd

1-3-4

- Player I wins if $n \not\equiv 0, 2 \pmod 7$.
- Player II wins if $n \equiv 0, 2 \pmod 7$

1-5-6

- Player I wins if $n \not\equiv 0, 2, 4 \pmod{11}$.
- Player II wins if $n \equiv 0, 2, 4 \pmod{11}$

1-7-8

Patterns for 1-a-$a+1$, a odd

1-3-4

- Player I wins if $n \not\equiv 0, 2 \pmod 7$.
- Player II wins if $n \equiv 0, 2 \pmod 7$

1-5-6

- Player I wins if $n \not\equiv 0, 2, 4 \pmod{11}$.
- Player II wins if $n \equiv 0, 2, 4 \pmod{11}$

1-7-8

- Player I wins if

Patterns for 1-a-$a+1$, a odd

1-3-4

- Player I wins if $n \not\equiv 0, 2 \pmod 7$.
- Player II wins if $n \equiv 0, 2 \pmod 7$

1-5-6

- Player I wins if $n \not\equiv 0, 2, 4 \pmod{11}$.
- Player II wins if $n \equiv 0, 2, 4 \pmod{11}$

1-7-8

- Player I wins if $n \equiv 0, 2, 4, 6 \pmod{15}$.

Patterns for 1-a-$a+1$, a odd

1-3-4

- Player I wins if $n \not\equiv 0, 2 \pmod 7$.
- Player II wins if $n \equiv 0, 2 \pmod 7$

1-5-6

- Player I wins if $n \not\equiv 0, 2, 4 \pmod{11}$.
- Player II wins if $n \equiv 0, 2, 4 \pmod{11}$

1-7-8

- Player I wins if $n \equiv 0, 2, 4, 6 \pmod{15}$.
- Player II wins if

Patterns for 1-a-$a+1$, a odd

1-3-4

- Player I wins if $n \not\equiv 0, 2 \pmod 7$.
- Player II wins if $n \equiv 0, 2 \pmod 7$

1-5-6

- Player I wins if $n \not\equiv 0, 2, 4 \pmod{11}$.
- Player II wins if $n \equiv 0, 2, 4 \pmod{11}$

1-7-8

- Player I wins if $n \equiv 0, 2, 4, 6 \pmod{15}$.
- Player II wins if $n \equiv 0, 2, 4, 6 \pmod{15}$

Patterns for 1-a-$a+1$, a odd

1-3-4

- Player I wins if $n \not\equiv 0, 2 \pmod 7$.
- Player II wins if $n \equiv 0, 2 \pmod 7$

1-5-6

- Player I wins if $n \not\equiv 0, 2, 4 \pmod{11}$.
- Player II wins if $n \equiv 0, 2, 4 \pmod{11}$

1-7-8

- Player I wins if $n \equiv 0, 2, 4, 6 \pmod{15}$.
- Player II wins if $n \equiv 0, 2, 4, 6 \pmod{15}$

1-a-$a+1$ for a ODD

Patterns for 1-a-$a+1$, a odd

1-3-4

- Player I wins if $n \not\equiv 0, 2 \pmod 7$.
- Player II wins if $n \equiv 0, 2 \pmod 7$

1-5-6

- Player I wins if $n \not\equiv 0, 2, 4 \pmod{11}$.
- Player II wins if $n \equiv 0, 2, 4 \pmod{11}$

1-7-8

- Player I wins if $n \equiv 0, 2, 4, 6 \pmod{15}$.
- Player II wins if $n \equiv 0, 2, 4, 6 \pmod{15}$

1-a-$a+1$ for a ODD

- Player I wins if

Patterns for 1-a-$a+1$, a odd

1-3-4

- Player I wins if $n \not\equiv 0, 2 \pmod 7$.
- Player II wins if $n \equiv 0, 2 \pmod 7$

1-5-6

- Player I wins if $n \not\equiv 0, 2, 4 \pmod{11}$.
- Player II wins if $n \equiv 0, 2, 4 \pmod{11}$

1-7-8

- Player I wins if $n \equiv 0, 2, 4, 6 \pmod{15}$.
- Player II wins if $n \equiv 0, 2, 4, 6 \pmod{15}$

1-a-$a+1$ for a ODD

- Player I wins if $n \not\equiv 0, 2, \ldots, a-1 \pmod{2a-1}$.

