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Restricted Ramsey Configurations

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1. BACKGROUND AND NOTATION

In recent years there has been great development in the area becoming known as Ramsey Theory. In this paper we explore some restricted Ramsey theorems, best illustrated by the following beautiful result of Nešetřil and Rödl.

We begin with the first nontrivial example of the original theorem of Ramsey: Given any two-coloring of the edges of K_6 there exists a monochromatic triangle. Let us use the notation $H \to (G)_c$ if any c-coloring of the edges of a graph H yields a monochromatic G. In this instance we write $K_6 \to (K_3)_2$. It was asked, by P. Erdös, what graphs H have the property $H \to (K_3)_2$, and whether such H exist if you restrict the clique number $w(H)$. In an elegant, though complex, paper Folkman [4] showed that there exist H satisfying $H \to (K_3)_2$, where $w(H) = 3$. A full generalization, using a totally different method, was given by Nešetřil and Rödl [6], who showed that for all G, c there exists a graph H so that $H \to (G)_c$ and $w(H) = w(G)$.

Notation.

$$[n] = \{1, 2, \ldots, n\},$$

$$[A]^s = \{B: B \subseteq A, \mid B \mid = s\},$$

$$[n]^s = \{B: B \subseteq [n], \mid B \mid = s\}.$$

If $\mathscr{F}$ is a family of sets, $\chi(\mathscr{F})$ = chromatic number of $\mathscr{F}$ = minimal c such that $U\mathscr{F}$ may be c-colored so that no $A \in \mathscr{F}$ is monochromatic.

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2. RESTRICTED VAN DER WAERDEN CONFIGURATIONS

In this section we prove a result on Van der Waerden's theorem analogous to the result of Nešetřil and Rödl. We recall the basic result:

VAN DER WAERDEN'S THEOREM [7]. *Let k, c be given positive integers. Then there is a positive integer $n = n(k, c)$ such that given any c-coloring of the set $[n]$ there exists a monochromatic arithmetic progression (m.a.p.) of k elements.*

We define a set of integers A to be a V_{kc} set (or V-set where k, c are understood) if any c-coloring of A yields a m.a.p. of size k.

THEOREM 1 (restricted Van der Waerden configuration). *For all k, c there exists a V-set A such that A contains no arithmetic progression of length $k + 1$.*

Proof. By the Hales–Jewett theorem [5] there is an integer n so that if the n-dimensional cube k^n (i.e., the set of points $(x_1, \ldots, x_n)$ $x_i = 0, 1, \ldots, k-1$) is c-colored, there must be a monochromatic line. Now let p be a prime, $p > k$ and set

$$A = \{a_0 + a_1 p + \cdots + a_{n-1} p^{n-1}, 0 \leqslant a_i < k\}.$$

(i) A is a V-set. Correspond A to k^n by $a_0 + \cdots + a_{n-1}p^{n-1} \leftrightarrow (a_0, \ldots, a_{n-1})$. The c-coloring of A corresponds to a c-coloring of k^n for which there is a monochromatic line which, in A, is an arithmetic progression.

(ii) A contains no arithmetic progression of length $k + 1$. For suppose $x, x + d, \ldots, x + kd$ were in A. Let $d = d_i p^i + d_{i+1} p^{i+1} + \cdots$ with $d_i \neq 0$ and $x = \sum x_j p^j$. Then, looking at the p^i coefficient in the base p expansion of $x + sd$, we see that $x_i + sd_i$ is equal to $0, 1, \ldots,$ or $k - 1$ modulo p for $0 \leqslant s \leqslant k$. However, p is prime and $d_i \not\equiv 0 \pmod{p}$ so the $x_i + sd_i$ are distinct modulo p, a contradiction. Q.E.D.

3. INDUCED VAN DER WAERDEN THEOREM

Let G, H be graphs. We write $G \xrightarrow{i} (H)_c$ if whenever the edges of G are c-colored there exists a monochromatic induced subgraph H. That is, there exists a color i and a subset of vertices V so that all edges on V are color i and those edges form the graph H. For all H, c it is known [1, 6] that there exists G so that $G \xrightarrow{i} (H)_c$. In this section we show the analogous result for Van der Waerden's theorem.

