The Book Review Column[1]
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In this column we review the following books.

1. A joint review of the following items. The reviews are by William Gasarch.
    (a) **Reality Conditions: Short Mathematical Fiction**, by Alex Kasman. This is a collection of short stories that use Math as a theme. It is written by a mathematician so it is factually correct. Its also surprisingly good. I mean no disrespect, but initially I expected that a mathematician writing fiction would have the same quality as a writer doing mathematics.
    (b) **Numb3rs**, TV show. No, its not a misprint, they use a '3' instead of an 'e'. This is a TV show where they solve crimes using Math. Is it interesting? Is the math accurate? Do they listen to their math consultants?
    (c) **Mathematical Apocryphia: Stories and Annecdotes of Mathematicians and the Mathematical** by Steven Kranz
    (d) **Mathematical Apocryphia Redux: More Stories and Annecdotes of Mathematicians and the Mathematical** by Steven Kranz. These are books of annecdotes about mathematicians. Are they interesting? A mixed bag.

2. A joint review of the following items. They are all books about particular numbers. The reviews are by Brian Blank. This joint review originally appeared in **College Math Journal** Volume 32, No. 2, March 2001, pages 155-160. It appears here with their permission.
    (a) **A History of Pi** by Petr Beckmann.
    (b) **The Joy of Pi** by David Blatner.
    (c) **The Nothing That Is** by Robert Kaplan.
    (d) **e: The Story of a Number** by Eli Maor.
    (e) **The story of i: An Imaginary Tale** by Paul Nahin.
    (f) **Zero: The Biography of a Dangerous Idea** by Charles Seife.

3. **The Square Root of 2: A Dialogue Concerning a Number and a Sequence** by David Flannery. Review in the form of a dialogue between William Gasarch, Alexander Kruskal, Justin Kruskal, and Rebecca Kruskal. (The Kruskal Triplets.) An entire book on $\sqrt{2}$. Is there enough there for a book? Is there enough there for a review? Read the review and decide.

Books I want Reviewed

If you want a FREE copy of one of these books in exchange for a review, then email me at gasarchcs.umd.edu

Reviews need to be in LaTeX, LaTeX2e, or Plaintext.

Books on Algorithms and Data Structures

1. *Design and Analysis of Randomized Algorithms: Introduction to Design Paradigms* by Hromkovic.
2. *Combinatorial Optimization: Packing and Covering* by Cornuejols.
3. *An Introduction to Data Structures and Algorithms* by Storer.
4. *Nonlinear Integer Programming* by Li and Sun.
5. *Handbook of Algorithms for Wireless Networking and Mobile Computing* edited by Boukerche.
6. *Solving Polynomial Equation Systems II: Macaulay's Paradigm and Grobner Technology* by Mora.

Books on Cryptography and Security

1. *Complexity and Cryptography: An Introduction* by Talbot and Welsh.
2. *Complexity Theory and Crytography: An Introduction to Cryptocomplexity* by Rothe.
3. *Crytographic Applications of Analytic Number Theory: Complexity Lower Bounds and Pseudorandomness* by Shparlinski.
4. *Cryptography and Computational Number Theory* edited by Lam, Shparlinski, Wang, Xing.
5. *Coding, Cryptography, and Combinatorics* edited by Feng, Niederreiter, Xing.
6. *Foundations of Computer Security* by Salomon.
7. *Coding for Data and Computer Communication* by Salomon.

Books on Coding Theory

1. *Introduction to Coding Theory* by van Lint.
2. *Block Error-Correcting Codes: A Computational Primer* by Xambo-Descamps.
3. *Coding Theory: A First Course* by Ling and Xing.
4. *Authentication Codes and Combinatorial Designs* by Dingyi Pei.
5. *Algebraic Coding Theory and Information Theory: DIMACS Workshop* Edited by Ashikhmin and Barg.

Game Theory

1. *An Introduction to Game-Theoretic Modelling* by Mestertson-Gibbons.

Combinatorics Books

1. *A Beginners Guide to Graph Theory* by Wallis.
2. *Graphs and Discovery: A DIMACS Workshop* Edited by Fajilowicz, Fowler, Hansen, Janowitz, Roberts. (About using software to find conjectures in graph theory.)
3. *Combinatorial Designs: Constructions and Analysis* by Stinson.

Logic Books

1. *Classical Mathematical Logic: The Semantic Foundations of Logic* by Epstein.
2. *Logicism Renewed: Logical Foundations for Mathematics and Computer Science* by Gillmore.
3. *Essay on Constructive Mathematics* by Edwards.
4. *Elements of Finite Model Theory* by Libkin.

Misc Books

1. *Computation Engineering: Applied Automata Theory and Logic* by Gopalakrishnan. (Looks like an automata theory textbooks.)
2. *Prediction, Learning, and Games* by Cesa-Bianchi and Lugosi.
3. *Combinatorial Auctions.* Edited by Cramton, Shoham, and Steinberg.
4. *Small Worlds: The Dynamics of Networks Between Orders and Randomness* by Duncan J. Watts.
5. *Domain Decomposition Methods– Algorithms and Theory* by Toseli and Widlund.
6. *Handbook of Computational Methods of Integration* by Kyther and Schaferkotter.
7. *Polynomials* by Prasolov.
8. *The Political Mapping of Cyberspace* by Crampton.
9. *An Introduction to Difference Equations* by Elaydi.
10. *Difference Equations: From Rabbits to Chaos* by Cull, Flahive, Robson.
11. *Diophantine Analysis* by Steuding.

