

What is the Consensus on $W(3, r)$?

Def 0.1 $W(k, r)$ is the least W such that for all r -colorings of $[W]$ there is a monochromatic arithmetic progression of length k .

The following is known about $W(3, r)$:

$$2^{\Omega((\ln r)^2)} \leq W(3, r) \leq 2^{O(r(\log r)^4)}.$$

The first inequality is a standard argument that was pointed out to me on a comment on a blog and which I later wrote up.

The second inequality follows from Bloom's paper *A quantitative Improvement for Roth's Theorem on Arithmetic* which is on arXiv. (An earlier paper by Sanders, also on arXiv's *On Roth's theorem for progressions* has the result with 4 replaced by 6.)

I am running a very small poll on what people who are interested in this problem think. I may make the results into a blog post. So please answer:

1. What do you think is the asymptotic behavior of $W(3, r)$?
2. Why do you think as you do?
3. When do you think it will be resolved?