THEOREM 2 (induced Van der Waerden theorem). *Let $e_0, \dots, e_{k-1} = 0$ or 1. For all c there exists a set A so that if A is c-colored there is an arithmetic progression of integers $\beta_0, \dots, \beta_{k-1}$ such that $\beta_i \notin A$ iff $e_i = 0$ and $\{\beta_i : e_i = 1\}$ is monochromatic.*

Essentially, we have in some color i an induced "pattern" given by $e_0, \dots, e_{k-1}$.

Proof. The theorem is trivial if none or one of the $e_i = 1$. Now let $i_0 < \cdots < i_{t-1}$ be those indices with $e_{i_j} = 1$. Let $S = \{i_0, \dots, i_{t-1}\}$. Define $f: S \to \{0, \dots, t-1\}$ by $f(i_r) = r$. By the Hales–Jewett theorem we find n so that every c-coloring of the n-dimensional cube t^n yields a monochromatic "special line." Here "special line" is a set $\mathbf{x}_0, \dots, \mathbf{x}_{t-1}$, with $\mathbf{x}_i = (x_{i1}, \dots, x_{in})$ so that for all j either $x_{0j} = x_{ij} = \cdots = x_{t-1,j}$ or $x_{0j} = 0, x_{1j} = 1, \dots, x_{t-1,j} = t-1$. (For example, in 3^2, (2, 0), (1, 1), (0, 2) is not a special line.) Now set

$$A = \{a_0 + a_1 k + \cdots + a_{n-1}k^{n-1} : a_i \in S\}.$$

We associate A with t^n by

$$a_0 + \cdots + a_{n-1}k^{n-1} \leftrightarrow (f(a_0), \dots, f(a_{n-1})).$$

A c-coloring of A induces a c-coloring of t^n which contains a monochromatic special line $\mathbf{x}_0, \dots, \mathbf{x}_{t-1}$. These correspond to $\alpha_0, \dots, \alpha_{t-1} \in A$, $\alpha_m = \sum_{j=0}^{n-1} a_{mj}k^j$, where for all j either $a_{0j} = \cdots = a_{t-1,j}$ or $a_{mj} = i_m$. Let T denote the set of coordinates j on which $a_{0j} = \cdots = a_{t-1,j}$. We extend $\alpha_0, \dots, \alpha_{t-1}$ to the arithmetic progression $\beta_0, \beta_1, \dots, \beta_{k-1}$, $\beta_m = \sum_{j=0}^{n-1} b_{mj}k^j$ defined as follows. For $j \in T$, $b_{0j} = b_{1j} = \cdots = b_{n-1,j} = a_{0j}$. For $j \notin T$, $b_{mj} = m$, $0 \leqslant m \leqslant k-1$. The sequence $\beta_0, \dots, \beta_{k-1}$ is the desired arithmetic progression.

4. RAMSEY FAMILIES

DEFINITION. Let $\mathfrak{A}$ be a family of sets, $U\mathfrak{A} = V$, $c \geqslant 2$. $\mathfrak{A}$ is called a c-Ramsey Family if given any c-coloring of $[V]^2$ there exists $A \in \mathfrak{A}$ so that $[A]^2$ is monochromatic. (We note that $\mathfrak{A}$ is c-Ramsey iff $\chi(\{[A]^2 : A \in \mathfrak{A}\}) > c$.) If $K_n \to (K_k)_c$ then $\mathfrak{A} = [n]^k$ is a c-Ramsey Family. This example might seem to indicate that c-Ramsey Families have their elements clustered together. Our theorem, however, is in the opposite direction.

