

Review of
Tales of Zero and One:
A Book About the Beauties of Binary
Author of Book: Harry Lewis
Author of Review: Bill Gasarch

**1 Do you have a relationship with the author.
If so, please explain.**

He was my advisor for my PhD that I got in 1985. We are still in email contact. I initially asked if that was a conflict and was told that it is not.

**2 What is the author hoping to accomplish,
and to what extent do you believe the work
will be successful, based on submitted ma-
terial.**

The author wants to show that just from 0 and 1 much mathematics and computer science can be derived. The author takes us on a wonderful tour of, not just math and computer science, but also history and logic. The author accomplishes what he set out to do.

**3 Are you currently teaching a course for which
this book might be used as a core text, or
have you taught such a course in the re-
cent past? (list course name, how often it
is offered, level and typical enrollment)**

No.

4 Is the work original and the scholarship sound?

The book's intent is to present known material. Nevertheless, the blend of mathematics and history is original as most math books either ignore history or do a shoddy job of it. The scholarship is excellent in that the history is well researched and tells the reader episodes they didn't know (such as that Shannon and Fano were unable to come up with the best prefix-free code that encoded one letter at a time).

5 What are the best textbooks already published on the subject, and how does this work compare with them?

1. This book is not really a textbook.
2. There are no other books like it. The combination of topics makes sense but has never been done before.
3. The book combines *correct well researched history* with *correct and interesting mathematics* which is rare.

6 What textbook(s) are you using for this course? What are the strength's and weaknesses of this book relative to those you are using?

This book is not really a textbook.

7 What is your opinion of the proposed manuscript's level, style, organization, and length? Based on the writing sample, will the manuscript require extensive copy editing to improve its readability

The books length is fine. For the next section of the review I answer two of your questions which both ask for suggestions for improvement. If the author does those suggestions then the book will not need extensive copy-editing.

8 Please list any technical or mathematical errors you encountered. How could this project be improved? Can you cite specific sections or chapters that should be added or reconsidered?

I list in this section all suggestions I have for improvement, whether or not they are errors.

Preface

The preface should say what the prerequisite for the book is and hence who the intended audience is.

1) Binary Codes

1. Page 3. The book says that the Inka Khipu of *pre-Colombian America* have a claim to have used binary. *pre-Colombian* is a rather long time. The book should say what years they are talking about.
2. Page 12. Paragraph three begins
The argument for eight-bit bytes
Page 12. Paragraph four begins
The problem with eight-bit bytes
There are two reasons paragraph four should begin

The argument against eight-bit bytes

- 1) Symmetry with paragraph 3
- 2) The paragraph's beginning makes the reader think that 8-bit bytes lost. In fact, it initially reads as if Fred Brooks lead the movement against eight bit bytes.
3. Page 13. The book seems to say that the unicode can code any symbol from any language. How many bits are used? The book mentions that the horse head symbols uses 17 bits. In the next section you say that its a variable length code; however, this should be brought up earlier since the book initially reads as if the unicode is fixed length.
4. Page 19. The book defines entropy and gives examples. The book wisely does not get into the mathematics of why its the right definition. But the book should say that there is a way to either derive the formula or prove it is correct using mathematics.
5. Page 22. The book states the lifting lemma informally and then proves it informally. Later the Lifting Lemma is referred to but hard to find. The lifting lemma should be stated by using the word **Lemma** or **Lifting Lemma**. With that, it is probably good to have an informal statement and definitely good to have an informal proof.
6. Page 25. The paragraph beginning *Finally we are in a position...* In this paragraph he says that we are going to use induction. This is a sophisticated notion. So I went to the Preface to see who the intended audience is. There is nothing about that. If the level is of a lower one than students who know induction then the author might want to have starred sections, or say *you can skip this proof* or some mechanism of that sort. If Induction proofs occur often in the book AND the target audience is below that level, then a short appendix that is an introduction to induction would be appropriate.
7. Page 27.

David Huffman avoided taking that final exam in 1951 and published his discovery a year later in a four page paper

- 1) The phrase *that final exam* makes the reader think that there must have been a prior reference to the exam. There isn't.

2) This sentence seems to assert two independent facts whose connection was on page 20 but the reader might have forgotten it by now.

