Review [1] of
The New Mathematical Coloring Book:
Mathematics of Coloring and the Colorful Life of its Creators
Second Edition
by Alexander Soifer
Springer, 2024
Approx 889 pages
EBook $169, Hardcover $220

Review by
William Gasarch gasarch@umd.edu

1 This is a Second Edition With a Lot More

In 2009 Alexander Soifer published *The Mathematical Coloring Book* which I reviewed for SIGACT News here:

`https://www.cs.umd.edu/~gasarch/bookrev/40-3.pdf`.

The book was around 600 pages. In 2024 Alexander Soifer published *The New Mathematical Coloring Book*. This new book is around 900 pages.

In this review I first republish the first two sections of my review of the first edition (with a few comments added), then summarize whats common to both books, and whats new in the new book.

Begin Excerpt from Prior Review

2 Introduction

I first had the pleasure of meeting Alexander Soifer at one of the *Southeastern International Conferences on Combinatorics, Computing, and Graph Theory*. If I was as careful a historian as he is, I would know which one. Over lunch he told me about van der Waerden's behavior when he was living as a Dutch Citizen in Nazi Germany. Van der Waerden later claimed that he opposed the firing of Jewish professors. Soifer explained to me that in 1933 the German government passed a law requiring Universities to fire all Jewish professors *unless they were veterans of WW I* (there were other exceptions also). Van der Waerden protested that veterans were being fired, in violation of the law. So he was objecting to *the law not being carried out properly* and not to *the law itself*. Alex told me that the full story would soon appear in a book he was writing on Coloring Theorems. I couldn't tell if the book would be a math book or a history book. It is both. (ADDED LATER: While both the first edition and the new book have much about van der Waerden and his actions under the Nazi regime, Alexander Soifer has a separate book on that, *The Scholar and the State* [2]. I reviewed that book for SIGACT News:

`https://www.cs.umd.edu/~gasarch/bookrev/FRED/vdwhistory.pdf`

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The second time I met Alex was at the next *Southeastern International Conference on Combinatorics, Computing, and Graph Theory*. Alex gave a talk on the following:

1. Prove that for any 2-coloring of the plane there are two points an inch apart that are the same color. (This is easy: I was able to do it in 2 minutes.)
2. Prove that for any 3-coloring of the the plane there are two points an inch apart that are the same color. (This is easy: I was able to do it in 3 minutes.)
3. ADDED COMMENT: D.N.J. de Grey showed, in 2018, that for any 4-coloring of the plane there are two points an inch apart that are the same color. This proof require the use of a computer.
4. Prove that there is a 7-coloring of the plane such that for all points p, q that are an inch apart, p and q are different colors. (This is easy: I was able to do it in 7 minutes.)
5. Find the number χ such that (1) for any $(\chi - 1)$-coloring of the plane there will be two points an inch apart that are the same color, and (2) there exists a χ-coloring of the plane such that for all points p, q that are an inch apart, p and q are different colors. (This is open: I was unable to do this in χ minutes.)

Alex uses the symbol χ for this quantity throughout the book; hence we will use the symbol χ for this quantity throughout the review.

The problem of determining χ is called the *Chromatic Number of the Plane Problem* and is abbreviated *CNP*. Alex told me *CNP is the most important problem in all of mathematics.* I think his point was that its important to work on problems that can be explained to the layperson and that he was using this as an illustration. Or maybe he really does think so. I hope he does—the world needs idealists.

After seeing Alex's talk I asked my colleague Clyde Kruskal what happens if only a subset of the plane is colored. For example, what is the largest square that can be 2-colored? 3-colored? Clyde then obtained full characterizations of 2 and 3-colorings for rectangles and regular polygons [1]. The paper contains the following marvelous result: an $s \times s$ square is 3-colorable iff $s \leq 8/\sqrt{65}$.