THEOREM 3. *For all k, c there exists a c-Ramsey Family $\mathfrak{A}$ such that all $A \in \mathfrak{A}$ have $|A| = k$ and $A, B \in \mathfrak{A} \to |A \cap B| < 2$.*

Note that the "2" is best possible as if for all $A, B \in \mathfrak{A}$, $|A \cap B| \leqslant 1$ then the sets $[A]^2$ would be disjoint so that $\mathfrak{A}$ would not be c-Ramsey even for $c = 2$. We prove this result for $c = 2$, the general case being nearly identical. We begin with a result on the number of monochromatic K_k. By Ramsey's theorem there exists m so that $K_m \to (K_k)_2$. Now let $n \geqslant m$.

Color K_n arbitrarily with two colors and let $\mathscr{C} = \{A \in [n]^k : [A]^2$ is monochromatic$\}$. Every $B \in [n]^m$ contains some $C \in \mathscr{C}$. There are $\binom{n}{m}$ sets in $[m]^n$ and each $C \in \mathscr{C}$ may be covered by at most $\binom{n-k}{m-k}$ of them so

$$|\mathscr{C}| \geqslant \binom{n}{m} \Big/ \binom{n-k}{m-k} = \binom{n}{k} \Big/ \binom{m}{k} = \alpha \binom{n}{k},$$

where $\alpha = \binom{m}{k}^{-1}$ is a positive number, independent of n.

Now we apply the probabilistic method as used by Erdös in [2] (or Erdös and Spencer [3]). The reader is forewarned that this method involves quite crude asymptotic analysis. A number of constants (2, 5) are picked merely to be specific; they by no means give the best proof. We make no attempt here to find the smallest $n = n(k, c)$ such that the desired $\mathfrak{A} \subseteq [n]^k$ exists.

Fix $\epsilon > 0$, small ($\epsilon = 0.1$ will do). Let n be "sufficiently large," so that the approximations we shall make will be accurate. Let $\mathfrak{A}$ be a *random* subset of $[n]^k$ where for each $A \in [n]^k$,

$$P[A \in \mathfrak{A}] = p = n^{2+\epsilon-k}, \qquad (1)$$

and these probabilities are mutually independent. More formally, we could define a probability space whose elements were subsets of $[n]^k$ (possible $\mathfrak{A}$'s) and with probabilities generated by (1). Call (A_1, A_2) an intersecting pair if $A_1, A_2 \in [n]^k$, $|A_1 \cap A_2| \geqslant 3$, $A_1 \neq A_2$. For any family $\mathfrak{A}$ let $I(\mathfrak{A})$ denote the number of intersecting pairs (A_1, A_2), $A_1, A_2 \in \mathfrak{A}$. For any fixed intersecting pair (A_1, A_2),

$$\text{Prob}[A_1, A_2 \in \mathfrak{A}] = p^2 = n^{4+2\epsilon-2k}.$$

There are at most $\binom{n}{k}\binom{k}{3}\binom{n}{k-3} \leqslant n^{2k-3}$ intersecting pairs. By the linearity of expected value,

$$E[I(\mathfrak{A})] = (\#\text{ of intersecting pairs}) \cdot p^2 \leqslant n^{1+2\epsilon}. \qquad (2)$$

(This seems to be not what we want, as we need to find $\mathfrak{A}$ with no intersecting pairs, but it is only the first step.) Let

$$C_i, 1 \leqslant i \leqslant 2^{\binom{n}{2}}$$

denote the possible 2-colorings of $[n]^2$. For each i let $D_i \subseteq [n]^k$ denote the family of k-sets S such that $[S]^2$ is monochromatic under C_i. We know

$$|D_i| \geqslant \alpha \binom{n}{k}, \qquad 1 \leqslant i \leqslant 2^{\binom{n}{2}}.$$

For each i,

$$\text{Prob}\,[D_i \cap \mathcal{A} = \varnothing] = (1-p)^{|D_i|} \leqslant (1-p)^{\alpha\binom{n}{k}},$$

which is very small. We wish to find $\mathcal{A}$ so that $D_i \cap \mathcal{A} \neq \varnothing$ for all i.