A Joint Review[2] of

1. **Reality Conditions: Short Mathematical Fiction**, by Alex Kasman. MAA 2005, 246 pp., $29.95, Softcover.
2. **Numb3rs**, TV show. CBS. Free. Currently running Friday's at 10:00PM.
3. **Mathematical Apocryphia: Stories and Annecdotes of Mathematicians and the Mathematical** by Steven Kranz. MAA, 2002, 225 pp., $39.95. Softcover.
4. **Mathematical Apocryphia Redux: More Stories and Annecdotes of Mathematicians and the Mathematical** by Steven Kranz. MAA, 1999, 225 pp., $39.95. Softcover.

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1 Introduction

I once asked a lesbian friend of mine "what is your favorite TV show?" She said it is "The L word" (a lesbian-themed show on HBO). She admitted that it really wasn't that good, but it was about people like her, and it was one of the few such shows. I approach fiction (and even nonfiction) about mathematicians in the same light. I give it some credit just for relating to us.

In this review I consider three such works and ask the following questions: (1) is it fun to read (or view) independent of the math? (2) does the math fit in well? (3) will it get people interested in math? (4) does it portray mathematics and mathematicians in an accurate light (and is that a good thing)?

2 Reality Conditions: Short Mathetmatical Fiction

Alex Kasman is a mathematician who teaches a course in mathematical fiction. He has compiled an impressive list of such at http://math.cofc.edu/faculty/kasman/MATHFICT/default.html. He then wrote his own math fiction which is the book under review.

The book is composed of short stories that use math in some way. Most of the characters in it are ordinary mathematicians at universities. They are not geniuses. Hence the stories are realistic. There are 16 stories of which about 3 are fantasy (really could not happen) and 2 are science fiction of the type that is unlikely to happen. The rest are quite realistic.

The stories integrate math into the plot in ways that are natural and make sense. The portrayal of both the math and the mathematicians is accurate. For fiction that uses math this is both rare and appreciated. The math used is Topology, Geometry, bio-comp, cryptography, and other areas. The reader does not need to understand the math, but does have to know what math is about in a general way. Hence the ideal audience would be anyone who knows and likes math at the level of an undergraduate senior or higher. For such a reader the book is a fun read. Note that it could not be used to get someone interested in math. The book will not teach you any math nor inspired you to look some up.

I *could* discuss each story individually. I choose not to. I enjoyed the book by reading it and not knowing what to expect, and for the readers who are going to buy the book, I recommend that they do the same.

3 Numb3rs

A crime is committed! Who do you call? A mathematician of course!

This show is about two brothers, one works for the FBI and one is a college professor who is a math genius. The college professors name is Charley. His field is ambigous at times in that he works on P vs NP, mathematics of Physics, and can glance at a sketch of a proof of the Riemann Hypothesis and declare that the ideas are 'elegant'. While there may be people who have worked in all of those areas, they are rare. I doubt they would also have time to solve crimes. There are also two main supporting characters- their father, and a Physics professor who is friend and mentor to Charley.

Independent of the mathematics, some episodes have interesting plots and some do not. This show is good but not great[3]. If it was not about a mathematician I would not be watching it.

The mathematics is very odd. They get a lot of details right (e.g., the Clay Prize, mentioning real mathematicians) and even make some jokes that few people would get ("Just a run of the Yang-Mills black Hole.") But if they need something ridiculous to be true for the plot to advance, they will do it. My favorite example is in an episode where they needed to assume that a solution to The Riemann Hypothesis would imply factoring is in P (not likely to be true, but not ridiculous) and that once you had the proof you could then *easily* come up with the actual code to factor (which is ridiculous).

Another problem is that the kind of math needed for police work would not seem to require a mathematical genius. They seem to mostly use probability and data mining, which can get tedious. In other episodes they name drop terms like "Euler cycle", "P vs NP" "Vornoi Diagram", "Game Theory" or "Mathematics of Negoiations" (is there such a thing?). But they do not seem to really use these concepts. They often sound like people speaking a non-native language they just barely learned. Sometimes they just say "The math tells us ..." without telling us what they used. This may be better than using terms they don't understand.

On the other hand, they are actually mentioning Vornoi Diagrams on TV! This has to be a plus. There are at least two websites that explain the math used in the show:

http://www.cbs.com/primetime/numb3rs/ti/activities.shtml

and

http://www.atsweb.neu.edu/math/cp/blog/

This show could inspire people to study mathematics as they see it being used to do something. The main character is likable, though he is somewhat stereotypical. TV guide said he is sexier than mathematicians really are, though my wife disagrees.

The readers of this column will not learn any math from this show. In fact, they may wince at times. But this show can inspire someone to study some math, which is certainly good. If your kids watch it, you should watch it with them so that they do not get any wrong ideas about math. Just like when you watch 'The L word' with your kids.

[3]My proofreader, Clyde Kruskal, disagrees. He thinks its "okay but not good."

