Review of[1]

Mathematics Galore:
The First Five Years of the St. Marks' Institute of Mathematics
by James Tanton
Publisher: MAA
$50.00 hardcover, 268 pages, Year: 2012

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1 Introduction

The author has gotten high school students into mathematics research via workshops and a newsletter that had expository math articles in it. This book is a collection of 26 of those newsletters. There are also several appendices some of which have original research done by the students.

Even though a high school students can understand the material it is interesting and sometimes surprisingly difficult. The chapters are well written and motivated.

2 Summary of Contents

I list some things I learned from the book that were both new and interesting (to me).
Chapter 1- Arctangents: Let $f_0 = 1$, $f_1 = 1$, $(\forall i \geq 2)[f_i = f_{i-1} + f_{i-2}]$, the familiar Fibonacci numbers. Then

$$\arctan\left(\frac{1}{f_n}\right) = \arctan\left(\frac{1}{f_{2n-1}}\right) + \arctan\left(\frac{1}{f_{2n-2}}\right).$$

Chapter 2: Benford's Law: Benford's law states that in many tables of numbers (e.g., log tables, IRS tax forms) more entries begins with 1 then with 2, with 2 then with 3, etc. Estimates are that 1 appears as the first digit around 30% of the time, 2 about 17% of the time. As stated above this is not rigorous. However, it is empirically true. In more well defined settings (e.g., look at the table of powers of a number) this can be made rigorous. The author proves Benford's law in some cases.

Chapter 5- Dots and Dashes: The following is trivial: The nth square is n^2. But what about the nth *non-square*? Its not hard to work out; however, I had never thought of the question. The answer is the $round(n + \sqrt{n})$ where $round(x)$ is the rounded version of x.

Chapter 6- Factor Trees: When I teach discrete math (Honors Section) I want to show them that unique factorization over the integers is not obvious, and then that its true (I could save time by just letting them think its obvious). One way to show them the UF is not obvious is to show them a domain where unique factorization is not true. I often use $D = \{a + b\sqrt{6} \mid a, b \in \mathsf{Z}$. One can show that $2, 3, \sqrt{6}$ are all primes (defined properly) and hence $6 = \sqrt{6} \times \sqrt{6} = 2 \times 3$. So D is not a UFD. This chapter gives an easier example, thought its not an Integral Domain. Just take the even integers. A prime p is such that if $p = ab$ then either $a = \pm 1$ or $b = \pm 1$. Note that $400 = 2 \times 2 \times 10 \times 10 = 2 \times 2 \times 2 \times 50$. Hence the even integers does not have unique factorization

Chapter 16- Personalized Polynomials: I give a problem that is not from this chapter but is inspired by it. Let $p(x)$ be a polynomial of degree d such that $p(0), p(1), \ldots, p(d)$ are all integers. Show that, for all integers n, $p(n)$ is an integer. Hint: view $p(x)$ as a linear combination of $\binom{n}{0}, \binom{n}{1}, \ldots, \binom{n}{d}$.

Chapter 20-Repunits and Primes: It is well known that there is no polynomial $p(x) \in \mathsf{Z}[x]$ such that and infinite number of $p(0), p(1), p(2), \ldots$ are prime. What about if $p(x) \in \mathsf{Q}[x]$? $\mathsf{R}[x]$? $\mathsf{C}[x]$? The same holds!

Chapter 22-Tessellations: The plane can be partitioned into an infinite number of regular 3-gone, 4-gons, or 6-gons. What about 5-gons? 7-gons? In this chapter it is shown that 3,4,6 are the ONLY numbers n such that the plane can be partitioned into n-gones.

Appendix III-The Mobius Function: This chapter *motivated(!)* the Mobius Function and the Mobius Inversion Formula. I won't state the function or the formula, but I will state two fun problems they use to motivate it. In both the scenario is an infinite set of lockers numbered $1, 2, 3 \ldots$ and an infinite set of students numbered $1, 2, 3, \ldots$. The lockers are originally all closed. When student i walks down the hall he changes the status of lockers $i, 2i, \ldots$. (1) If the students walk down the hall in order: 1,2,3,... then which lockers are open at the end? (2) If you want to have only locker 1 open then which students do you send down the hall?

3 Opinion

While reading it I kept asking myself *Who would benefit from reading this book?* This question is actually two questions: (1) who would understand this book, and (2) who would find things interesting and new in it. As I showed in the last section, a college professor (at least me) did find many things new and interesting. (I think 'new and interesting' is equivalent to 'interesting and new'.)

A very bright high school student could benefit a lot from this book if he has some guidance in reading it, which I assume the original audience for this material did. A college professor will find some things new and interesting in this book.

4 Books of this Type

This is the 18th book of math essays on a variety of topics that I have reviewed. The other 17 are:

1. **Martin Gardner in the Twenty-First Century** Edited by Michael Henle and Brian Hopkins.
2. **Selected Papers on Fun & Games** by Donald Knuth.
3. **Six Proceedings from the Gatherings for Gardner Conference**. Edited by a variety of people.

4. **Dude, Can You Count?** by Christian Constanda.
5. **Mathematical Treks: From Surreal Numbers to Magic Circles** by Ivars Peterson.
6. **Professor Stewart's Cabinet of Mathematical Curiosities** by Ian Stewart,
7. **Five Minute Mathematics** by Ehrhard Behrends,
8. **Aha Gotcha!- Aha Insight!** by Martin Gardner,
9. **Origami, Eleusis, and the Soma Cube** by Martin Gardner,
10. **Hexaflexagons, Probability Paradoxes, and The Tower of Hanoi** by Martin Gardner,
11. **Group Theory in the Bedroom and Other Mathematical Diversions** by Brian Hayes.

There are other books that are borderline in that they were either too hard (e.g., Proceedings of he Erdos Centennial) or too focused (e.g., Three books on *Games of no chance*). Having said that, the borderline between "recreational" and "real" math is a fractal.

The reviews of these books all have the same basic format: I say how well written they are, I describe some sample mathematics from the book, and I say it would be wonderful for your (1) great niece, (2) bright high school students, (3) bright undergraduates, (4) some combination of the above. This is not cynical- these books have been well written and are appropriate for some combination of (1),(2),(3).

How different are the books? I looked over all of my reviews and found, much to my surprise, that there is *very little overlap.* There is much math of interest that one can learn before it gets hard; some if it will be new and interesting to you!