We actually find $\mathcal{A}$ so that $|D_i \cap \mathcal{A}|$ is "large" for all i. (This stronger result will be necessary since (2) is weaker than desired.) For fixed i, $|D_i \cap \mathcal{A}|$ is a random variable with binomial distribution $B(|D_i|, p)$. It is "smallest" where $|D_i|$ is minimal. We calculate, for fixed i,

$$\text{Prob}\,[|D_i \cap \mathcal{A}| < 5n^{1+2\epsilon}] \leqslant \text{Prob}\left[B\left(\alpha\binom{n}{k}, p\right) \leqslant 5n^{1+2\epsilon}\right]$$

$$\leqslant \binom{\alpha\binom{n}{k}}{5n^{1+2\epsilon}}(1-p)^{\alpha\binom{n}{k}-5n^{1+2\epsilon}}.$$

We make the gross approximations

$$\binom{\alpha\binom{n}{k}}{5n^{1+2\epsilon}} \leqslant (n^k)^{5n^{1+2\epsilon}} = \exp n^{1+2\epsilon+o(1)},$$

$$(1-p)^{\alpha\binom{n}{k}-5n^{1+2\epsilon}} \sim \exp\left[-p\alpha\binom{n}{k}\right] = \exp[-n^{+2+\epsilon+o(1)}].$$

So

$$\text{Prob}[|D_i \cap \mathcal{A}| \leqslant 5n^{1+2\epsilon}] < \exp[-n^{2+\epsilon+o(1)}].$$

Our next step makes clear why such "infinitesimal" probabilities were necessary.

$$\text{Prob[for some } i, |D_i \cap \mathcal{A}| \leqslant 5n^{1+2\epsilon}] \leqslant \sum_{i=1}^{2^{\binom{n}{2}}} \text{Prob}[|D_i \cap \mathcal{A}| \leqslant 5n^{1+2\epsilon}]$$

$$\leqslant 2^{\binom{n}{2}} \exp[-n^{2+\epsilon+o(1)}]. = o(1)$$

For n sufficiently large,

$$\text{Prob}[|\mathcal{A} \cap D_i| > 5n^{1+2\epsilon} \text{ for all } i] > 0.9.$$

From (2),

$$\text{Prob}[I(\mathcal{A}) \leqslant 2n^{1+2\epsilon}] > 0.5.$$

We may, therefore, find a specific $\mathcal{A}_0$ such that

(i) For any coloring C_i of $[n]^2$ there are at least $5n^{1+2\epsilon}$ monochromatic $S \in \mathcal{A}_0$.

(ii) There are at most $2n^{1+2\epsilon}$ intersecting pairs in $\mathcal{A}_0$.

Select a set $\mathcal{A}_1 \subseteq [n]^k$ containing at least one member from each intersecting pair in $\mathcal{A}_0$. Clearly we may find $\mathcal{A}_1$ of cardinality $\leqslant 2n^{1+2\epsilon}$. Set $\mathcal{A}^* = \mathcal{A}_0 - \mathcal{A}_1$. Then

(ii′) $\mathcal{A}^*$ has no intersecting pairs. (As all the pairs in $\mathcal{A}_0$ have been "broken up").

(i′) For any coloring C_i of $[n]^2$ there is a monochromatic $S \in \mathcal{A}^*$ (as there were $5n^{1+2\epsilon}$ such S and at most $2n^{1+2\epsilon}$ have been eliminated.)

$\mathcal{A}^*$ is the desired 2-Ramsey Family. Q.E.D.

A strong result, analogous to [2], can also be shown by the same method. We say $\{A_1, \ldots, A_t\}$, $A_i \in [n]^k$ is a *t-cycle* if

$$\left|\bigcup_{i=1}^{t} A_i\right| < k + (t-1)(k-2).$$

(A 2-cycle is, in our notation, an intersecting pair.)

THEOREM 4. *For all k, c, t if n is sufficiently large there exists a k-family $\mathcal{A} \subseteq [n]^k$ which is c-Ramsey but contains no s-cycles, $2 \leqslant s \leqslant t$.*

(We omit the proof, as it follows the lines of Theorem 5.)