4 Mathematical Apocrypha and Mathematical Apocrypha Redux

This is a collection of *true* stories about mathematicians. The stories really do seem true- they don't have that polished feel of a story that has been made funnier or more interesting by bending the truth. But by the same token, the stories often are not that funny or interesting. Those that are funny or interesting are more so since they are true. Those that are not funny or interesting are often pointless.

The chapters of *Mathematical Apocrpha* are (1) Great Foolishness, (2) Great Affrontery, (3) Great Ideas, (4) Great Failures, (5) Great Pranks, (6) Great People, (7) Futher Reading. After a while the themes get repetive: Absent-mindedness, inability to do some easy things, ability to do very hard things, and some stories where the fact that the people involved are in math is irrelevant. The stories on pranks tended to be funny, while those on people were interesting but not funny. More generally, in a random sample of 55 stories, 7 were funny and interesting, 20 were interesting but not funny, 8 were funny but not interesting, and 20 were not funny or interesting. Realize that some of this is a matter of taste.

There is very little math in this book. Anyone could read it, but I suspect you need to be an academic (any field) to appreciate it. You will learn some history, and you will learn that mathematicans are people too; however, you probably already know that.

The second book *Mathematical Apocrpha Redux* is surprising in that it is just as good as the first book. Maybe even better. But it has the same pros and cons.

A Joint Review of

1. **A History of Pi**, by Petr Beckmann. St. Martins's Press, 1976, 200 pp., $11.95; Barnes and Noble Books, 1989, $14.95.
2. **The Joy of Pi**, by David Blatner. Walker & Co., 1997, 144 pp., $18.00.
3. **The Nothing That Is**, by Robert Kaplan. Oxford University Press, 1999, 225 pp., $22.00.
4. **e: The Story of a Number**, by Eli Maor, Princeton University Press, 1998, 232 pp., $14.95.
5. **An Imaginary Tale**, by Paul Nahin, Princeton University Press, 1998, 248 pp., $24.95.
6. **Zero: The Biography of a Dangerous Idea**, by Charles Seife. Viking Press, 2000, 248 pp., $24.95.

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Note: This review originally appeared in **College Math Journal** Volume 32, No. 2, March 2001, pages 155-160. It appears here with their permission.

We are in the midst of an onslaught of books devoted to particular numbers. In this article we will be concerned with several recent and not-so-recent books about π, e, i, and 0; of the quantities in the famous equation $e^{\pi i} + 1 = 0$, only 1 does not yet have its book. Given the trend, we can look forward to its appearance, possibly followed by volumes on "+" and "=".

The practice of devoting a book to a particular number is by no means a new development. In 1913 Ernest William Hobson, a distinguished mathematician and expositor, wrote a brief but excellent account of π [12]. Featuring serious history and serious mathematics, Hobson's Squaring the Circle, elementary though it is, inevitably became forbidding as a popular reference. By 1970 the time was ripe for a book on π that did not have quite so stiff an upper lip. Who better to write such a book than an electrical engineer?

Freed from the "dispassionate aloofness of the historian and the tiresome rigor of the mathematician," Petr Beckmann fashioned A History of Pi for a new age. Light and breezy, it is the kind of book that you can devour during a short-hop flight. True to his word, Beckmann does not "complicate" his explanations by "excessive mathematical rigor." Nevertheless, there is just enough mathematical detail to keep the book respectable. In particular, Beckmann provides a first rate (but no longer up-to-date) glimpse into the methods of the digit hunters. There can be no question that Beckmann struck an agreeable balance. His book has been deservedly successful, remaining in print for more than thirty years.

Of course, just as there was a shift in paradigm between 1913 and 1970, so has there been a shift between 1970 and the present. Nowadays who is better qualified to write a book on π than an expert in computer publishing, especially if he has already written books on digital imaging and virtual reality? Enter David Blatner and The Joy of Pi. When I saw this book's price listed as \$18.00($5.737(\pi)$) on the inside cover flap, I was prepared to offer it a hearty welcome. Alas, a reviewer is not allowed to judge a book by its cover: as I turned over the first page, disillusionment set in. The back cover blurb trumpets that The Joy of Pi is "beautifully designed" and "whimsically formatted" with a "creative graphic layout." Indicative of that creative layout is an interesting straightedge and compass diagram. It seems to be an exquisite historical example of the circle-squarer's art, but that is just my guess: although the figure appears ten times throughout the book it is described nowhere.

There are many other instances where the book could stand to be less whimsically formatted. Not one of the formulas is labeled or even referred to. On page 71, for example, there appears a truly remarkable infinite series for π. Since it is given in the chapter on the Chudnovsky brothers, the reader may suspect that it is their discovery and indeed it is. But surely the reader should not have to guess. The author tells us that the computation of 707 digits of π in 1873 "was hailed throughout the civilized world as the unveiling of a great mathematical truth," but he does not inform us that each additional term of the Chudnovskys' series yields an additional 14 digits of π. It is a telling oversight.

Whereas Hobson expected his readers to stick with him as he led them through a demonstration of the transcendence of π, Beckmann, who attempted nothing nearly as arduous, found it necessary to advise his readers to skip over the mathematics that they found too difficult. By contrast, The Joy of Pi resembles grade school science texts that have been shorn of all meaningful content so as to furrow no brow. Although we learn, for example, that Snell "believed firmly in his theories [but] he was never able to prove them," we do not learn exactly what those theories were.