2) Binary Numerals

1. Page 33. In the description of how to multiply 23 by 27 the book says *Start with 1 and repeatedly double —*

$$1, 2, 4, 8, 16$$

*—and stop there since the next doubling, to 32, would exceed 23. At the same time, start with 27 and keep doubling **it** the same number of times—*

$$27, 54, 108, 216, 432.$$

Then take the numbers in the first sequence that add up to 23:

$$1 + 2 + 4 + 16 = 23$$

*and add up the numbers **in the corresponding positions of the second sequence:***

$$27 + 54 + 108 + 432 = 621$$

to get the answer: $23 \times 27 = 621$, using only doubling and addition.

At first reading I had a hard time figuring out what **in the corresponding positions of the second sequence** meant. By reverse engineering I think you mean:

Write 23 in binary 10111.

The places in the number that are 1's are, reading left to right, 1st, 2nd, 3rd, and 5th. Hence you add the 1st, 2nd, 3rd, and 5th elements of the sequence

$$27, 54, 108, 216, 432.$$

But that was not clear from what was written.

2. Page 33 (this may be dealt with later). The usual method people do for multiplication is $O(n^2)$ time for two n -digit numbers. Do the Rhind Papyrus method and the Russian Peasants method for multiplication do better? If not, then why were they used? I suspect that they were used before we had our current system of numbers and hence our algorithm would be hard to do then.
3. Page 41. A faster way to add. The book says that in principle a human could use this algorithm. That may be technically true but I can't see a human ever using the algorithm (though computers of course do). The book should point this out.
4. Page 43. The book makes the point that we can't just put a negative sign in front of a number to make it negative because then $+0$ and -0 are two different representations for 0. But base 10 has the same problem. This is not a base-2 problem. I think this is a computer problem—if we used base-10 for computers we would have that issue. So please clarify that this is a computer-issue, not a base 2-issue.
5. Page 43. The 2's-complement representation of -5 is 11111011. But this is also the representation of $2^0 + 2^1 + 2^3 + 2^4 + 2^5 + 2^6 + 2^7 = 251$. This seems ambiguous. The book should say why 1111101 does not have two interpretations. Are you only allowing 7-bit numbers? If so then see next point.
6. Page 43. If you are only allowing 7-bit numbers then what happens if you add together two large number so the sum is 8 bits?
7. Page 50. Second paragraph. *On the same sheet with the binary multiplication shown above, Harriot wrote the weight of a "glass of water," as shown in the illustration, of the empty glass, and of the difference.*
I can't tell if you are quoting Harriot or paraphrasing him, and also what the *illustration* refers to—is it in this book or is in something Harriot wrote, or what?
8. Page 50. You note that Harriot's sheets had no influence. Did Leibniz's work on binary have any influence?
9. Page 56. The story of how Shannon took his knowledge of logic (from a class in philosophy) and used it to revolutionize computing is interesting

but a lot is left out. Had the systems already gone from analog to digital? What were the systems doing and how did he improve them.

10. Page 60. The last line. The book says that *extreme miniaturization has again caused computer hardware faults to be commonplace*. Why would that be?
11. Page 61. Second paragraph. The book refers to the parity bit for message of length 8 bits as *a hidden ninth bit*. In what sense is it hidden? Its transmitted and used. I think you want to say that its not part of the message being transmitted, but its not hidden.
12. Page 62. First paragraph. The book says that if a bit is transmitted four times then you could use majority rule. But you can't: what if you receive 0011?
13. Page 62. In the first paragraph you refer to the 4-bit code as detecting that there were 2 errors. In the second paragraph you refer to the (7,4) code detecting that there were 2 errors. This does not seem like a win: you are back to knowing there were errors but not knowing what to do about them. You should point out that you have detected errors but can't fix them, and then you can ask the sender to resend. This is fine if it does not happen often.
14. Page 63-64. You seem to have defined Hamming Codes in two different way that I assume are equivalent (and thats really interesting).
Way 1: given a string of length 7 find he closest string to it in the code.
Way 2: Parity check for its 3-5-7 and 3-6-7 and 5-6-7.
If these (or something like it) is equivalent they you should mention that.
15. Page 67. The first sentence does not parse well:
Now if you have a Gray code of any length, say n , it's a simple matter to get a Gray code of length one greater, $n + 1$.
The *length one* looks like you are going to get a Gray code of length one.
Better:

Now if you have a Gray code of any length, say n , it's a simple matter to get a Gray code of length $n + 1$.

