Sublinear Algorithms for Processing Massive Datasets

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Publications

- Externalities and Fairness [WWW'19]
- Streaming and Massively Parallel Algorithms for Edge Coloring [ESA'19]
  (also appeared in [DISC'19] as a brief announcement)
- Massively Parallel Algorithms for String Matching with Wildcards [arXiv]
- Computational Analyses of the Electoral College: Campaigning is Hard But Approximately Manageable [preprint]

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Sublinear Space

Have you ever had a dataset so big that it doesn't fit in the memory?

Sublinear Space Algorithms!

$$o(n)$$

Sublinear Space

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RAM model

$$o(n)$$

Sublinear Space

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Sublinear Space Algorithms!

Alternative models

$$o(n)$$

Alternative Models

of Computation

Massively Parallel Computation

- Modern frameworks for Large-scale parallel/distributed data processing: MapReduce, Hadoop, Spark.

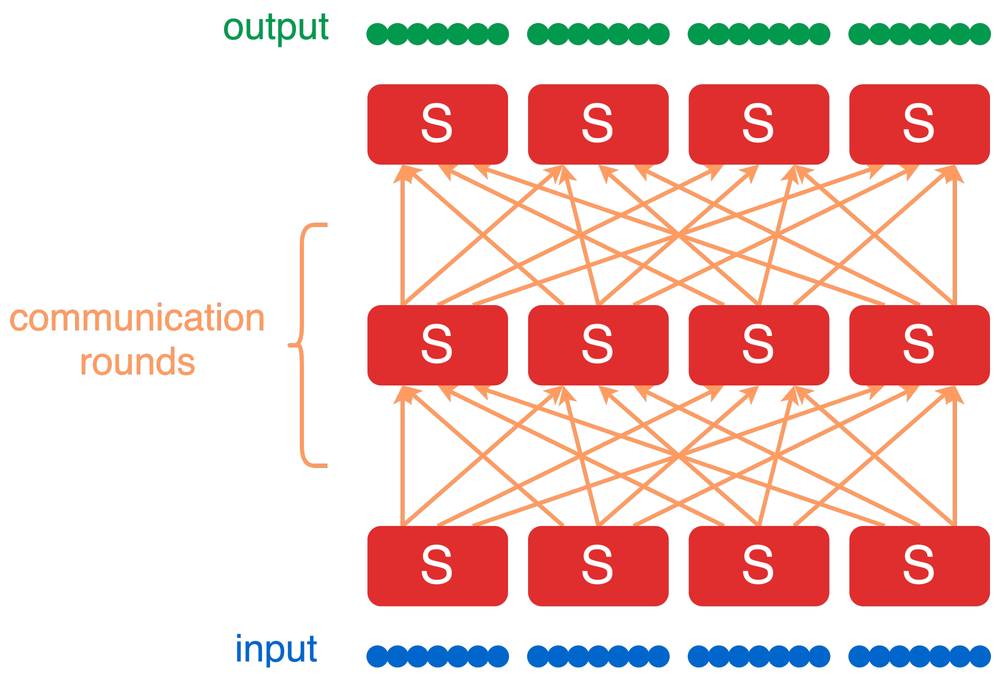
- Key Idea: distribute the workload among several machines.

- The MPC model: A theoretical model to abstract out the computational power of these frameworks.

[Karloff et. al 2010]
[Goodrich et. al 2011]
[Beame et. al 2013]
[Andoni et. al 2014]