The Joy of Pi is amusing in places and has some trivia that readers might find curious. Nevertheless, I would advise those readers of this journal who want to learn more about π and its history to turn to Beckmann supplemented by [3], [6], [12], and [16]. Having done so they will find Blatner's book utterly superfluous. Indeed, since π has been so well taken care of, we may safely turn our attention to other numbers: the number e, for example.

In retrospect, a book on e was a natural. With the longevity of A History of Pi so plainly in view, one wonders that so many years passed before such a book appeared. Patterned on Beckmann's successful model, Eli Maor's e: The Story of a Number rolled off the presses in 1994. A very slightly augmented paperback printing was released in 1998.

Persuading readers that the number e is a natural subject for study is no easy task, but historical exposition is perfect for the job. Unfortunately, Maor is not entirely convincing when he discusses the origins of e. In his preface Maor suggests that e first appeared in connection with the formula for compound interest, an assertion he repeats in his third chapter. Is there any evidence for his conclusion? The leap from daily compounding to continuous compounding would have been astounding at the time the number e was introduced. Yet not even one case of any type of subannual compounding is documented in Maor's book. Furthermore, although records of compound interest date back to antiquity, the appearance of e was exactly contemporaneous with the introduction of the Napierian logarithm. How can Maor's hypothesis be reconciled with the chronology?

Exacerbating these problems, Maor's analysis of Napier's contribution is confusing. He writes that Napier chose "for a base a number small enough so that its powers will grow reasonably slowly." The entire discussion is predicated on the notion of a base. Yet Maor writes in a footnote to this very discussion, "As we have seen, he [Napier] did not think in terms of a base, a concept that developed only later." We have a sure guide to Napier's discovery of logarithms: Napier's own account of his thinking. That report will leave the reader in awe. Not only does it reveal the role of e as natural base, it leaves us in admiration for its author, an amateur mathematician who hit upon an essential kinematical consideration of calculus even before the creation of the coordinate plane and analytic geometry. Of course, few will find it pleasurable to read a technical work written in archaic language. Maor provides a sketchy reconstruction of Napier's method that he hides in an appendix. More thorough accounts of Napier's construction of logarithms may be found in [1], [5], and [9].

Several historical nits can be picked. Maor writes that Johannes Kepler formulated his three laws of planetary motion in Germany. In fact, during Kepler's lifetime there was no state that could be referred to as "Germany." Even if there were, Kepler derived and formulated his laws during the twenty-six years he worked in Prague and Linz. Referring to the twentieth century discovery of The Method of Archimedes, Maor asserts that "J. L. Heiberg found a medieval manuscript in Constantinople." In fact, Heiberg identified that manuscript and others. He was actually in Copenhagen when he came across a catalogue description of a then unidentified palimpsest (containing The Method and several other manuscripts) in Constantinople. These inaccuracies concern facts that are unimportant to the author's story but they are not trifles, as I will argue later.

As is to be expected, the mathematical portion of Maor's tale is generally sound. In one uncharacteristic lapse, Maor illustrates the solution of the Brachistochrone Problem with a sketch of a cycloid generated by a circle rolling on top of the x-axis with the positive y-axis pointing upward. For a correct diagram accompanied by a proof, as well as mathematical treatments of many topics that Maor only talks about, the delightful book of Simmons, Calculus Gems, is recommended [18]. Indeed, Calculus Gems and the supplemental references [1], [5], and [9] remain my favorite resources for the topics that Maor covers. However, I am sure that others may well find Maor's book an attractive and convenient choice.

With histories of Jr and e already in the bag, you can confidently predict what was bound to come along next. Sure enough, Paul Nahin's An Imaginary Tale: The Story of (square root of -1) made its appearance in 1998. Like Beckmann, Nahin is a professor of electrical engineering. Unlike either Beckmann or Maor, Nahin had the ambitious plan of making mathematical computation an integral part of his narrative. He is certainly to be applauded for his concept. I will not say much about the execution of the concept. For one thing, An Imaginary Tale has already received a thorough review [2]-I wrote it myself! Writing for the professional mathematician I felt obliged to expose a litany of historical, mathematical, and pedagogical problems. There is much to like about An Imaginary Tale but the bottom line is that it has too many serious flaws to merit a general recommendation.

Mathematics is filled with curious and interesting numbers. Taken together, with a paragraph or two devoted to each, they constitute the basis for a charming and interesting book [20]. The problem is, how do you jump on the bandwagon and build an entire book around a single number? Take the number 0, for instance. What would you have to say about it? Well, we now know what Robert Kaplan, a retired high school teacher, and Charles Seife, a science journalist, would say and we are scarcely the wiser for it. Robert Kaplan's The Nothing That Is: A Natural History of Zero arrived from Oxford University Press late in 1999. Its appearance must have been mortifying for Charles Seife, whose own Zero: The Biography of a Dangerous Idea, was scheduled for release (or rushed to press?) in February 2000.