After talking to Alex I very much looked forward to his book. I first got my hands on it at the SODA (Symposium on Discrete Algorithms) conference of 2009. The Springer-Verlag book vendor let me read parts of it during the coffee break. I later got a copy and read the whole thing.

3 What Kind of Book is this?

When I first read the book I noticed something odd. The first sentence is *I recall April of 1970.* **Most of the book is written in the first person, like a memoir or autobiography!** The only parts that are not written in first person is when someone else is doing the talking.

In Alex's honor my review is written in his style.

Ordinary math books are not written in the first person; however, this is no ordinary math book! I pity the Library of Congress person who has to classify it. This book contains much math of interest and pointers to more math of interest. All of it has to do with coloring: Coloring the plane (Alex's favorite problem), coloring a graph (e.g., the four color theorem), and of course Ramsey Theory. However, the book also has biographies of the people involved and scholarly discussions of who-conjectured-what-when and who-proved-what-when. When I took Calculus the textbook had a 120-*word* passage about the life of Newton. This book has a 120-*page* passage about the life of van der Waerden.

Is this a math book? YES. Is this a book on history of Math? YES. Is this a personal memoir? YES in that the book explicitly tells us of his interactions with other mathematicians, and implicitly tells us of his love for these type of problems.

Usually I save my opinion of the book for the end. For this book, I can't wait:

This is a Fantastic Book! Go buy it Now!

End of Excerpt of Prior Review

4 Something Old Something New, Something Borrowed, Something k-Colored

The first edition had eleven parts and 49 chapters. The new book has thirteen parts and 68 chapters. For a detailed description of what was in the first edition, see my review. For now, I list topics that are in both books. Note that for all topics listed one should add ... *and variants and history.*

1. The Chromatic Number of the Plane.
2. The Four-color Theorem and variants.
3. Ramsey Theory on graphs.
4. Ramsey Theory on the natural numbers (van Der Waerden's Theorem).
5. Euclidean Ramsey Theory
6. (My favorite part) Using Ramsey Theory on the integers to obtain results about coloring the plane.

The list above does not capture the breadth and depth of the book. Putting that aside, whats in the new book thats not in the old book?

4.1 The Chromatic Number of the Plane

Let χ be the least number so that there is a χ-coloring of the real plane such that no two points of the same color are an inch apart. For 68 years it was known that $4 \le \chi \le 7$. While people (including Soifer) studied variants of the problem there was no progress on the original problem.

Until 2018.

In that year D.N.J. de Grey showed that $\chi \ge 5$. The proof used a computer program. Soifer gives the history and context of the result since he had the best seat in the house to the events.

Recall that the proof of the 4-color theorem (both the original proof and a simpler one done later) used a computer program. Hence the comments on this kind of proof in the chapter on the 4-color theorem are more interesting now then they were in the original book.

4.2 There is a triangle-free graph Unit Distance Graph on 17 Vertices

De Bruin and Erdos

The first edition recounted the following competition:

4.3 The Nevanlinna Prize is now the IMU Abacus Medal

The first edition

5 Opinion

Who *could* read this book? The upward closure of the union of the following people: (1) an excellent high school student, (2) a very good college math major, (3) a good grad student in math or math-related field, (4) a fair PhD in combinatorics, or (5) a bad math professor.

Who *should* read this book? Anyone who is interested in math or history of math. This book has plenty of both. If you are interested in math then this book will make you interested in history of math. If you are interested in history of math then this book will make you interested in math. Any researcher in either mathematics or the history of mathematics, no matter how sophisticated, will find many interesting things they did not know.

References

[1] C. Kruskal. The chromatic number of the plane: the bounded case. *Journal of Computer and System Sciences*, 74:598–627, 2008. `www.cs.umd.edu/~kruskal/papers/papers.html`.

[2] A. Soifer. *The scholar and the state: In search of Van der Waerden*. Springer-Verlag, New York, Heidelberg, Berlin, 2014.