1 Introduction

Consider a final round of Jeopardy! with players Alice and Betty. We assume that going into this round Alice has the highest monetary total. The procedure is as follows:
1. The players hear the category of the final question.
2. The players place bets.
3. The host gives them the clue in the form of an answer and the players attempt come up with the correct response in the form of a question.
4. The correct response is revealed.
5. If a player responds correctly, she adds the amount she bet to her total. If she responds incorrectly, she loses that amount from her total.
6. The player with the most money takes home her total. In Jeopardy! the second-place player ends the game with 2000 dollars. However, for the purposes of this paper we will assume the losing player has payoff a of 0 dollars. This simplification may favor a safer betting strategy in extreme edge cases. However, in these cases it simply serves to model slightly loss-averse players.
7. If there is a tie, the players answer tie-breaking questions (without betting) until exactly one of the players responds correctly. This player wins the game and takes home her money. The other player leaves with no money.

For the purposes of this paper, Alice will always be defined as the player with the most money. Betty will always trail Alice going into the final round.

In the first section of our paper, this payoff is strictly defined as the expected amount of money that the player will bring home on that day. We will refer to our exploration of this payoff as our single-day analysis. We determine optimal betting strategies, defined by the maximin bet, for Alice and then for Betty.

Our maximin strategies and solutions for this single-day scenario may be generalized to other simultaneous single-action betting games. However, we next move to considering a slightly different, more Jeopardy!-specific problem. In Jeopardy!, the winning player goes on to play in later games. Therefore, we consider a scenario in which each player’s payoff includes the expected amount
of money she would win on following days, given that she triumphs in the current game. In order to provide concrete suggestions to players, we make some simplifying assumptions based upon historical Jeopardy! data.

\section*{2 Previous Work}

Gilbert and Hatcher \cite{1} defined the structure of the two-player payoff matrix which we utilize in this paper. In their paper, they define the game’s payoff as the players’ probabilities of winning and identify equilibrium points and mixed strategies based on this payoff. Ferguson and Melolidakis \cite{2} utilized this same payoff matrix in their 1997 paper to identify minimax points.

While both these papers define the payoff as a player’s probability of winning the game, in this paper payoff will be defined as the expected amount of money the player will win. As a result, our payoff matrix is continuous and discontinuous in the same areas as previous matrices, but the payoffs themselves are different. Our paper will more closely follow Ferguson and Melolidakis’s minimax strategy than Gilbert and Hatcher’s equilibrium strategy. However, unlike in Ferguson and Melolidakis’s paper, our payoffs are not zero-sum, and therefore we utilize a maximin strategy instead of a minimax strategy.

Our distinct payoff definition leads to significant strategical differences between our work and both previous papers. For example, when Alice has more than twice the amount of money that Betty has, she can always bet $0$ dollars to ensure herself a win. A strategy focused on win probability will always recommend this bet given both the equilibrium and minimax strategies. However, according to our payoff definition, in which Alice wants to maximize her expected monetary winnings, there are cases in which she should bet all her money instead.

The Gilbert and Hatcher paper and the Ferguson and Melolidakis paper have both excluded a third player from their calculations, and we follow this model. While Jeopardy! is always played with three players, often the third player has a non-positive sum of money and therefore cannot participate in Final Jeopardy! Ferguson and Melolidakis note that given their primary analysis of a three-player strategy, adding a third player with the least money is unlikely to significantly alter the strategy. We similarly make this simplification. However, some of our maximin proofs can be extended to a three-player game, as will be noted in those proofs.
3 Variables

Table 1: Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_A )</td>
<td>Alice’s money coming into the final question</td>
</tr>
<tr>
<td>( M_B )</td>
<td>Betty’s money coming into the final question</td>
</tr>
<tr>
<td>( P_A )</td>
<td>Alice’s probability of responding to the final question correctly</td>
</tr>
<tr>
<td>( P_B )</td>
<td>Betty’s probability of responding to the final question correctly</td>
</tr>
<tr>
<td>( Z_A )</td>
<td>The amount of money Alice bets</td>
</tr>
<tr>
<td>( Z_B )</td>
<td>The amount of money Betty bets</td>
</tr>
<tr>
<td>( T_A )</td>
<td>Alice’s probability of winning a tiebreaker</td>
</tr>
<tr>
<td>( T_B )</td>
<td>Betty’s probability of winning a tiebreaker</td>
</tr>
</tbody>
</table>

4 The Maximin Strategy

In order to find optimal strategies for players, we consider what a player can bet to maximize her minimum payoff. The bet which accomplishes this is her maximin bet, and the smallest possible payoff given that bet is her maximin.