Not surprisingly, the two books have much in common. Each book fleshes out its slender premise with the same sort of slightly off-topic discussion. They share, for example, similar treatments of infinitesimals and the development of the differential calculus. Both books are written informally with an individual style not usually found in mathematics books. And both books are chock-full of misconceptions, half-truths, outright lies, and mumbo-jumbo. Consider this composite passage for a brief taste of their flavor: "If you look at zero you see nothing; but look through it and you will see the world. It provides a glimpse of the ineffable and the infinite. Zero shaped humanity's view of the universe-and of God." The numerous similarities notwithstanding, the two books are not equivalent. In fact, one is merely mediocre whereas the other is truly vile.

Let us dispose of the latter with the candor it deserves. Charles Seife's intent is to sensationalize at every turn: if the truth suffers, then so be it. He begins his story with a rousing bait-and-switch: "Zero hit the USS Yorktown like a torpedo ... Though it was armored against weapons nobody had thought to defend the Yorktown from zero ... No other number can do such damage." With these words Seife is alluding to a division by zero error that crashed the computers that controlled the missile cruiser's engines. Although the Yorktown incident is of vital interest for several reasons, division by zero is not one of them. Go ahead-divide by zero on your calculator. Did you break it? As to Seife's assertion that no other number can do such damage, I leave the refutation as an exercise with a hint: an equally dangerous floating point exception can be found in the title of one of the books I have discussed.

Is Seife guilty of nothing more than journalistic exuberance? No! According to Seife, "every time mathematicians tried to deal with the infinite or with zero they encountered trouble with illogic." Seife tells his readers: "If you were to throw a dart at the number line it would never hit a rational number, Never." He asserts that the series 1 - 1 + 1 - 1 + 1 - 1 + 1 "can equal 0 and 1 at the same time." Ironically, Seife repeats a favorite old myth, passed down from one popularizer to the next, that Euler was easily bamboozled by divergent series. "The careless manipulation of zero and infinity led him astray," says Seife. Popularizers hold onto their cherished fables as tenaciously as the circle-squarers hold onto their delusions. You cannot tell them otherwise. On the matter of divergent series Hardy tried more than fifty years ago [10]: "It is a mistake to think of Euler as a 'loose' mathematician ... his language sometimes suggests a point of view far in advance of the general ideas of his time. Here as elsewhere, Euler was substantially right." There are many other examples of the popularizer's knack for confabulating the historical record. Seife joins Maor in propagating the myth of the Greek horror infiniti. Read [11] as an antidote.

Why does Seife adopt a snide, smirky, spiteful tone for his book? Why must he refer to Martin Luther as a "constipated German monk"? He describes Kronecker as "the mathematician who would hound Cantor into a mental institution." This is a serious charge that is based on nothing more than the fictional writing of E.T. Bell. I could suggest [81 for a well-researched account but that would only be a start: there is not enough space in this review to set right all that is wrong with Seife's book.

Turning to Robert Kaplan's book is something of a relief. In his hands the story of zero is initially made to seem worth the telling. For the first eight chapters I was not tempted to conclude that Kaplan had filled a much-needed gap in the literature (to borrow a quip from classics professor Moses Hadas). Right from the start, however, I found Kaplan's florid prose to be an obstacle. The phrase "recursive abstracting" is listed five times in the index. I am still not sure what it means but Kaplan assures us that it "is the very stuff of mathematics." To make the matter clearer he adds that it is "this abbreviating the sweep of landscape you have just taken in to an apercu for a higher order of seeing."

Language and style may be a matter of taste but facts are facts. Referring to a mathematician whose name is associated with a well-known limit theorem of calculus, Kaplan asks "Shall we call him Guillaume Francois Antoine, Marquis de l'Hopital?" No we shan't! Guillaume de l'Hopital was the Marquis de Sainte-Mesme. In a similar vein, Gauss's first name is spelled as "Karl" instead of "Carl." Mathematical facts are also garbled. Kaplan describes Ramsey Theory as "a branch of mathematics that studies rapidly growing functions." Later Kaplan gives a construction of what he calls the Farey sequence but which is in reality the Stern-Brocot sequence. The oft-repeated "history" of Farey sequences that Kaplan resurrects was debunked five years ago when Bruckheimer and Arcavi consulted the primary sources [4].

Given that Ramsey Theory, Farey sequences, and the names of mathematicians have little to do with the history of zero, it is reasonable to wonder if any of this matters. Let me be clear on this issue: these errors are of the utmost importance. In mathematics we read an author's assertion and his proof of it; in doing so we are able to ascertain validity. That is not how we as mathematicians receive history. If we cannot rely on the say-so of a historian, then his writing is worthless to us. In Kaplan's case we have grounds to question his understanding of mathematics and reason to reject his standards of accuracy. In short, why should we be convinced by what he has to say? Let me illustrate.