Patterns for 1-a-$a+1$, a odd

1-3-4

- Player I wins if $n \not\equiv 0, 2 \pmod 7$.
- Player II wins if $n \equiv 0, 2 \pmod 7$

1-5-6

- Player I wins if $n \not\equiv 0, 2, 4 \pmod{11}$.
- Player II wins if $n \equiv 0, 2, 4 \pmod{11}$

1-7-8

- Player I wins if $n \equiv 0, 2, 4, 6 \pmod{15}$.
- Player II wins if $n \equiv 0, 2, 4, 6 \pmod{15}$

1-a-$a+1$ for a ODD

- Player I wins if $n \not\equiv 0, 2, \dots, a-1 \pmod{2a-1}$.
- Player II wins if

Patterns for 1-a-$a+1$, a odd

1-3-4

- Player I wins if $n \not\equiv 0, 2 \pmod 7$.
- Player II wins if $n \equiv 0, 2 \pmod 7$

1-5-6

- Player I wins if $n \not\equiv 0, 2, 4 \pmod{11}$.
- Player II wins if $n \equiv 0, 2, 4 \pmod{11}$

1-7-8

- Player I wins if $n \equiv 0, 2, 4, 6 \pmod{15}$.
- Player II wins if $n \equiv 0, 2, 4, 6 \pmod{15}$

1-a-$a+1$ for a ODD

- Player I wins if $n \not\equiv 0, 2, \ldots, a-1 \pmod{2a-1}$.
- Player II wins if $n \equiv 0, 2, \ldots, a-1 \pmod{2a-1}$

Patterns for 1-a-$a+1$, a even

1-2-3

Patterns for 1-a-$a+1$, a even

1-2-3

- Player I wins if $n \not\equiv 0 \pmod{4}$.

Patterns for 1-a-$a+1$, a even

1-2-3

- Player I wins if $n \not\equiv 0 \pmod 4$.
- Player II wins if $n \equiv 0 \pmod 4$)

Patterns for 1-a-$a+1$, a even

1-2-3

- Player I wins if $n \not\equiv 0 \pmod 4$.
- Player II wins if $n \equiv 0 \pmod 4$

1-4-5

Patterns for 1-a-$a+1$, a even

1-2-3

- Player I wins if $n \not\equiv 0 \pmod 4$.
- Player II wins if $n \equiv 0 \pmod 4$

1-4-5

- Player I wins if

Patterns for 1-a-$a+1$, a even

1-2-3

- Player I wins if $n \not\equiv 0 \pmod 4$.
- Player II wins if $n \equiv 0 \pmod 4$

1-4-5

- Player I wins if $n \not\equiv 0, 2 \pmod 8$.

Patterns for 1-a-$a+1$, a even

1-2-3

- Player I wins if $n \not\equiv 0 \pmod 4$.
- Player II wins if $n \equiv 0 \pmod 4$

1-4-5

- Player I wins if $n \not\equiv 0, 2 \pmod 8$.
- Player II wins if

Patterns for 1-a-$a+1$, a even

1-2-3

- Player I wins if $n \not\equiv 0 \pmod 4$.
- Player II wins if $n \equiv 0 \pmod 4$

1-4-5

- Player I wins if $n \not\equiv 0, 2 \pmod 8$.
- Player II wins if $n \equiv 0, 2 \pmod 8$

Patterns for 1-a-$a+1$, a even

1-2-3

- Player I wins if $n \not\equiv 0 \pmod 4$.
- Player II wins if $n \equiv 0 \pmod 4$

1-4-5

- Player I wins if $n \not\equiv 0, 2 \pmod 8$.
- Player II wins if $n \equiv 0, 2 \pmod 8$

1-6-7

Patterns for 1-a-$a+1$, a even

1-2-3

- Player I wins if $n \not\equiv 0 \pmod 4$.
- Player II wins if $n \equiv 0 \pmod 4$

1-4-5

- Player I wins if $n \not\equiv 0, 2 \pmod 8$.
- Player II wins if $n \equiv 0, 2 \pmod 8$

1-6-7

- Player I wins if

Patterns for 1-a-$a+1$, a even

1-2-3

- Player I wins if $n \not\equiv 0 \pmod 4$.
- Player II wins if $n \equiv 0 \pmod 4$

1-4-5

- Player I wins if $n \not\equiv 0, 2 \pmod 8$.
- Player II wins if $n \equiv 0, 2 \pmod 8$

1-6-7

- Player I wins if $n \equiv 0, 2, 4 \pmod{12}$.