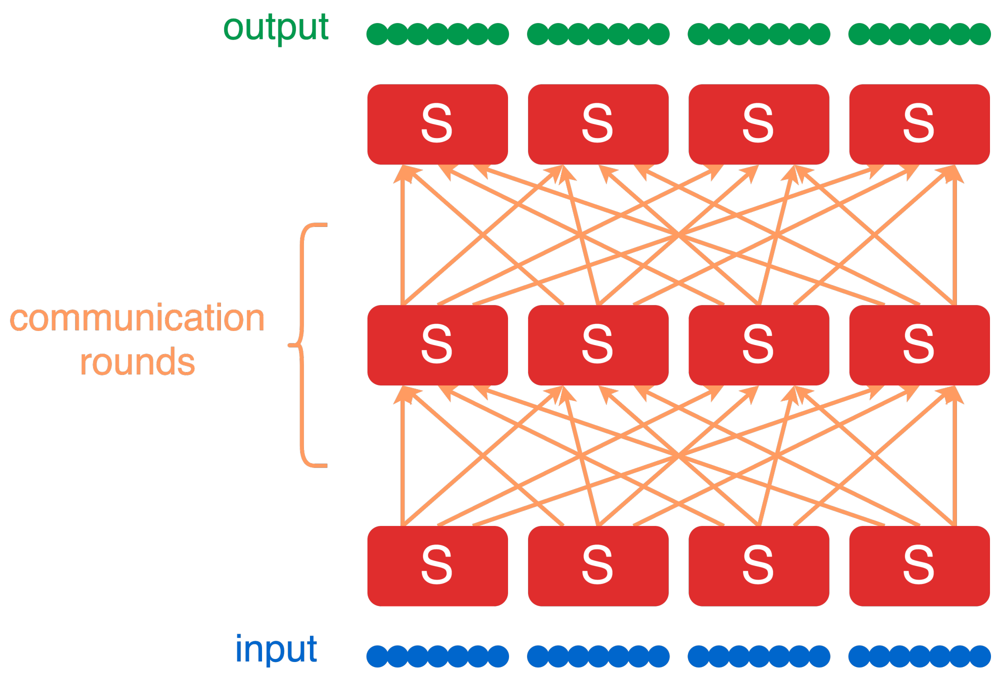
Massively Parallel Computation

- There are $\boldsymbol{M}$ machines each with memory $\boldsymbol{S}$.
- The input of length $\boldsymbol{N}$ is initially (randomly) distributed among the machines.
  - Sublinear space $\boldsymbol{S} = o(\boldsymbol{N})$.
  - Usually $\boldsymbol{N} = O(\boldsymbol{S} \cdot \boldsymbol{M})$.
- The data is processed in several **synchronous rounds**.



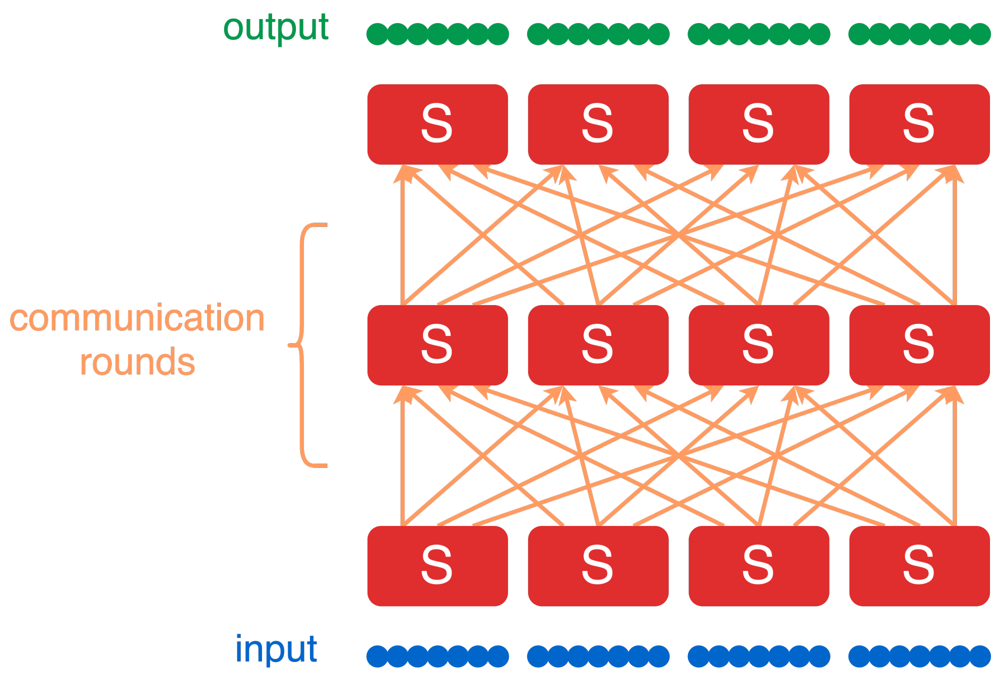
Massively Parallel Computation

- There are M machines each with memory S.
- The data is processed in several **synchronous rounds**. In each round,
  - Machines perform arbitrary computation on their **local** data.
  - Machines **communicate** with each other. Total **incoming/outgoing** messages of each machine is bounded by $O(S)$ words.



Massively Parallel Computation

- There are M machines each with memory S.
- The data is processed in several **synchronous rounds**.
- Main bottleneck: **Communication**. We wish for algorithms with very small number of rounds.
  - Often **sub-logarithmic** rounds.