One of the key historical problems concerning zero is the extent of the influence that the ancient Babylonian zero had on the Hindu zero. This is a question that the experts have batted around for some time without coming to agreement amongst themselves. Ifrah, for example, concludes that India came upon zero on its own [13, p. 3411. Freudenthal, on the other hand, has argued that the Hindu zero originated with the Babylonians. After presenting Freudenthal's arguments, Van der Waerden [19, p. 57] will allow no more than that Freudenthal's scenario "is quite possible." Menninger [14, p. 399] concurs that the events of Freudenthal's theory could have happened "and with this 'could have' we shall have to be satisfied." Now Kaplan, who comes out unwafflingly for Feudenthal's hypothesis, dismisses the opposing point of view with nothing more than these words "these disputes ... are paved wall to wall with fallacies of negative, presumptive and possible proofs, fallacies pragmatic and fallacies aesthetic." Perhaps, but based on Kaplan's track record why should the reader accept his authority?

Although The Nothing That Is has been decorated with attractive art drawn by the author's wife, nobody thought to include any maps. One of the most important "documents" in the history of zero is a tablet that was unearthed at Kish. Would the reader not want to know that this ancient city, long since vanished, was situated at the narrowing of the gap between the Tigris and Euphrates rivers, south of present-day Baghdad. Avicenna is said to have been born in Bukhara. Will that be meaningful to readers outside of Uzbekistan? Not even an atlas will help locate the many Arab mathematicians who are cited but for whom no geographic information is given.

Although I would not steer the general reader with a casual interest in the history of numbers away from Kaplan's book, I would recommend that he try alternatives such as [7] first. Readers of this journal who are interested in the development of the concept of zero should bypass The Nothing That Is and seek out works of greater scholarship ([13], [14], [15], and [19]).

There! I let slip the word "scholarship." There was a time when that was the mission of the university presses. They were the publishers of last resort for serious work of scholarly value but specialist appeal. Compare the books of original research that the university presses used to publish ([12], [14], [15], [19]) with the recent crop of popular rehashings. Nowadays it would seem that the harvesting of jacket fodder has assumed priority over even routine copy editing. Princeton University Press, for example, boasts of Nahin's literary accomplishments on the outside cover of his book, yet on the inside we are treated to the following not atypical sentence (p. 82): "In fact, Gauss had been in possession of these concepts in 1796 (before Wessel) and had used them to reproduce, without Gauss' [sic] knowledge, Wessel's results." When Princeton reprinted Maor's book in 1998, four pages concerning a new Mersenne prime were added. Could the footnote referring to the "unpublished" proof of Fermat's Last Theorem not have been updated? Despite many reprintings since 1974, Beckmann's book still refers to the "unproven four color conjecture.

All of this is as nothing compared to the assault on scholarship spearheaded by Oxford University Press. You will search in vain for notes or a bibliography in Kaplan's book. That is because Oxford entirely removed these off-putting accouterments of scholarship, electing to post them on a website. It is ironic that Kaplan (p. 7) chortles that Sumerian clay tablets have outlasted computer punchcards of the 1960s. Of what value will his notes and bibliography be in five years when the "portable document format" (or pdf) has yielded to the next improvement in electronic archiving? Of what value will his documentation be in ten years when Oxford finds it unprofitable to waste disk space on a file pertaining to an out-of-print book? For the time being, no one should be deceived into thinking that the web is a place for anything but ephemera. And no one should be deceived into thinking that the imprimatur of a leading university conveys authority and scholarship.

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Review [4] of

The Square Root of 2:
A Dialogue Concerning a Number and a Sequence
Author: David Flannery
Publisher: Copernicus Books, 2006
$25.00, Hardcover

Reviewers: William Gasarch, Alexander Kruskal, Justin Kruskal, Rebecca Kruskal

Professor Clyde Kruskal, who works in parallelism, has triplets: Alexander, Justin, and Rebecca Kruskal. They are in 9th grade, have already taken elementary algebra, and are taking Geometry now (this was written in Spring 2006). Professor William Gasarch spoke to them about the square root of 2. What follows is an *interpretation* of their conversation.

Act I: The irrationality of $\sqrt{2}$

Gasarch: What are the most important numbers in mathematics?
Alexander: π.
Rebecca: I was going to say that!!
Justin: I've seen the number e on a calculator.
Gasarch: You are missing some easier ones.
Rebecca: Zero.
Justin: But that's just nothing (all laugh).
Gasarch: Rebecca is right— 0 is an important number. And there is one more that's important.
Alexander: One.
Justin: What about negative one?
Gasarch: OH, yes, that's important also.
Rebecca: But you said there was just one more left!! If I had known there were two I would have gotten that one!!
Gasarch: I'm glad I don't have kids of my own. Now, there is also i which is the square root of -1 which we won't go into today, but you may see later in High School. There have been books written about $0, \pi, e$, and i.
Rebecca: What a boring life that is to write a book on a number.
Gasarch: You are both right and wrong.
Rebecca: Huh?
Gasarch: You are wrong because these books embody math and history of interest. But you also right because, as the review by Brian Blank indicates, the books aren't that good. However, you are wrong because these people did not spend their entire lives on these books.
Rebecca: (Confused) So am I right or wrong!!?
Alexander: (teasing) You're wrong! You're wrong!
Gasarch: Knock it off. OKAY, let me tell you why we are here. There have been books written about 0, e, π, i. My plan was to review these books for my column. But I found a marvelous review of the $0, e, \pi$ and i books already, which I got permission to reprint. I then got a book on