The maximin strategy is a common game theoretical approach for simultaneous single-action games, as it provides a worst-case payoff guarantee regardless of other players’ actions. This strategy simulates a scenario in which players have no knowledge of others’ bets and do not have multiple rounds of betting to reach an equilibrium point.

For example, let us consider the scenario in which \( M_A = 7 \), \( P_A = 0.3 \), \( M_B = 4 \), and \( P_B = 0.5 \). The following payoff matrix represents all possible combinations of bets for Alice and Betty, where the row represents Alice’s bet and the column represents Betty’s bet. The first value in each cell represents Alice’s payoff and the second represents Betty’s payoff given the corresponding bets. We round payoffs to the nearest hundredth of a dollar in this example. Here, we assume a player’s chances of winning a tiebreaker, and thus winning the game given a tie, to be proportional to her relative monetary success thus far in the game. Thus, Alice’s chance of winning given a tie is \( \frac{M_A}{M_A + M_B} \). Later, we refer to this tie value with the variable \( T \) to maintain generality.
Table 2: Payoff Matrix

<table>
<thead>
<tr>
<th>Alice’s bet</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(7, 0)</td>
<td>(7, 0)</td>
<td>(7, 0)</td>
<td>(5.73, 1.27)</td>
<td>(3.5, 4)</td>
</tr>
<tr>
<td>1</td>
<td>(6.6, 0)</td>
<td>(6.6, 0)</td>
<td>(5.83, 0.76)</td>
<td>(4.5, 2.45)</td>
<td>(4.06, 3.24)</td>
</tr>
<tr>
<td>2</td>
<td>(6.2, 0)</td>
<td>(5.56, 0.64)</td>
<td>(4.45, 2.1)</td>
<td>(4.45, 2.45)</td>
<td>(4.45, 2.8)</td>
</tr>
<tr>
<td>3</td>
<td>(4.78, 1.02)</td>
<td>(4.4, 1.75)</td>
<td>(4.4, 2.1)</td>
<td>(4.4, 2.45)</td>
<td>(4.4, 2.8)</td>
</tr>
<tr>
<td>4</td>
<td>(3.3, 2.8)</td>
<td>(3.97, 2.13)</td>
<td>(4.35, 2.1)</td>
<td>(4.35, 2.45)</td>
<td>(4.35, 2.8)</td>
</tr>
<tr>
<td>5</td>
<td>(3.6, 2.8)</td>
<td>(3.6, 2.8)</td>
<td>(4.05, 2.35)</td>
<td>(4.3, 2.45)</td>
<td>(4.3, 2.8)</td>
</tr>
<tr>
<td>6</td>
<td>(3.9, 2.8)</td>
<td>(3.9, 2.8)</td>
<td>(3.9, 2.8)</td>
<td>(4.12, 2.58)</td>
<td>(4.25, 2.8)</td>
</tr>
<tr>
<td>7</td>
<td>(4.2, 2.8)</td>
<td>(4.2, 2.8)</td>
<td>(4.2, 2.8)</td>
<td>(4.2, 2.8)</td>
<td>(4.2, 2.8)</td>
</tr>
</tbody>
</table>

Alice can only impact her own bet, and has no control over Betty’s choice. Thus, she can choose which row of probabilities to accept, but cannot control which value within that row will represent her actual payoff. Similarly, Betty can choose the column of the bet combination, but not the row. In this case, since Alice aims only to maximize her worst-case payoff, she will choose to bet 2 dollars, since this row provides the largest worst-case payoff, leaving Alice at least a payoff of 4.45 dollars. Alice can ensure at least a payoff of 4.45 dollars no matter Betty’s bet. Thus, 4.45 is Alice’s maximin and 2 dollars is her maximin bet. By examining the minimum payoff for Betty in each column, one can see that Betty’s maximin is 2.8 dollars corresponding to her maximin bet of 4 dollars.