Related **distributed**/**parallel** models

- The **PRAM** model (shared memory)
  - Any PRAM algorithm running in time $t = t(n)$ can be simulated in $O(t)$ MPC rounds.

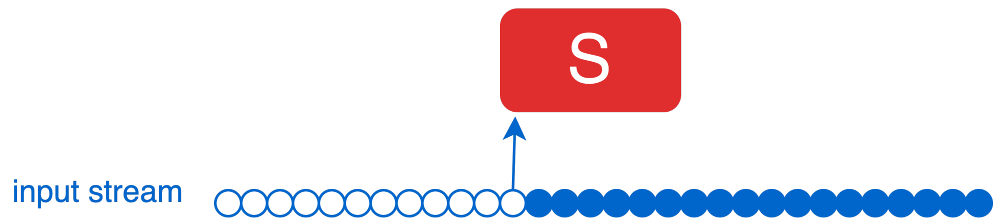
[Karloff et. al 2010]

- The **LOCAL**, **CONGEST**, and **congested-clique** models
  - Similar techniques can be used for both MPC and these distributed models.
  - Congested-clique is almost equivalent to MPC (in terms of the number of rounds).

[Behnezhad et. al 2018]

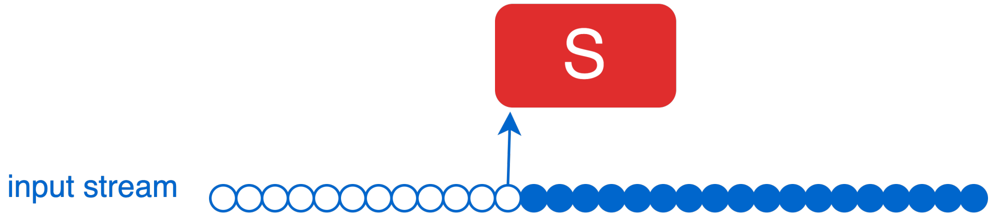
Streaming

- There is a single machines with memory S.
- The input of length N is streamed into the machine.
  - Sublinear space $S = o(N)$.
- The data is processed in several **passes**.



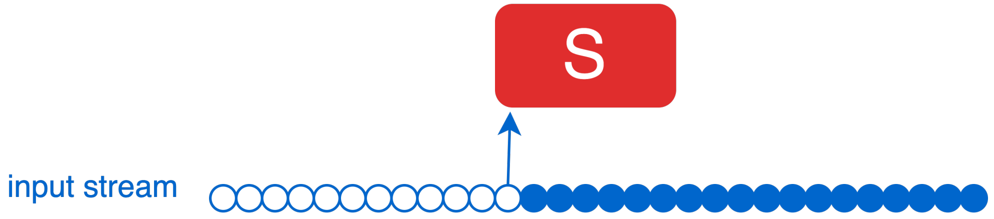
Streaming

- There is a single machines with memory S.
- The data is processed in several **passes**. In each pass:
    - The input entries arrive **sequentially** and **one by one** in a specific order.
    - The order can be either **random** or **adversarial**.



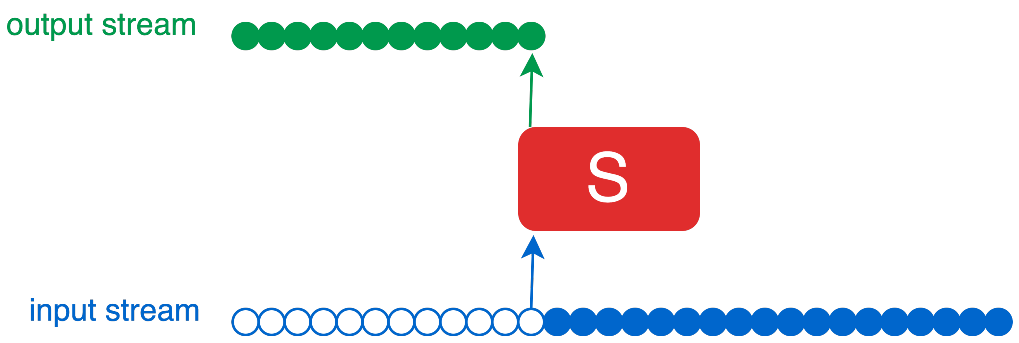
Streaming

- There is a single machines with memory S.
- The data is processed in several **passes**.
- There is a **trade-off** between the number of passes and the space of the machine.