$\sqrt{2}$. It seems more pitched to High School Students so I decided to read it, talk to you about it, and see if it really is good for High School Students.
Justin: Most high school students won't care about $\sqrt{2}$.
Gasarch: The three of you like math, so the question really is "will high school students who like math like this book?" Anyway, after we talk about it I'll essentially write down what we all said and that will be the review, and you'll all be co-authors.
Rebecca: Cool!
Gasarch: So, lets start. Consider a square that is 1 by 1 (draws on board). What is the length of the diagonal (draws the diagonal).
Justin: You use that theorem we learned. Some Geek Guy.
Gasarch: I think you mean some Greek guy. His name was Pythagoras.
Alexander: You do that square thing. You know, $a^2 + b^2 = c^2$.
Gasarch: Yes, (does the algebra on the board) so we get that the this length is such that, if you square it, you get 2.
Rebecca: Lets see, $1^1 = 1$ and $2^2 = 4$, so there is no such number.
Justin: You're forgetting about fractions.
Rebecca: Oh yeah. (Tries some fractions.) None of these work. There is no such number.
Gasarch: But I have on the board a line segment that has that length.
Alexander: It's irrational.
Gasarch: OH, you are saying that there is no fraction with this property. Did you learn that somewhere.
Alexander: I think I heard it someplace.
Gasarch: Can you prove it?
Alexander: No, but isn't it one of those things that everyone kind of knows is true but nobody has proven? Like that the primes are infinite.
Gasarch: (not quite sure how to respond) Uh, lets go around and see what you think about the following two questions: Is there a fraction what when squared it is 2, and in either case, is there a proof?
Justin: I think that $\sqrt{2}$ is irrational and the proof is really, really, really, really hard.
Rebecca: I think that $\sqrt{2}$ is irrational and nobody knows how to prove it.
Alexander: I think that $\sqrt{2}$ is irrational and nobody knows how to prove it.
Rebecca: Copycat!
Gasarch: Why do you think it's hard to prove?
Justin: No matter how many places you compute, you'll never know if $\sqrt{2}$ will stop. If it does stop, you'll know it's rational. If it hasn't stopped yet, you don't know whether it's going to, and you can't just compute the whole infinite number of places and then note that it didn't stop. (EDITORIAL NOTE: We then discussed that even if a decimal expansion goes on forever it may still be a rational. We omit this from the review as it is not relevant to the book.)
Gasarch: Back to the topic at hand. You all think that $\sqrt{2}$ is irrational and the proof is either hard or unknown. It turns out that you are right that $\sqrt{2}$ is irrational, but actually the proof is easy enough that I can show it to you right now. (Gasarch does the proof on the blackboard and they understand it.)
Gasarch: You all thought it would be hard to prove, but it was not. So, why did you think so, and what was the error in your thinking?
Justin: This is the first time we saw a proof that something could not be done.

Alexander: Actually, this is the first time we saw a proof at all! In Geometry they have these dumb 'two-column proofs' which don't seem like proofs at all.
Gasarch: Realize that a proof is a chain of reasoning. The actual form is not important. The proofs you've seen in Geometry are chains of reasoning, though they may not seem that way. Alexander- you had referred to things that "everyone kind of knows are true but there is no proof". There are very few of these things. But more important is, do you find the result and the proof interesting? And be honest— the point of this meeting is to see if this would interest high school students who already like math.
Justin: Yes, it was kind of cool to see that you could prove there is no fraction for $\sqrt{2}$.
Alexander: Yes. Its interesting to prove that you can't do something.
Rebecca: Yes, it's good to see a proof so that we know things are true.
Gasarch: The book we are reviewing does not have this proof until page 42.
Alexander: What do they do until that point?
Gasarch: The book is written as a conversation between a professor and a student. So their conversation was much like ours, discussing $\sqrt{2}$, having the student try to find a rational number for it, etc.
Rebecca: But 42 pages. That sounds really boring, though I like reading books in conversation form.
Gasarch: That's the problem I have with the book. I'm not sure who wants to READ the book. For someone like me the book is too elementary. For someone like you the book may be too boring in that they spend a long time getting anywhere. On the other hand, I did learn some stuff in there that is worth telling you.

Act II: Tiling Problems

Alexander: What else is there to even say about $\sqrt{2}$? It's just a number.
Gasarch: Lets go to a different problem. Say you have a $\sqrt{2}$ by 1 rectangle (Draws on board). Can you cover it with squares that are x by x. Such a covering is called a tiling.
Justin: Yes you can- just make x small enough.
Rebecca: No you can't.
Alexander: Yes you can.
Rebecca: Can I change my vote? I want to say yes you can.
Gasarch: Yes you can change your vote. Now, I'll go around and ask you each again, and ask for your reasons. Justin.
Alexander: Once the square is small enough, of course you can make it all work out. Its obvious.
Justin: Yeah. What he said.
Rebecca: Yeah. What he said.
Gasarch: It turns out that you can't. And I'll prove it to you. (The proof is done on the blackboard and they understand it. Gasarch then showed them that every rectangle that has rational sides can be tiled.)
Rebecca: You two made me change my vote!! I would have been right if it you didn't make me change my vote!!
Gasarch: Rebecca, the point of these votes is to get you thinking about stuff. If you vote wrong and then see what the truth is you are enlightened.
Alexander: What if you vote correctly? Are you still enlightened?