In the first part of our single-day analysis, we will examine when a dominant strategy bet exists for the leading player Alice. A bet that is a dominant strategy must be a maximin bet, because by definition a dominant strategy is optimal in every case. Therefore the scenario resulting in the dominant bet’s lowest possible payoff results in at least as low payoffs for all other bets. If a dominant strategy exists, then, this is the maximin bet.

When no dominant strategy exists, we find maximins by utilizing our structural knowledge of the two-player payoff matrix.

5 The Payoff Matrix

Table 2’s values can be generalized when \( M_A, M_B, P_A, \) and \( P_B \) are instead variables. This general payoff matrix has a predictable structure.

This matrix has distinct regions defined by their own payoff equations and are discontinuous from region to region. These different regions relate to the
four possible outcomes of the players responding to the final question: both players respond correctly (CC), neither player responds correctly (II), only Alice responds correctly (CI), or only Betty responds correctly (IC). Whether the outcome is a win, loss, or a tie for a player in each case depends on how much each player bets. Each region of the matrix represents combinations of bets that have the same winner for each outcome.

The general shape of the payoff matrix can be seen in Figure 1 with the winner in each of the four Final Jeopardy! outcomes (CC, II, IC, and CI, respectively) listed along with each region. ‘A’ indicates that Alice wins, ‘B’ indicates that Betty wins, ‘T’ that there is a tie, and ‘L’ that, as they both end with 0 dollars, both players lose. For example, region 6 is labeled ABBA because Alice ends the game with the most money in the CC and CI case and Betty does otherwise. The diagonal regions 1, 3 and 5 are "tying lines," places in which some outcome results in a tie. Regions 2 and 4 are intersections of tying lines where more than one outcome results in a tie. These regions each exist only for a single bet combination. The shape of the payoff matrix changes depending on the ratio between $M_A$ and $M_B$, but the regions below remain relatively consistent when $M_B < M_A < 2M_B$.

Figure 1: General Structure of the Payoff Matrix

The following table identifies the payoff equations for Alice within each region.
### Table 3: Payoff Equations by Matrix Region

<table>
<thead>
<tr>
<th>Region</th>
<th>Alice’s Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$P_A(M_A + Z_A) + T_A * (1 - P_A)(1 - P_B)(M_A - Z_A)$</td>
</tr>
<tr>
<td>2</td>
<td>$T_A * (1 - P_A)(M_A - Z_A) + P_A(M_A + Z_A)$</td>
</tr>
<tr>
<td>3</td>
<td>$P_A(M_A + Z_A) + (1 - P_A)((1 - P_B)(M_A - Z_A) + T_A(P_B)(M_A - x))$</td>
</tr>
<tr>
<td>4</td>
<td>$(1 - P_B)(P_A(M_A + Z_A) + (1 - P_A)(M_A - Z_A)) + T_AP_B((P_A)(M_A + Z_A) + (1 - P_A)(M_A - Z_A))$</td>
</tr>
<tr>
<td>5</td>
<td>$T_A(P_A)(M_A + Z_A) + (1 - P_A)(1 - P_B)(M_A + Z_A) + P_A(1 - P_B)(M_A + Z_A)$</td>
</tr>
<tr>
<td>6</td>
<td>$P_A(M_A + Z_A)$</td>
</tr>
<tr>
<td>7</td>
<td>$P_A(M_A + Z_A) + (1 - P_A)(1 - P_B)(M_A - Z_A)$</td>
</tr>
<tr>
<td>8</td>
<td>$P_A(M_A + Z_A) + (1 - P_A)(M_A - Z_A)$</td>
</tr>
<tr>
<td>9</td>
<td>$(1 - P_B)(P_A)(M_A + Z_A) + (1 - P_B)(1 - P_A)(M_A - Z_A)$</td>
</tr>
<tr>
<td>10</td>
<td>$P_A(M_A + Z_A)$</td>
</tr>
</tbody>
</table>