W-streaming

- There is a single machines with memory S.
- What if the **output** also doesn't fit in the memory?
  - We use the **W-streaming** model.
  - The output is also streamed.



Semi-streaming and Semi-MPC

- In many graph problems, we assume $S = O(|V|)$, where the input graph is given as $G = (V, E)$.
- The variant of the streaming model in which $S = O(|V|)$ is called Semi-streaming.

[Feigenbaum et. al 2005]
[McGregor 2014]

- This restriction is stricter on dense graphs and less strict on sparse graphs.
- The standard variants are either too trivial or too hard on sparse graphs.
- Similarly, the Semi-MPC model is also defined.

Edge Coloring

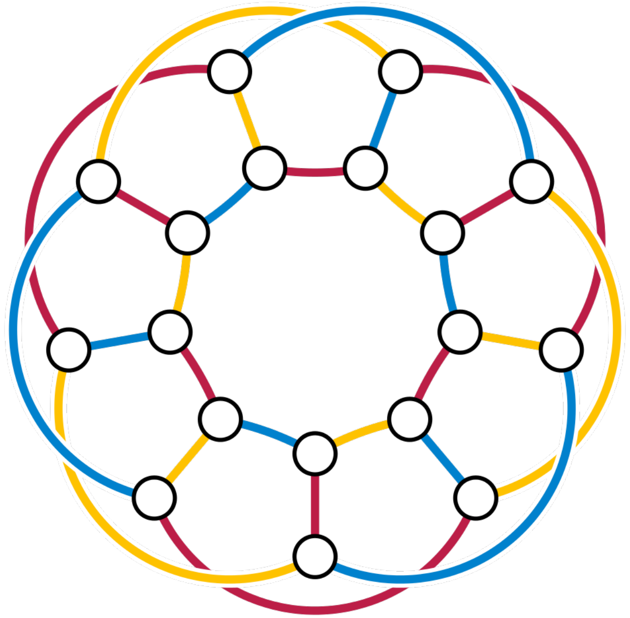
Streaming and Massively Parallel Algorithms for Edge Coloring **[ESA'19]**

Behnezhad, Derakhshan, Knittel, Hajiaghayi, **me**

The **Edge Coloring** problem

- Given a graph $G = (V, E)$, a **valid** "edge-coloring" is a function $\mathbf{COL}: E \rightarrow [\boldsymbol{\Psi}]$ so that no two **incident edges** have a common color.

- We wish to minimize $\boldsymbol{\Psi}$, i.e., the number of colors.



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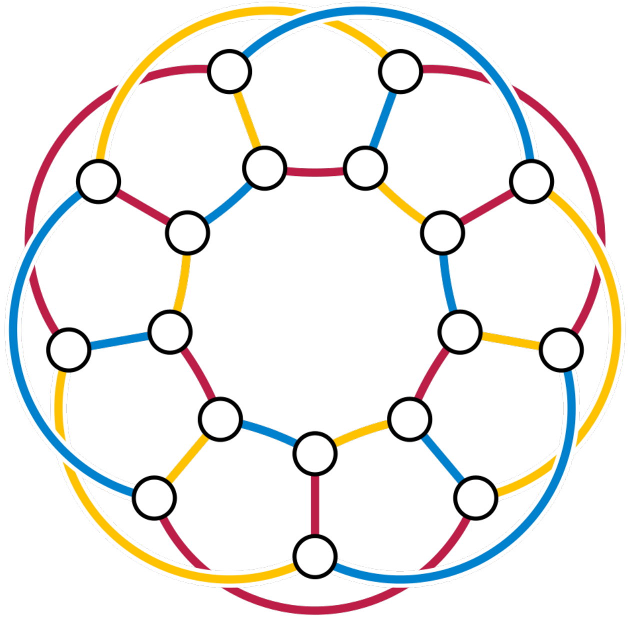
- Given a graph $G = (V, E)$, a **valid** "edge-coloring" is a function $\mathrm{COL}: E \to [\Psi]$ so that no two **incident edges** have a common color.

- Let Δ be the maximum degree of graph G.

- Then, we know $\Delta \leq \Psi \leq \Delta + 1$.

[Vizing 1964]

- An odd cycle ($\Delta = 2$) needs 3 colors.

