Review of[1]
The Cult of Pythagoras: Math and Myths
by Alberto A. Martinez
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1 Introduction

Excerpt from a popular Math history book from 3000 AD written by Milt Peel Creble.
Monty Hall was a great Mathematician in the late 1900's who worked in probability. He challenged some of the hidden assumptions in probability. He used various paradoxes to test his theories. One became quite popular and was called The Monty Hall Paradox. *Alas, mathematicians of his day counted his popular work against him. Hence he never received the Nobel Prize in Mathematics.*

What is wrong with this narrative?

- Monty Hall is not a mathematician.
- Monty Hall did not invent the Monty Hall Paradox. Steve Selvin did.
- The Monty Hall Paradox is not really a paradox. It was never used to test anything nor is it problematic.
- There was no Nobel Prize in mathematics until the year 2500.

Could this absurd and clearly false story ever become a well known story? Alas yes. The book under review shows the several oft-told stories about Pythagoras are as absurd. He also discusses other myths.

Another excerpt from a popular Math history book from 3000 AD written by Milt Peel Creble.
Monty Hall discovered the compact numbers which were used to resolve his paradox. At the time some old fashioned mathematicians claimed incorrectly that the compact numbers do not exist; however, much like the complex numbers, the quaternions, and the surreals, the compact numbers are now accepted by all mathematicians.

What is wrong with this narrative (aside from the issues raised in the last narrative)?

1. The Monty Hall Paradox does not need to be resolved.
2. There are no such thing as compact numbers. At least not yet.
3. The author of the book under review would say that the compact numbers were *invented* to resolve the paradox. To say they were *discovered* seems very very odd.

The author discusses whether math is invented or discovered. His own opinion is that much more of it is invented than we tell our students (or perhaps than we even realize). He also urges us to not hide these issues and historical debates from students.

2 Pythagoras

The information we have on Pythagoras is very slim. So-called historians in the past would use phrases like *as is well known Pythagoras provided the first rigorous proof of the Pythagorean Theorem* when this was not well known and there was no evidence for it. Similarly there is no evidence for the story that when one of his followers discovered a number that was not rational (the hypotenuse of a right triangle with both legs of length 1) that follower was (1) put to death or (2) accidentally drowned, which the Pythagorean thought was a sign from God. (I had heard both versions.)

The author also speculates some on how these stories could have evolved. Alas, there is also scant evidence to go on here as well. My own opinion is that history is not just written by the winners, but is also written to make stories neat and clean and have a point, even if there is no such point in those stories. I give a non-math example of a story I heard in my youth:

Charles R. Drew was an African-American doctor who helped perfect blood transfusions. He died in 1950 when, after a car accident, the hospital he was at refused to give him a blood transfusion because he was black.

This story is not true. It was common in the 1950's for a hospital to only treat whites, and there was a notion (without any scientific foundation) that it was bad (morally? healthy-wise?) to give a black man white blood or vice-versa. Hence the story is great at conveying these points and to make us feel superior since we don't think that way anymore. But, alas, its just not true.

Pythagoras permeates the entire books. Bertrand Russell criticizes Pythagoras for introducing Platonism (which he did not). Carl Boyer (a historian) comes close to claiming that Pythagoras invented or discovered (hard to tell which) infinitesimals (which he did not).

3 Gauss and Galois

One of the most well-told tales is that in first grade Gauss summed the numbers $\{1, 2, \ldots, 100\}$ quickly using a trick that is now well known. There are many versions of the story. While the story has very little foundation at least (unlike the Pythagoras story) the point it's trying to make—that Gauss was exceptional at math at a young age— is true. Or so I've been told.

Galois did indeed die at a young age in a duel. Much else you've heard about him is speculation. And no, he did not write down all of his math the night before the duel.

4 Different Types of Numbers

There had been a myth that Complex Numbers confused Euler (I had not heard that one– but I did hear a myth that Euler was confused by infinite series.) This book debunks that and takes us on a tour of some nice mathematics as well. The issue at hand is how to define $\sqrt{a}$. Even for $a \geq 0$ this is not clear- is it the positive square root, or is it multivalued? No matter how you define it some standard rule of mathematics will fail.

This brings up the other point of this book: Math has had controversies. We should teach them! Math is more interesting and more accurate when we tell students that math did not arise in the pristine form that it ends up in textbooks. The story of Complex Numbers and Quaternions are examples that are discussed.

Are questions of number systems all settled now? Surprisingly no! What is 1/0? Infinity? Undefined? Or as some computers say NaN (Not a Number). There are still competing systems. Computers have made this more relevant since 1/0 may come up naturally and not as a mistake. Even in pure math it may be relevant since projective geometry has a point at infinity.

The author also writes of the very different views of infinitesimals. The most amusing one is that Leibniz thought they were a useful fiction, but that others agreed but didn't want this known. The author writes only of the controversy about infinitesimals taking place in the north (England and Germany) where it was mostly a mathematical debate. The book *Infinitesimals* by Amir Alexander tells the story of the controversy over infinitesimals in Italy where it was mixed in with the Catholic Church who were against infinitesimals and banned their use.

The author invents his own number system where $(-1) \times (-1) = (-1)$. His claim is not that we should use it, but that we should not automatically reject it. Note that complex numbers, Quaternions, and transfinite numbers were at one time rejected. The author does mention surreals; however, they seem to have never caused controversy.

The author ponders if π really would be the best number to beam over to an alien civilization. He does not mention the τ-movement, that a better constant for math would be τ which is the ratio of the circumference to the diameter, actually 2π. Some believe we should use τ since 2π occurs in formulas more than π does[2]

5 Geometry

There is an entire chapter on non-euclidean Geometry. The history is interesting. The point is that Geometry was once thought us as being an absolute— so theorems in it were thought to be discovered; however, now that we know there are many geometries, it might be better to think that geometry is invented.

6 Summary

The author has a strong point of view on the nature of mathematics and presents it with authority. Even if you disagree with him, in fact especially if you disagree with him, this book is worth reading.

His point of view is that some parts of math are discovered (that is, exist independent of our knowing about them) but many are invented. While I don't quite know where he draws the line, it's far earlier than I do— he seems to think that $(-1)(-1) = 1$ is invented.

I used to draw the line at large cardinals. My wife thinks the Banach-Tarski paradox proves math is broken, so she would say the Axiom of Choice was surely invented and is false.

One question that is not quite addressed: why do some systems win out? Is it because they are more useful? Is it politics? This would be a study unto itself.

I close by saying when I first had thoughts like the author. I noticed that $5^{1/2} = \sqrt{5}$ and wondered if there was some sense in which we are multiplying 5 by itself 1/2-a-time. There is not! We define $5^{1/2} = \sqrt{5}$ so that the rule $a^b \times a^c = a^{b+c}$ is true. In my humble opinion *there is no other reason to define it that way.* I have no problem defining it that way, but we should admit that it's a convention we invented so that an old rule still holds, and not a discovered law of nature.

[2] When I blogged about this Terry Tao wondered if $2\pi i$ might be even better. He went on to win the Fields Medal and the Breakthrough award, though not for that observation.

Milt Peel Creble is an anagram of a real person's name. I discovered (invented?) this anagram. I challenge the reader to determine who it is an anagram of.