The results of Nešetřil and Rödl plus Theorem 4 lead us to pose the following

Question 1. For all k, c, t does there exist for some n a graph $H \subseteq [n]^2$ such that $\{A\colon A \in [n]^k, [A]^2 \subseteq H\}$ is c-Ramsey with no s-cycles, $s \leqslant t$?

The most important case is $c = t = 2$. We may rephrase the question as follows.

Question 1′. For all k is there a graph H such that $H \to (K_k)_2$ and yet H does not contain two complete subgraphs on k vertices with more than two points in common?

5. Van der Waerden Families

Let $k \leqslant n$ be positive integers. We define

$$S = S_{kn} = \{A \in [n]^k\colon A \text{ an arithmetic progression}\}.$$

Van der Waerden's theorem may be phrased as follows. For all k, c if n is sufficiently large the hypergraph S_{kn} has chromatic number $> c$. Let $\mathcal{A} \subseteq S$. We say $\mathcal{A}$ is a c-Van der Waerden family of arithmetic progressions if the hypergraph $\mathcal{A}$ has chromatic number $> c$. From Van der Waerden's theorem for all k, c there exist such $\mathcal{A}$, namely one may take $\mathcal{A} = S_{kn}$ for n sufficiently large.

Definition. A set $\{A_1, \ldots, A_t\} \subseteq S_{kn}$ is called a t-cycle if

$$\left| \bigcup_{i=1}^{t} A_i \right| \leqslant k + (t-1)(k-1).$$

(This is *different* from the definition in Section 4. Here we deal with vertex colorings whereas in Section 4 we were interested in edge colorings.)

Theorem 5. *For all k, c, t there exists n and a family $\mathcal{A} \subseteq S_{kn}$ such that $\mathcal{A}$ is c-Van der Waerden and yet $\mathcal{A}$ contains no s-cycles for $s \leqslant t$.*

We only indicate the proof as it follows the lines of Theorem 3. We let n be "sufficiently large" and $\mathcal{A}$ be a random subset of S_{kn} where each $A \in S_{kn}$ is in $\mathcal{A}$ with probability $p = n^{-1+\epsilon}$. Here ϵ is independent of n, $0 < \epsilon < 1/t$. By a simple counting argument S_{kn} has Cn^t t-cycles. (The C's are constants, not necessarily equal, independent of n.) The expected number of t-cycles in $\mathcal{A}$ is at most $(Cn^t)\,p^t = Cn^{\epsilon t} = o(n)$. The s-cycles, $s < t$, are also few in expected number.

Let m be the "Van der Waerden number" such that any c-coloring of $[m]$ yields a monochromatic arithmetic progression of length k. The family S_{kn} of all k-element arithmetic progressions contains $C_1 n^2$ sets. Color $[n]$ arbitrarily with c colors and let $\mathscr{C}$ be the family of monochromatic $A \in S_{kn}$. Every arithmetic progression of length m contains at least one $A \in \mathscr{C}$; an $A \in \mathscr{C}$ may be extended to an arithmetic progression of length m in at most C_2 ways; hence $|\mathscr{C}| \geqslant C_3 n^2$. For the c^n possible colorings of $[n]$ let D_i be the set of monochromatic arithmetic progressions in the ith coloring. The random variable $|D_i \cap \mathcal{A}|$ has the binomial distribution $B(|D_i|, p)$ so (after some calculation)

$$\operatorname{Prob}[|D_i \cap \mathcal{A}| < n] < \exp[-n^{1+\epsilon+o(1)}].$$

As there are "only" c^n colorings, with probability $1 - o(1)$,

$$|D_i \cap \mathcal{A}| \geqslant n$$

for all colorings. Since the expected number of small cycles in $\mathcal{A}$ is $o(n)$, with probability $1 - o(1)$, $\mathcal{A}$ has $< n$ small cycles. We select $\mathcal{A}_0$ satisfying these two properties then delete an $A \in \mathcal{A}_0$ out of each small cycle, leaving a family $\mathcal{A}^*$ with the desired properties. This completes the sketch of Theorem 5.