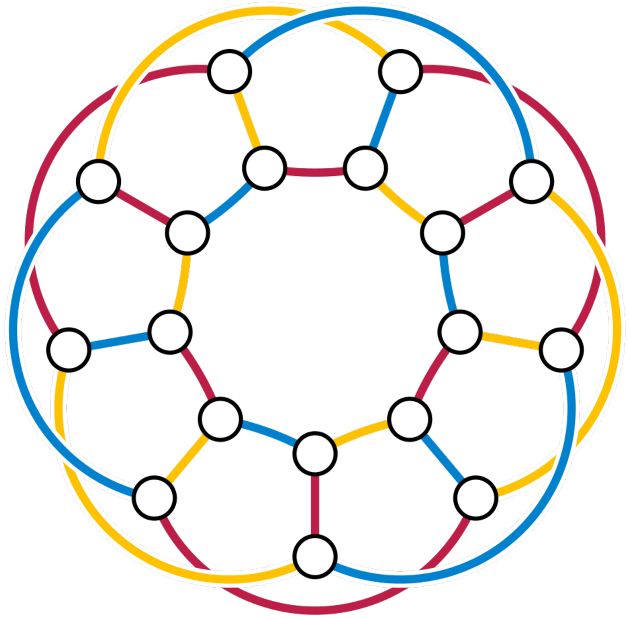


The **Edge Coloring** problem

- Given a graph $G = (V, E)$, a **valid** “edge-coloring” is a function $\mathbf{COL}: E \to [\Psi]$ so that no two **incident edges** have a common color.

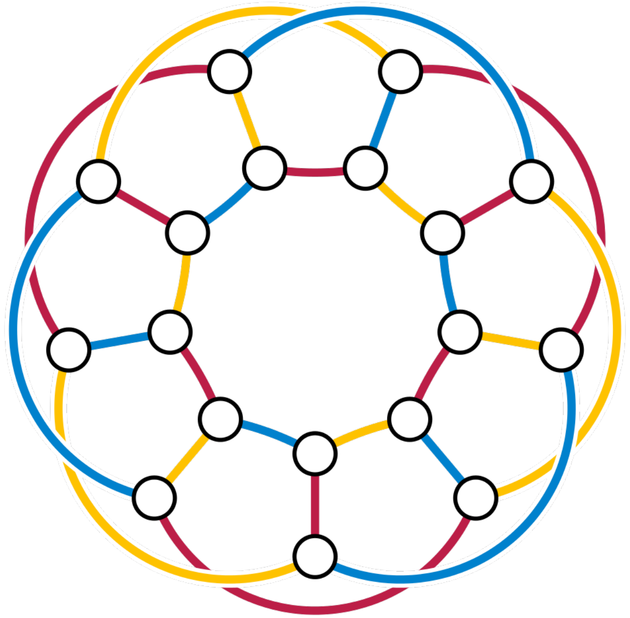
- However, $(\Delta + 1)$-coloring algorithms are highly **sequential**.

- There is a greedy $(2\Delta - 1)$-coloring algorithm.



The **Edge Coloring** problem

- Given a graph $G = (V, E)$, a **valid** "edge-coloring" is a function $\mathbf{COL}: E \to [\Psi]$ so that no two **incident edges** have a common color.
- There is a greedy $(2\Delta - 1)$-coloring algorithm.
  - Process edges in an arbitrary order.
  - Color each edge with an available color.



Related work

- Vertex Coloring: Assadi et. Al
- Distributed Ghaffari et al
- Harvey et. Al
- Streaming?

