Now by Theorem 1 we see that $m = 1$. Hence $o(R-0,\cdot) = p$ implies that $(R-0,\cdot)$ is cyclic and thus a commutative group.

Finally we need to show that the right distributive law holds in R. But this follows easily from the fact that $(R,\cdot)$ is commutative. Hence R is a field.

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ON THE EXISTENCE OF ABSOLUTE PRIMES

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An absolute prime base ten is a prime number such that every permutation of its digits is also prime. Our method of establishing the existence of nontrivial examples consists of utilizing a computer program search among the first million positive integers. We present a list of those found in this range. We believe that the list may not be exhaustive even for primes that involve only two distinct digits in their presentation. Indeed the problem of existence of absolute primes that employ just two distinct digits could be, to the best of our knowledge, as difficult as the enumeration of the Mersenne primes.

An absolute prime with more than one digit can employ only 1, 3, 7, or 9 as digits. We exhibit the list of absolute primes we found, and we prove that no absolute prime may utilize all four of the numbers 1, 3, 7, 9 as digits. We wish to thank the referee for a thorough study of our first draft.

We found the following absolute primes. They are: 2, 3, 5, 7, 11, 13, 17, 31, 37, 71, 73, 79, 97, 113, 131, 199, 311, 337, 373, 733, 919, 991. Note that twelve on this list have no repeated digits.

THEOREM. *There exists no absolute prime utilizing all four digits* 1, 3, 7, *and* 9.

Proof. Upon division by 7 the numbers 1379, 1793, 3719, 7913, 7193, 3197, and 7139 have remainders 0, 1, 2, 3, 4, 5, and 6, respectively. Hence, an integer N having all these digits in it may be permuted into $N_1 = K + 1379$. If one divides K by 7, let $K = 7q + r$ $(0 \leqq r < 7)$. Then 1379 may be permuted so that its remainder is $7 - r$ upon division by 7 and so N may be permuted to the form

$$N_2 = 7q + r + 7r + (7 - r)$$

and so 7 divides N_2.