

Remaining corrections

- 1) **Page 7, new zeta-function paragraph and note.** Several small errors were introduced in the new material:

- (a) “the Riemann zeta function is $\mathbb{C}$ defined by” should be “the Riemann zeta function is defined by.”
- (b) “It’s trivial roots are the are the negative even integers” should be “Its trivial roots are the negative even integers.”
- (c) “From this equation you an derive” should be “From this equation you can derive.”

The mathematical idea in the note is right: at negative even integers, the sine factor in the functional equation vanishes while the other relevant factors are finite.

DONE

- 2) **Page 8, RH statement and logarithmic integral.**

- (a) The RH statement contains an old fragment after the completed sentence: “that are not negative even integers are on the line . . .” Delete that entire fragment.

DONE but I also changed it a bit.

- (b) The note after $\text{Li}(x)$ says that $1/\ln t$ has a singularity at $t = 0$. The relevant singularity is at $t = 1$. Also, use “principal value,” not “Principle Value.” Thus write:

$$\text{Li}(x) = \text{PV} \int_0^x \frac{dt}{\ln t}, \quad \text{where the principal value is needed because of the singularity at } t = 1.$$

DONE except that I use *principle value*. The memory device for principle vs principal is

The principal is your pal.

- 3) **Page 9, Lagarias criterion.** The divisor sum is missing its summand in both copies of the displayed inequality. Define

$$\sigma(n) = \sum_{d|n} d$$

$$\sigma(n) \leq H_n + \exp(H_n) \log(H_n),$$

with equality only for $n = 1$. In Note 7.5, the sentence “We speculate if this would lead” is incomplete. Since the same question is asked immediately afterward in Open Problem 7.6, I suggest deleting that incomplete sentence.

DONE

- 4) **Pages 8–9, two RH usage/style points.** Of the RH occurrences I checked, two would read more consistently as follows:

- (a) “The RH seems like a continuous problem” $\rightarrow$ “RH seems like a continuous problem.”
- (b) In the final question of Open Problem 7.6, write “ M halts iff the RH is false,” matching the two preceding displays.

DONE

5) Pages 11–12, new Collatz definitions and Korec paragraph.

- (a) Delete the final comma in $\text{Colmin}(a_0) = \min\{a_0, a_1, a_2, \dots, \}$.
- (b) “One way to rephrase the conjectures” should be “One way to rephrase the conjecture.”
- (c) Use ≈ 0.792 , not ~ 0.792 , in $\frac{\log 3}{\log 4} \approx 0.792$; this is a numerical approximation, not an asymptotic relation.

DONE. I did not know about the $\approx$ command! Great!

6) Page 12, new Tao paragraph. Two mathematical details need adjustment. Tao’s hypothesis is not merely that f is increasing; it is that $\lim_{N \rightarrow \infty} f(N) = +\infty$. Also, the logarithmic-density statement should assert the limit, not say “if the limit exists.” For the set S defined in the paper, write

$$\lim_{x \rightarrow \infty} \frac{1}{\log x} \sum_{\substack{n \leq x \\ n \in S}} \frac{1}{n} = 1.$$

For the comparison sentence, I suggest:

Korec obtained a power bound for a set of natural density one. Tao obtained an arbitrarily slowly growing bound, but with logarithmic density one rather than natural density one.

DONE- a bit diff from what you said.

7) Page 21, proof of Theorem 13.2 and Definition 13.3.

- (a) “We use this proof to show to prove $\text{CON}(T)$ in T ” should be “We use this proof to prove $\text{CON}(T)$ in T .”
- (b) The padding paragraph is mathematically in the right place and answers the earlier question. In the source, however, it uses raw T and N_T instead of the paper’s macros $\backslash\text{TT}$ and $\backslash\text{NN_TT}$. This renders correctly, so this is only optional source consistency cleanup.

DONE

8) Page 22, Open Problem 13.5.

- (a) “Such a method may lead to smaller state count” should be either “Such a method may lead to a smaller state count” or “Such methods may lead to smaller state counts.”
- (b) In item 2, replace “there is an s -state TM with M halts iff $\neg \text{CON}(T)$ ” with “there is an s -state TM M such that M halts iff $\neg \text{CON}(T)$.”

DONE

9) Page 22, acknowledgments.

- (a) “their proofread of the paper” should be “their proofreading of the paper.”
- (b) Usage/style: “11 pages of corrections that greatly improved the paper” reads more naturally than “11 pages of corrections which greatly improved the paper.”

DONE

10) Pages 24–25, bibliography.

- (a) The DMR entry still has “Yuri Matijasevic.” Use “Yuri Matiyasevich.”

DONE

- (b) The Lagarias and Guy entries print *American Mathematics Monthly*. The journal’s name is *The American Mathematical Monthly*.

DONE- and I also corrected many *American Mathematical Monthly* entries in my files.

- (c) The BBC-2024 entry still renders as “The bbcahllenge Collaborative-Website.” The word *bbchallenge* is misspelled, and the unwanted “Collaborative-Website” remains. If the intended institutional author is The bbchallenge Collaboration, use

`author = {{The bbchallenge Collaboration}},`

DONE

Answers to your questions

- 1) **Broader reformulations / Zoom.** No hidden set of corrections or Zoom call was implied by my earlier sentence. I omitted the broader reformulations because they were not clear-cut corrections and were not necessary for this proofreading pass.

Source checks

- Lagarias criterion: <https://arxiv.org/abs/math/0008177>.
- Tao’s Collatz theorem: <https://arxiv.org/abs/1909.03562>.
- xurl documentation: <https://ctan.org/pkg/xurl>.