Edge Coloring

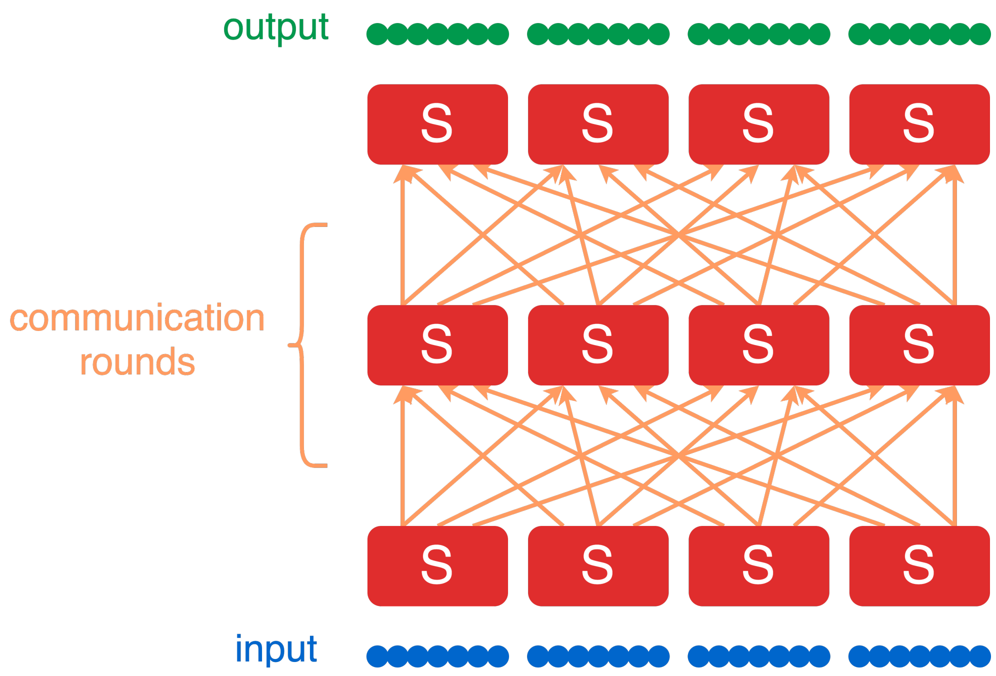
Massively **P**arallel **C**omputation

MPC Edge Coloring

- A graph $G = (V, E)$ is given where $|V| = n$ and $|E| = m$, i.e. $N = O(n + m)$.

- There is a **constant**-round MPC algorithm, with $S = O(n)$ and $S \cdot M = O(m)$, which computes an edge-coloring so that $\Psi = \Delta + \tilde{O}(\Delta^{3/4})$.

[BDKHS 2019]



Semi-MPC Edge Coloring

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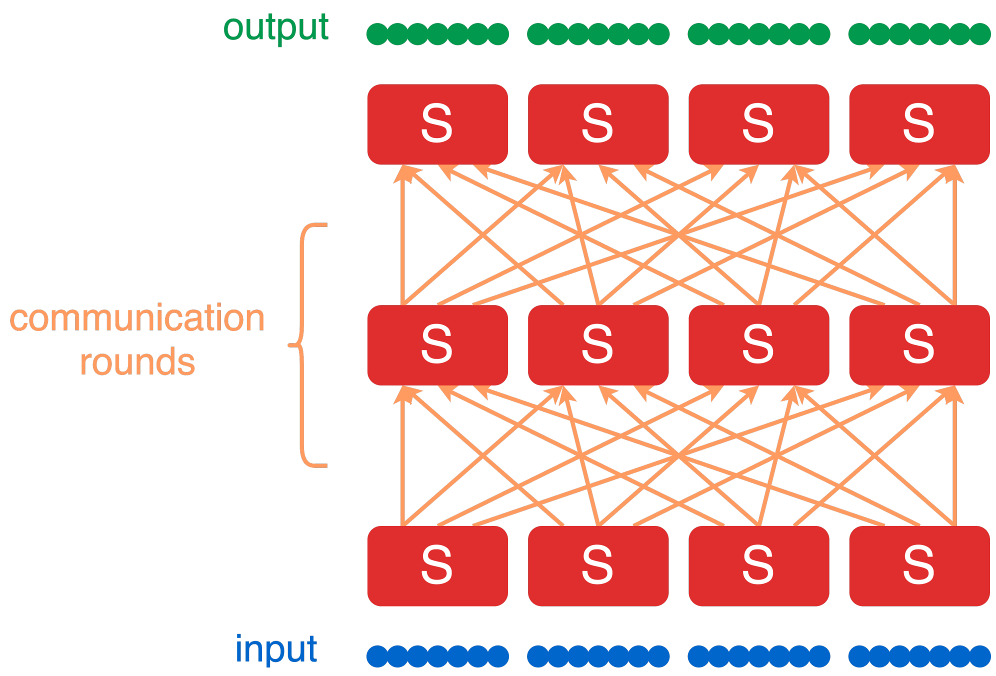
  [BDKHS 2019]

- It is technically a **Semi-**MPC algorithm.