16. Page 69. Theorem 6 is interesting in that one would think (indeed—I did think) that the Gray code has no simple relation to binary numbers. However, is it important? Its fine (and should still be included) if its not important, but the author should say if the connection is an interesting tidbit or actually used.
17. Page 70-72. On Page 70 the title of the section is *Gray Codes and the towers of Hanoi*. But the section does not seem to mention Gray codes.
18. Page 71. The algorithm for Towers of Hanoi is about spindles a, b, c . But the picture uses spindles 1, 2, 3. They should both use the same notation.

3) Axel Thue's Beautiful String

1. Page 76. The paragraph beginning *But that doesn't rule out* mentions *two* different strings:
 - A square-free infinite string on a 3-letter alphabet.
 - A cube-free infinite string on a 2-letter alphabet.

That is the order they are presented in.

Then the book discusses these two strings in the reverse order. This was confusing. One way to help this confusion is to have a subsection labelled

An Infinite Cube-Free String on a 2-letter Alphabet

And later have a (short) section titled

An Infinite Square-Free String on a 3-letter Alphabet

2. Page 85. Theorem 18. If I only care about subsets of $\{0, \dots, 2^n - 1\}$ then I can get this theorem without the Thue string

Example: $\{0, 1, 2, 3, 4, 5, 6, 7\} = \{0, 7, 1, 6\} \cup \{2, 5, 3, 4\}$.

Hence the Thue string is probably more needed for the squares case.

It would be nice to put in the squares theorem, but that is probably to hard.

4) The Bit String that Answers All Questions

1. Page 89. The last paragraph

It was during that world's fair that the eminent mathematician David Hilbert rose at the Sorbonne to address the international congress of Mathematicians.

The word *during* is ambiguous here. Does the book mean that it was in the same time frame or that he actually gave the address *at* the world's fair (which I would find strange, though perhaps the common person knew more math then than they do now.)

2. Page 90. The book should say how many problems Hilbert proposed which was 23.
3. Page 90. The book should make clear that *Hilbert Problems* and *The Hilbert Program* are different, though one could interpret some of the Hilbert Problems as being in the Hilbert program.
4. Page 92. ... *other Jewish scholars* ... makes it seem as though Godel or Hilbert were Jewish, which they were not.
5. Page 93. ... *British fifty-pound note, shown here.*

If possibly you should number your figures and pictures and hence say things like

shown in picture 4.1

6. Page 93-94. Turing worked in logic on computability, and also with actual computers for the war effort. Did either one influence the other?
7. Page 97. Some reals have two representations as infinite binary strings like

0.011111...

and

0.1...

It is possible that you use one of the representations and the diagonal proof gives the other one.

The book should mention this point, if only to say that it can be dealt with.

8. Page 102. The paragraph beginning *We'll skip* has two typos:
uncomputabiltyy and *Forma poof*
9. Page 104. Typo at the beginning of Section 4.6.
... *since Alonzo Church proved it first in less convincing way*
should be
... *since Alonzo Church proved it first in a less convincing way*
10. Page 104. People were unimpressed that HALT was undecidable since HALT is a problem about computation, or perhaps a contrived problem. On Page 91 you talk about Godel's Theorem. Were there some people who were unimpressed since the theorem proven unprovable is unnatural (far more unnatural than HALT)?
11. Page 104. You should mention that Emil Post lost an arm in a car accident when he was 12.
12. Page 105. The sentence that begins
To formulate an undecidable ...
is too long. Break it into two sentences.
13. Page 106. The rules $u_i \rightarrow v_i$ look like you are just replacing one letter with another letter. Make it clear that u_i, v_i are strings of any length.
14. Page 111. Hilbert's tenth problem is thought of as having been solved by Davis-Putnam-Robinson and Matiyasevich.

9 How could this project be improved? Can you cite specific sections or chapters that should be added or reconsidered?

The improvements to this project are in Section 8. All of the corrections are local. There are no chapters that should be removed. There is no body of material that would lead to a new chapter.

10 Based on the limited content available, are there any chapters/sections that stand out as being particularly weak or particularly strong?

The parts of each chapter on history stand out as being particularly strong since every reader, novice or expert, will learn something they didn't already know.

The chapter on Thue's wonderful string is wonderful since, even though its mathematics, the novice will understand it, and many mathematicians do not know the material.

The chapter on The Bit String that answers is good for the novice but is well known to experts.

None of the chapters should be omitted, added to, or shortened.

11 What is your overall reaction to this proposal?

My overall reaction is positive. This is a great book that ties together many threads.