Gasarch: Yes, but in a different way. You are enlightened to know why you were right. Now back to Tiling- what to you think of the fact that you can't tile.
Justin: Wow. So you can't tile. What can you tile?
Gasarch: If the sides are rational then you can tile it. (We all do the proof of one case together- I assign the general case for Homework.)

Approximations to $\sqrt{2}$

Gasarch: The book contains a table that looks like this

n	n^2	$2n^2$
1	1	2
2	4	8
3	9	18
4	16	32
$\vdots$	$\vdots$	$\vdots$

We can use this table to approximate $\sqrt{2}$.
Rebecca: How?
Gasarch: Can there be a number that appears in the second and third columns?
Rebecca: Sure.
Alexander: Yeah, why not.
Justin: Yeah. We just haven't found it yet. Who cares?
Gasarch: Actually you cannot (Does the proof).
Rebecca: Okay, so you can't. Who cares?
Gasarch: If we found two numbers that are the same, that would give us that $\sqrt{2}$ is rational. But if we found two numbers in the table that are very close together, that would give us a very good approximation to $\sqrt{2}$. I want you to find numbers in the second and third column that differ by one.
Rebecca: 8 and 9.
Alexander: 49 and 50.
Justin: 288 and 289.
Gasarch: And we can use these to get the following good approximations to $\sqrt{2}$. (After some writing on the board.) We now see that
$\frac{1}{1}, \frac{3}{2}, \frac{7}{5}, \frac{17}{12}$
form better and better approximations to $\sqrt{2}$. This sequence has a pattern to it that will enable you to find the next element. Can you figure out the pattern? (After several tries they do not figure it out, though Rebecca claims she had it and lost it. I leave it to the reader to discover a recurrence for the numerator and denominator.)
Rebecca: How much of the book have we covered.
Gasarch: To be fair, they do more formal proofs and they also show that the fractions generated by this recurrence are in lowest form and are also all of the numbers you ever get by looking at the columns and seeing when they are 1 apart. And they also show that even if you didn't start with $\frac{1}{1}$ the sequence would still approximate $\sqrt{2}$.
Alexander: But isn't all of that obvious?

Gasarch: No, these things require careful proof. Remember that a while back it was obvious to you that you could tile the 1 by $\sqrt{2}$ rectangle with small enough tiles. And you were wrong.
Justin: So does the book teach the need for proof?
Gasarch: Not really, but they do proofs.

Continued Fractions

Gasarch: Note the following: $\sqrt{2} = 1 + \frac{1}{1+\sqrt{2}}$
Alexander: (does the algebra) Yes, that's true.
Gasarch: Now note that I have an equation that has $\sqrt{2}$ in terms of itself. SO, we can plug in the expression for $\sqrt{2}$ and get
$\sqrt{2} = 1 + \frac{1}{2+\frac{1}{1+\sqrt{2}}}$
Alexander: Is that really a fraction?
Gasarch: Yes it is. And I can do it again to get

$$\sqrt{2} = 1 + \frac{1}{2 + \frac{1}{2+\frac{1}{1+\sqrt{2}}}}.$$

Justin: That's a really weird looking fraction.
Gasarch: I can keep doing this forever to get

$$\sqrt{2} = 1 + \frac{1}{2 + \frac{1}{2+\frac{1}{2+\cdots}}}.$$

Alexander: What if you get tired of this and stop working it out?
Gasarch: Glad you asked! What if you just ignored alot of the terms? What if you just do

- 1
- $1 + \frac{1}{2}$
- $1 + \frac{1}{2+\frac{1}{2}}$
- $\vdots$

Rebecca: We get $\frac{1}{1}, \frac{3}{2}, \frac{7}{5}, \frac{17}{12}$.
Gasarch: Does the pattern look familiar?
Alexander: Yes, it's that other pattern we got.
Gasarch: Right. So two different ways to approximate the $\sqrt{2}$ end up being the same way.
Justin: Does the book do that?
Gasarch: Yes, in fact the book has three sequences that end up being the same that all approximate $\sqrt{2}$. Note that the book's title is "The square root of 2: A Dialogue concerning a number and a sequence." The sequence is that sequence.

Conclusions

Gasarch: We did four topics today: (1) $\sqrt{2}$ is irrational, (2) tiling, (3) approximations for $\sqrt{2}$, (4) Continued fractions, which lead to more approximations. Did you find these topics interesting.

Rebeeca: Yes, but not enough to like, you know, read.
Alexander: It was good to see some proofs not in that stupid form we do in Geometry.
Justin: I liked those weird fractions. But 42 pages to get to $\sqrt{2}$ being irrational? That seems so weird.
Alexander: But if it wasn't for the book we wouldn't be here listening to you talk about this stuff.
Rebecca: How long was the book? (she picks it up). 242 pages! That's too long. How much of it did you cover today?
Gasarch: I covered about half of it. Probably less if you want to think about how much rigor I left out.
Justin: So . . . is it a good book or not?
Gasarch: The book is a good place to get ideas on what to tell high school students about the $\sqrt{2}$ and is a launching point for many topics of interest. Some High School students might like to read it themselves, but it may be too slow paced. Reading this review may be just as good.