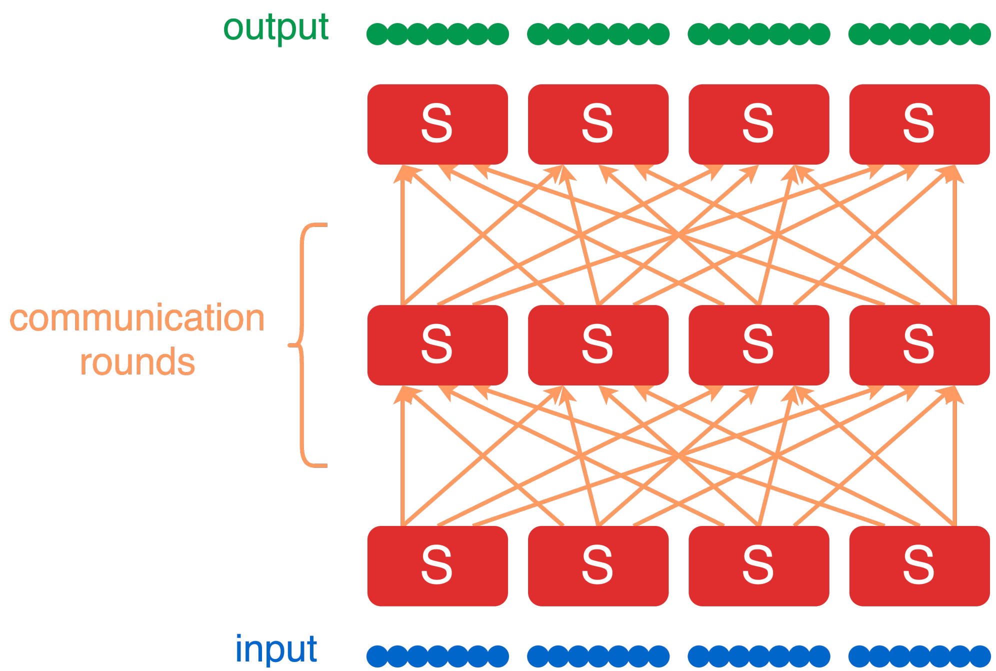
Semi-MPC Edge Coloring

- It is technically a **Semi-**MPC algorithm, but we can achieve $o(n)$ space in dense graphs.

- The exact space-per-machine is equal to

$$S = O\left(\frac{n\Delta}{k^2} + \frac{n}{k}\sqrt{\frac{\Delta}{k}\log n}\right)$$

- Set $k = \sqrt{\Delta} + \log n$.



Vertex Partitioning

- Set $k = \sqrt{\Delta} + \log n$.
- Random vertex partitioning: $V = V_1 \cup V_2 \cup \ldots \cup V_k$.

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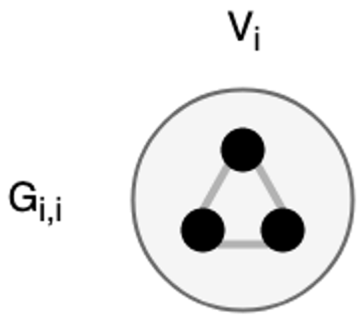
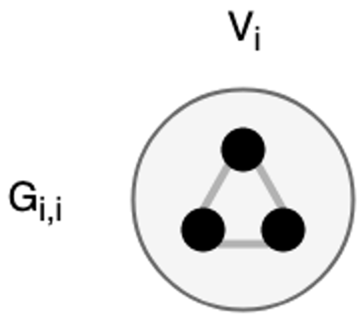
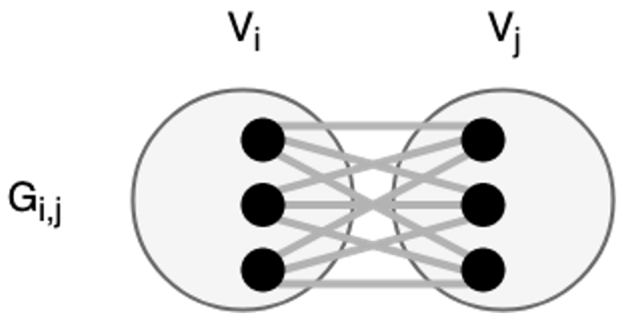
- Random vertex partitioning : $V = V_1 \cup V_2 \cup \dots \cup V_k$.
- $G_{i,i} = \{ (u,v) \mid u \in V_i \wedge v \in V_i \}$: the **induced** subgraph of each partition