### 6 Payoff Matrix Maximin Strategy

As the expected amount of money a player gains depends not only on the cases in which the player wins but how much money the players bet, payoffs are not constant within each region. However, the payoffs are still defined by continuous equations in each region. Furthermore the payoff equations for a player never include the other player’s bet as a variable. Therefore, given a player’s, let’s say Alice’s, bet, her payoff is impacted only by which region Betty makes the bet combination fall in, not what specific value Betty bets. To illustrate this concept, let us refer to Table 2. If Alice bets 3 dollars, Betty could cause the bet combination to fall into region 2 or 7. When Betty bets more than 0 dollars given Alice’s bet, Alice’s payoff will be 4.4 dollars. Similarly, if Betty bets 0 dollars, Alice can cause the bet combination to fall into region 8, 2, or 6. If Alice bets more than 3 dollars, the bet combination will fall into region 6 and Betty’s payoff will be 2.8 dollars regardless of Alice’s specific bet.

Regardless of Betty’s specific bet, Alice’s payoff will be 4.4 dollars. Similarly, if Betty bets 0 dollars, Alice can cause the bet combination to fall into region 8, 2, or 6. If Alice bets more than 3 dollars, the bet combination will fall into region 6 and Betty’s payoff will be 2.8 dollars regardless of Alice’s specific bet.

The number of payoff possibilities for a player that could result given a single bet is therefore equal to the number of distinct regions which exist in the row or column that she bets.

Given a player’s single bet, $Z$, $P_A$, $P_B$, $M_A$, and $M_B$ are all constant. Therefore, the payoff equations that this bet could fall into can be compared directly.
In order to find the lowest possible payoff for a player’s single bet, we can determine the least favorable payoff equation. As the players win, lose or tie in different combinations in each region, it is often clear which payoff equation is inferior. For example, region 6’s winner results are ABBA, while region 7’s are AABA. Alice wins in a case in region 7 where she loses in region 6. Therefore, given a certain bet by Alice, if these are the two potential payoff equations, region 6 would certainly offer her the lower payoff.

Once lowest payoff(s) for each of the player’s potential bets are found, the next step is to maximize them. Within the bets which share the same min region of 6, for example, the best choice is the bet where the payoff in region 6 is the highest. Region 6’s payoff equation is optimized when Alice bets \(M_A\). Therefore if the maximin is in region 6, it will be when Alice goes all in.

In this way, the number of potential maximins can be reduced down to at most a handful. From there, it may be necessary to directly compare payoff equations to determine the maximin and therefore the maximin betting strategy.

Overall, this technique for finding the maximin for a player involves finding the region with the worst payoff equation for each of her bets, maximizing the payoffs within those least desirable regions, then comparing those payoffs to find the one that affords the largest value, which is the maximin. We use this payoff matrix technique when using dominant strategies is not a possibility.

7 Alice’s Strategy

Before utilizing the payoff matrix maximin strategy, we will examine when general dominant strategies can be found.

7.1 When \(P_A \geq \frac{1}{2}\)

When Alice’s chance of answering the question correctly is greater than 50%, there is always a dominant strategy for her regardless of how much money each player has or the other players’ probabilities of winning. We will find her maximin bet by finding this dominant strategy.

We can upper bound Alice’s payoffs for all bets with the assumption that she wins the game in all Final Jeopardy! outcomes. This is expressed by the equation:

\[
E_{Z_A} \leq P_A(M_A + Z_A) + (1 - P_A)(M_A - Z_A)
\]

\[
= M_A + Z_A(2P_A - 1)
\]

When Alice bets \(M_A\) dollars, and therefore \(Z_A = M_A\), she will always lose if she responds to the question incorrectly, so her payoff is

\[
2P_A M_A
\]

\[
= M_A + 2P_A M_A - M_A
\]
= MA + MA(2PA - 1)

As ZA = MA, this equation is exactly equal to the equation

MA + ZA(2PA - 1)

Under the assumption that all other bets are equal to the upper bound described above, there is a linear relationship between ZA and EZA, expressed by the equation EZA = MA + ZA(2PA - 1).

When PA ≥ \(\frac{1}{2}\), 2PA - 1 ≥ 0, so we can maximize the payoff by maximizing ZA to MA. As betting MA yields a higher payoff than the upper bounds of all other bets when PA ≥ \(\frac{1}{2}\), this bet is a dominant strategy. Therefore Alice’s maximin bet is MA when PA ≥ \(\frac{1}{2}\).

Note that as long as Alice remains the leading player, this proof extends to the three-player case without loss of generality.