The juxtaposition of Theorems 1 and 5 and Question 1 leads us to pose the following

Question 2. For all k, c, t does there exist for some n a set $V \subseteq [n]$ such that

$$\{A\colon A \subseteq V, |A| = k, A \text{ an arithmetic progression}\}$$

is c-Van der Waerden with no s-cycles, $s \leqslant t$?

Theorem 6. *Question 2 is true for $t = 2$. That is, given k, c there is a set V which when c-colored yields a monochromatic arithmetic progression of length k and for which furthermore, if $A, B \subseteq V$ are arithmetic progressions of length k then $|A \cap B| \leqslant 1$.*

Sketch of Proof. As in Theorem 1 we find, by the Hales–Jewett theorem, an n such that if the n-dimensional cube k^n is c-colored there must be a monochromatic line. Now let p be prime, $p > 2k$, and set

$$V = \{a_0 + a_1 p + \cdots + a_{n-1} p^{n-1}\colon 0 \leqslant a_i < k\}.$$

V is the desired set. We associate $V = \sum_{i=0}^{n-1} a_i p^i$ with vector $\mathbf{v} = (a_0, \ldots, a_{n-1})$. A c-coloring of V yields a monochromatic line $\mathbf{v}_0, \ldots, \mathbf{v}_{k-1}$ which corresponds to a monochromatic arithmetic progression $v_0, \ldots, v_{k-1}$. Hence V is c-Ramsey. To show that V has no 2-cycles we note that $v_0, \ldots, v_{k-1}$ form an arithmetic progression iff the corresponding vectors $\mathbf{v}_0, \ldots, \mathbf{v}_{k-1}$ form an arithmetic progression. However, in the *cube k^n* any two lines clearly intersect at most one point.

We note that Theorem 5.2 does not appear to easily extend to the case $t = 3$. The cube k^n does indeed have 3-cycles, e.g.,

$$\{(0, k)\colon 0 \leqslant i \leqslant k-1\}, \qquad \{(i, k-1)\colon 0 \leqslant i \leqslant k-1\},$$
$$\{(i, i);\, 0 \leqslant i \leqslant k-1\}.$$

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Hadamard Matrices from Relative Difference Sets

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1. INTRODUCTION

A Hadamard matrix H is a square matrix of order n with entries ± 1 and which satisfies $HH^T = nI$, where H^T is the transpose of H and I is the identity matrix. It is easily shown that for H to exist n must be 1, 2, or a multiple of 4 [2]. The converse problem of constructing Hadamard matrices of all possible orders is much more difficult. Many authors have made contributions in an effort to find a solution, the results being many and varied (a list of all the constructions known in 1972 is contained in [5]). It is the purpose of this paper to add yet another class of Hadamard matrices to the ever growing list. More precisely, we prove the following results.

(i) Let n and $n - 2$ both be prime powers. If $n \equiv 1 \pmod 4$ there exists a Hadamard matrix of order $4n$, while if $n \equiv 3 \pmod 4$ there exists a Hadamard matrix of order $8n$.

(ii) Let m be an odd prime power for which there exists an integer $s \geqslant 0$ such that $(m - (2^{s+1} + 1))/2^{s+1}$ is an odd prime power. Then there exists a Hadamard matrix of order $4m$.

The following new orders ($\leqslant 4000$) of Hadamard matrices are obtained: 292 (recently obtained in [3]), 356, 404, 436, 596, 772, 964, 1016, 1028, 1108, 1208, 1268, 1396, 1412, 1556, 1588, 1604, 1636, 1732, 1796, 1828, 1844, 2116, 2164, 2228, 2264, 2276, 2564, 2692, 2836, 3076, 3284, 3524, 3704, 3716.

The main tools used in the construction of these matrices are a particular class of relative difference sets, as defined by Elliott and Butson [1], and a method of Whiteman using supplementary difference sets.

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