Patterns for 1-a-$a+1$, a even

1-2-3

- Player I wins if $n \not\equiv 0 \pmod 4$.
- Player II wins if $n \equiv 0 \pmod 4$

1-4-5

- Player I wins if $n \not\equiv 0, 2 \pmod 8$.
- Player II wins if $n \equiv 0, 2 \pmod 8$

1-6-7

- Player I wins if $n \equiv 0, 2, 4 \pmod{12}$.
- Player II wins if

Patterns for 1-a-$a+1$, a even

1-2-3

- Player I wins if $n \not\equiv 0 \pmod 4$.
- Player II wins if $n \equiv 0 \pmod 4$

1-4-5

- Player I wins if $n \not\equiv 0, 2 \pmod 8$.
- Player II wins if $n \equiv 0, 2 \pmod 8$

1-6-7

- Player I wins if $n \equiv 0, 2, 4 \pmod{12}$.
- Player II wins if $n \equiv 0, 2, 4 \pmod{12}$

Patterns for 1-a-$a+1$, a even

1-2-3

- Player I wins if $n \not\equiv 0 \pmod 4$.
- Player II wins if $n \equiv 0 \pmod 4$

1-4-5

- Player I wins if $n \not\equiv 0, 2 \pmod 8$.
- Player II wins if $n \equiv 0, 2 \pmod 8$

1-6-7

- Player I wins if $n \equiv 0, 2, 4 \pmod{12}$.
- Player II wins if $n \equiv 0, 2, 4 \pmod{12}$

1-a-$a+1$ for a even

Patterns for 1-a-$a+1$, a even

1-2-3

- Player I wins if $n \not\equiv 0 \pmod 4$.
- Player II wins if $n \equiv 0 \pmod 4$

1-4-5

- Player I wins if $n \not\equiv 0, 2 \pmod 8$.
- Player II wins if $n \equiv 0, 2 \pmod 8$

1-6-7

- Player I wins if $n \equiv 0, 2, 4 \pmod{12}$.
- Player II wins if $n \equiv 0, 2, 4 \pmod{12}$

1-a-$a+1$ for a even

- Player I wins if

Patterns for 1-a-$a+1$, a even

1-2-3

- Player I wins if $n \not\equiv 0 \pmod 4$.
- Player II wins if $n \equiv 0 \pmod 4$

1-4-5

- Player I wins if $n \not\equiv 0, 2 \pmod 8$.
- Player II wins if $n \equiv 0, 2 \pmod 8$

1-6-7

- Player I wins if $n \equiv 0, 2, 4 \pmod{12}$.
- Player II wins if $n \equiv 0, 2, 4 \pmod{12}$

1-a-$a+1$ for a even

- Player I wins if $n \not\equiv 0, 2, \ldots, a-2 \pmod{2a}$.

Patterns for 1-a-$a+1$, a even

1-2-3

- Player I wins if $n \not\equiv 0 \pmod 4$.
- Player II wins if $n \equiv 0 \pmod 4$

1-4-5

- Player I wins if $n \not\equiv 0, 2 \pmod 8$.
- Player II wins if $n \equiv 0, 2 \pmod 8$

1-6-7

- Player I wins if $n \equiv 0, 2, 4 \pmod{12}$.
- Player II wins if $n \equiv 0, 2, 4 \pmod{12}$

1-a-$a+1$ for a even

- Player I wins if $n \not\equiv 0, 2, \ldots, a-2 \pmod{2a}$.
- Player II wins if

Patterns for 1-a-$a+1$, a even

1-2-3

- Player I wins if $n \not\equiv 0 \pmod 4$.
- Player II wins if $n \equiv 0 \pmod 4$

1-4-5

- Player I wins if $n \not\equiv 0, 2 \pmod 8$.
- Player II wins if $n \equiv 0, 2 \pmod 8$

1-6-7

- Player I wins if $n \equiv 0, 2, 4 \pmod{12}$.
- Player II wins if $n \equiv 0, 2, 4 \pmod{12}$

1-a-$a+1$ for a even

- Player I wins if $n \not\equiv 0, 2, \ldots, a-2 \pmod{2a}$.
- Player II wins if $n \equiv 0, 2, \ldots, a-2 \pmod{2a}$