7.2 When PA < \(\frac{1}{2}\)

When PA < \(\frac{1}{2}\), Alice’s optimal bet has multiple cases:

7.2.1 When MA > 2MB

When the leading player Alice has more than twice Betty’s money, we can again find a dominant strategy. Alice will always win the game when betting 0 dollars. Therefore when ZA = 0 her payoff is MA. This value can again be expressed by the equation

MA + ZA(2PA - 1)

In this case, therefore, a bet of 0 is exactly equal to the upper bound equation of all bets. Let us assume that all other bets from 1 to MA also have payoffs equal to this upper bound. Under this assumption, there is a linear relationship between ZA and EZA, expressed by the equation EZA = MA + ZA(2PA - 1).

When PA ≤ \(\frac{1}{2}\), 2PA - 1 ≤ 0, so we can maximize EZA by minimizing ZA to 0. As betting 0 yields a higher payoff than the upper bounds of all other bets when PA ≤ \(\frac{1}{2}\), this bet is a dominant strategy. Therefore Alice’s maximin bet is 0 when PA ≤ \(\frac{1}{2}\) and MA > 2MB.

Note that as long as Alice has at least twice as much money as all other players, this proof extends to the three-player case without loss of generality.

7.2.2 When \(\frac{3}{4}MB < MA < 2MB\)

For the remaining cases, there are no clear dominant strategies for Alice in a three-player game. Therefore we will find Alice’s maximins utilizing the payoff matrix maximin strategy. The matrix’s structure changes depending on the relationship between MA and MB, and in the case where \(\frac{3}{4}MB < MA < 2MB\), the bottom of region 9 is at least one row above the top or region 6.
Figure 2: Structure of the Payoff Matrix when $\frac{3}{2}M_B < M_A < 2M_B$

Given this relationship between $M_A$ and $M_B$, Alice has three potential maximin bets.

If she bets less than $2M_B - M_A$, Alice’s worst region is region 9. As $P_A < \frac{1}{2}$, this equation is maximized when $Z_A = 0$. This will yield a minimum payoff of $(1 - P_B)M_A$ when betting 0.

If Alice bets exactly $2M_B - M_A$ dollars, her worst region is region 5. If Alice bets exactly $M_A - M_B$ dollars, her worst region may be either region 2 or region 7.

If Alice bets more than $M_A - M_B$, her worst region is region 6. This is maximized when Alice bets $M_A$ dollars, yielding a min payoff of $2P_AM_A$.

Finally, if Alice bets such that $2M_B - M_A < Z_A < M_A - M_B$, her worst region is region 7. We refer to these rows in which region 7 is the minimum payoff for Alice as min-7. As seen in Table 3, region 7’s payoff equation is

$$P_A(M_A + Z_A) + (1 - P_A)(1 - P_B)(M_A - Z_A)$$

When $P_A > (1 - P_A)(1 - P_B)$, this equation is optimized by maximizing $Z_A$ within min-7, and otherwise it is optimized by minimizing $Z_A$ within min-7. We must determine which of the potential maximin bets discussed above yield the highest payoff. We will define two different cases given the two optimizations of region 7.

**Case 1:** $P_A > (1 - P_A)(1 - P_B)$

Here, the higher that $Z_A$ is in region 7’s payoff equation, the greater the payoff.
is. However, to stay in min-7 Alice can only bet as much as $M_A - M_B - 1$. This means one potential maximin bet for Alice is $M_A - M_B - 1$.

However, this payoff is always lower than that of going all in. As mentioned previously, the minimum payoff equation for Alice when going all in is

$$P_A(M_A + Z_A)$$

When $Z_A = M_A$, this can be written as

$$P_A(M_A + Z_A) + (1 - P_A)(1 - P_B)(M_A - Z_A)$$

as $M_A - Z_A = M_A - M_A = 0$. This is the payoff equation in region 7.

Therefore we can consider Alice betting $M_A$ as a continuation of region 7. As Alice wants to maximize $Z_A$ given region 7’s payoff equation, she would rather bet $M_A$ than any other value. Therefore the potential maximin when betting $M_A$ always has a higher payoff than betting the most in min-7.

Betting $M_A - M_B$ is also inferior to going all in for Alice. When region 2 has a higher payoff than region 7, it is not even the minimum in its row. When it has a lower payoff, it is inferior to some payoff equation in region 7, which we have shown is itself inferior to going all in.

Betting $M_A$ is also superior to betting $2M_B - M_A$ to have a minimum in region 5, using similar logic.

Finally, betting $M_A$ is also a better strategy than betting nothing. As in this case $P_A > (1 - P_B)(1 - P_A)$ and $1 - P_A > \frac{1}{2}$:

$$P_A > (1 - P_B) \cdot \frac{1}{2}$$

Which by algebra, is equivalent to:

$$2P_A M_A > (1 - P_B) M_A$$

The value on the left is Alice’s minimum payoff when betting $M_B$ and the value on the right is her minimum payoff when betting 0.

Therefore when $P_A > (1 - P_A)(1 - P_B)$ and $P_A < \frac{1}{2}$, Alice’s minimum payoff given a bet of $M_A$ is higher than the minimum payoffs for all her other bets. Alice’s maximin, and therefore her optimal bet, occurs when Alice goes all in.

**Case 2:** $P_A < (1 - P_A)(1 - P_B)$

In this case, we would like to minimize $Z_A$ in min-7 to $2M_B - M_A + 1$. We can again consider the maximin when going all in as a continuation of region 7’s payoff equation, and therefore when region 7 should be minimized, going all in is clearly inferior to any bet in region 7 itself.

Region 2 is never a maximin in this case, for similar reasons as in the former case.

Therefore we must compare three payoffs for Alice: betting $2M_B - M_A + 1$ in region 7, $2M_B - M_A$ in region 5, or 0 in region 9. Without making further assumptions, all of these bets have cases in which they are the maximin bet.
Please refer to the code in the appendix for the specific numerical comparisons necessary.

We note that only in cases with both extremely high values of $T_A$ and extremely low values of $P_A$, $P_B$ and $M_B$ is it possible for region 5 to be the maximin, so for practical purposes players may be better off disregarding this option.

### 7.2.3 When $\frac{3}{2}M_B > M_A$

Figure 3: Structure of the Payoff Matrix when $\frac{3}{2}M_B > M_A$

In this case, any bet from Alice could result in at least one of region 6 or region 9, as the regions overlap vertically. These are the two potential maximin regions in the matrix. As we have previously shown, the payoff equation for region 9 is maximized to $(1 - P_B)M_A$ when $Z_A = 0$. The payoff equation for region 6 is maximized when $Z_A = M_A$, so the payoff is $2P_AM_A$.

These two payoffs represent potential maximins payoffs for Alice. She should bet 0 when:

$$(1 - P_B)M_A > 2P_AM_A$$

$$= (1 - P_B)/2 > P_A$$

And when $(1 - P_B)/2 < P_A$ she should bet $M_A$. If the two equations are equal, these bets are equally optimal payoffs for Alice.

### 8 Betty’s Strategy

Let us consider the optimal strategies for Betty, the trailing player.

#### 8.1 When $M_A > 2M_B$

In this scenario, when Alice bets 0 dollars, she can guarantee that the result is in region 8. Therefore all bets for Betty lead to the same maximin of 0 dollars,
so her bet does not matter. This is an accurate maximin strategy, as Betty makes no assumptions about Alice’s bet. However, if we assume that Alice is following the maximin strategy we have recommended, we can provide Betty with a more meaningful strategy.\footnote{As Alice’s bet in these cases is not only a maximin strategy but a dominant strategy, an optimal strategy for Betty given Alice’s optimal bet is in fact an equilibrium point.}

8.1.1 When $P_A \geq \frac{1}{2}$

Let us consider the situation where $P_A \geq \frac{1}{2}$. In this case, Alice will bet $M_A$ dollars\footnote{Alice could also bet 0 dollars if $P_A = \frac{1}{2}$. However, in this edge case Betty will have to hope that Alice goes all in, as this is her only hope in being able to win the game, and bet accordingly.}. We can write an equation for Betty’s payoff when Alice goes all in:

$$(1 - P_A)(1 - P_B)(M_B - Z_B) + (1 - P_A)(P_B)(M_B + Z_B)$$

In order to optimize this expression, $Z_B$ should be maximized when $P_B \geq \frac{1}{2}$, and minimized when $P_B \leq \frac{1}{2}$. We can conclude that if $P_A \geq \frac{1}{2}$, Betty has an optimal bet of $M_B$ if $P_B \geq \frac{1}{2}$ and of 0 if $P_B \leq \frac{1}{2}$ if we assume that Alice is betting her dominant strategy.

8.1.2 When $P_A < \frac{1}{2}$

In the scenario in which $P_A < \frac{1}{2}$, even when considering Alice’s strategy, Betty is out of luck. Alice will bet 0 dollars. As $M_A > 2M_B$, Betty cannot overcome Alice. Therefore all bets for Betty in this circumstance not only lead to the same payoff of zero dollars.

8.2 When $M_A < 2M_B$

If Betty bets less than $2M_B - M_A$, then her maximin is certainly in region 8, which leads to a payoff of 0. For any bet greater than this, her worst payoff equation is region 7. Therefore Betty’s maximin is in region 7. Thus, Betty would like to maximize her payoff in region 7. The payoff equation in this region is

$$(1 - P_A)(P_B)(M_B + Z_B)$$

Maximizing $Z_B$ to $M_B$ will maximize this equation to $(1 - P_A)(2P_B M_B)$. Betty should therefore go all in to secure her maximin bet in this circumstance.

9 Cheat Sheet for Alice

We will now restate the theorem for Alice’s optimal bet.

9.1 $P_A \geq \frac{1}{2}$

Alice’s maximin bet is $M_A$. 
9.2 \( P_A < \frac{1}{2} \)

9.2.1 \( M_A > 2M_B \)

Alice’s maximin bet is 0.

9.2.2 \( \frac{3}{2}M_B < M_A < 2M_B \)

Case 1: \( P_A > (1 - P_A)(1 - P_B) \)

Alice’s maximin bet is \( M_A \).

Case 2: \( P_A < (1 - P_A)(1 - P_B) \)

We must compare three payoffs for Alice: betting \( 2M_B - M_A + 1 \) in region 7, \( 2M_B - M_A \) in region 5, or 0 in region 9. Refer to the code in the appendix.

9.2.3 \( \frac{3}{2}M_B > M_A \)

Alice’s maximin bet is 0 when \( \frac{1 - P_B}{2} > P_A \) and \( M_A \) otherwise.

10 Cheat Sheet for Betty

10.1 \( M_A > 2M_B \)

When strictly considering maximins, every bet is equally bad for Betty, but if we assume Alice is betting her maximin:

10.1.1 \( P_A \geq \frac{1}{2} \)

Betty’s optimal bet is \( M_B \) if \( P_B \geq \frac{1}{2} \) and 0 if \( P_B \leq \frac{1}{2} \).

10.1.2 \( P_A < \frac{1}{2} \)

All bets lead to a maximin of 0 and a payoff of 0 dollars given Alice’s maximin strategy.

10.2 \( M_A < 2M_B \)

Betty’s maximin bet is \( M_B \).

11 Maximizing Overall Winnings: A Jeopardy!-Specific Scenario

In Jeopardy!, players who win a game come back to play the next day. Thus, if a player aims to maximize her payoff from Jeopardy! as a whole, not just on one specific day, she should consider her expected future winnings. We define "win benefit" to be the amount of money a player expects to win in future
Jeopardy! games should she win the game on the current day. In the following section, we consider a realistic perspective of a leading player going into Final Jeopardy! who hopes to take home the most possible money from all the games she participates in.

11.1 Simplifications and Statistical Assumptions

In order to realistically model the perspective of Alice, a leading player about to bet in Final Jeopardy!, we make some simplifications based on historical Jeopardy! data.

We assume Alice knows how likely she is to respond to the final question correctly based on her knowledge of the category of the final question. However, she likely does not know how knowledgeable Betty is on the topic of the final question. Thus, we use historical Jeopardy! data to estimate Betty’s chances of responding to the final question correctly and advise Alice’s bet based on this estimate. In the past 10 seasons of Jeopardy!, the average correct response rate on the final question is about 49.6% with a standard deviation of about 3.6% [3]. Thus, without knowing anything about Betty’s knowledge of the topic of the final question, Alice’s best assumption is that she has approximately a $\frac{1}{2}$ chance of responding correctly. We assume $P_B = \frac{1}{2}$ for the remainder of the paper.

We also use historical Jeopardy! data to quantify the win benefit. The average winnings on Jeopardy! are about 20,000 dollars per game. Furthermore, the returning champion wins Jeopardy! in about 46.26% of games [4]. Thus, using $\frac{a}{1-r}$, where $a = 0.4626$ and $r = 0.4626$, the equation for the sum of an infinite geometric series, a player can expect to win Jeopardy! about 0.86 more times after winning the first game. Thus, we multiply these values together to estimate that a player can expect to win about 17,000 dollars in future Jeopardy! games if she wins a current game. We use 17,000 dollars as the win benefit for the remainder of this paper.

We consider the cases in which Alice has between 0 and 50,000 dollars going into Final Jeopardy! The largest one-day winnings in Jeopardy! history is 77,000 dollars from a game in which the winning player entered Final Jeopardy! with 47,000 dollars [5]. Thus we assume Alice is unlikely to enter Final Jeopardy! with more than 50,000 dollars and provide advice to Alice accordingly.

We will not consider tying lines in this section. As we have shown in the previous section, these ties rarely impact a player’s optimal strategy.

11.2 Strategy

When considering the payoff with the win benefit, the first two steps for finding the maximin for a player using the payoff matrix strategy do not change. These steps are finding the region with the worst payoff equation for each of her bets and maximizing the payoffs within those least desirable regions. This is because the payoff equation for each region when a win benefit is added in
can be expressed as:

\[
\text{expected winnings that day} + (\text{probability of winning that day}) \times (\text{win benefit})
\]

We previously compared the payoffs of different regions given a single bet by comparing the outcomes in which the regions afforded each player a win. The expectation of the win benefit also scales by the number of cases in which a player wins.

In all cases where we determined that one region was strictly worse than another in a given row or column, it was because this region afforded the player in question equal or worse win/tie/loss results for every outcome. Therefore these regions also have lower win probabilities and thus their payoffs are still lower given a single bet.

However, adding in the win benefit can impact the final step, which is comparing the potential maximin values. In determining in which cases each minimum payoff is optimal, we utilized a combination of algebra and the Desmos online graphing calculator. As many of these comparisons are trivial but tedious, some of our inequalities have been solved only graphically.

11.3 \( M_A > 2M_B \)

Figure 4: Minimax Bets for Alice when \( M_A > 2M_B \)

Given this relationship between \( M_A \) and \( M_B \), the three potential maximins for Alice are region 8’s top row, region 8 in the lowest row which contains only region 8, and region 6 in the bottom row. The shaded areas indicate when each payoff is the greatest. Given our assumptions about \( P_B \) and the win benefit, this depends only on \( M_B \) and \( P_A \).
The intuition for why Alice’s minimax in this scenario is influenced by $M_B$ and not $M_A$ is as follows: Alice’s min payoff when betting $M_A$ is affected only by the value of $M_A$, while her payoff when betting $M_A - 2M_B - 1$ is affected by the relationship between $M_A$ and $M_B$. As $M_A$ increases, both payoffs increase and the relationship between the mins stays the same. When $M_B$ increases, however, betting $M_A - 2M_B - 1$ becomes worse and betting $M_A$ has the same payoff.

11.4 $\frac{3}{2}M_B < M_A < 2M_B$

Figure 5: Minimax Bets for Alice when $\frac{3}{2}M_B < M_A < 2M_B$

Given this relationship between $M_A$ and $M_B$, the three potential maximins for Alice are the top row of region 9, the bottom row of min-7 (the lowest row containing only region 7), and the bottom row of region 6. The shaded areas indicate when each payoff is the greatest. Given our assumptions about $P_B$ and the win benefit, this depends only on $M_A$ and $P_A$.

11.5 $M_A < \frac{3}{2}M_B$

Given this relationship between $M_A$ and $M_B$, the two potential maximins for Alice are the top row of region 9 and the bottom row of region 6. The shaded areas indicate when each payoff is the greatest. Given our assumptions about $P_B$ and the win benefit, this depends only on $M_A$ and $P_A$. 

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Figure 6: Minimax Bets for Alice when $M_A < \frac{3}{2}M_B$

References