Vertex Partitioning

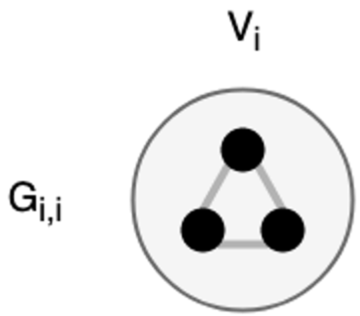
- Random vertex partitioning : $V = V_1 \cup V_2 \cup \ldots \cup V_k$.
- $G_{i,i} = \{ (u, v) \mid u \in V_i \wedge v \in V_i\}$: the **induced** subgraph of each partition
- $G_{i,j} = \left\{ (u, v) \,\middle|\, u \in V_i \wedge v \in V_j\right\}$: the **bipartite induced** subgraph of pairs of partitions

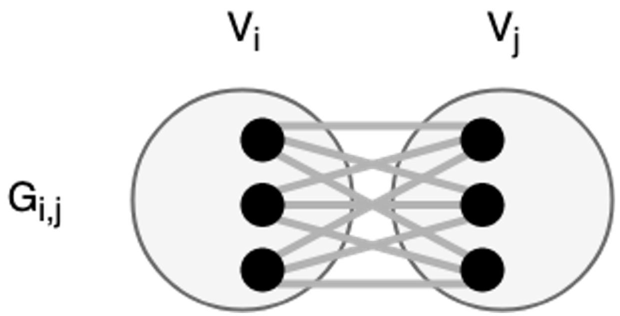




Local Coloring

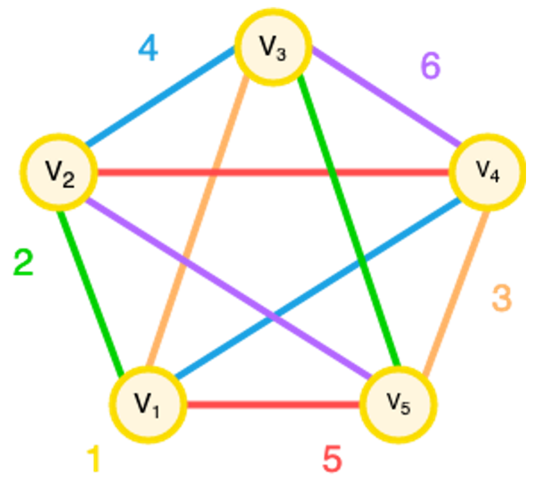
- $G_{i,i} = \{\, (u,v) \mid u \in V_i \wedge v \in V_i \}$: the **induced** subgraph of each partition
- $G_{i,j} = \left\{\, (u,v) \,\middle|\, u \in V_i \wedge v \in V_j \right\}$: the **bipartite induced** subgraph of pairs of partitions
- Run Vizing's $(\Delta + 1)$-coloring algorithm on each machine.





Merging Colored Subgraphs

- $k+1$ disjoint color **palettes** are enough.
- The number of colors in each **palette** needs needs to be as large as the maximum degree in subgraphs.
- The maximum degree in each subgraphs is **concentrated**.
- For $k = \sqrt{\Delta} + \log n$, we have $\Psi = \Delta + \tilde{O}(\Delta^{3/4})$.



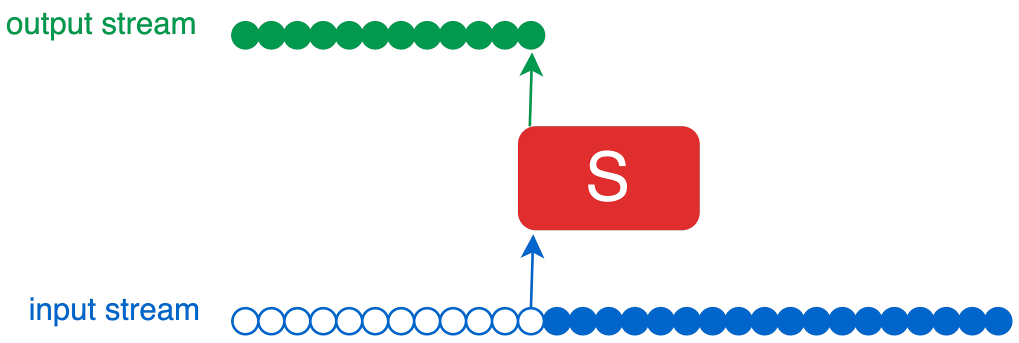
Edge Coloring

Random Streaming

Random Streaming Edge Coloring

- A graph $G = (V, E)$ is given where $|V| = n$ and $|E| = m$, i.e. $N = O(n + m)$.
- There is a one-pass W-streaming algorithm, with $S = O(n)$, which streams a valid edge-coloring so that $\Psi = (2e + \epsilon)\Delta$.

[BDKHS 2019]



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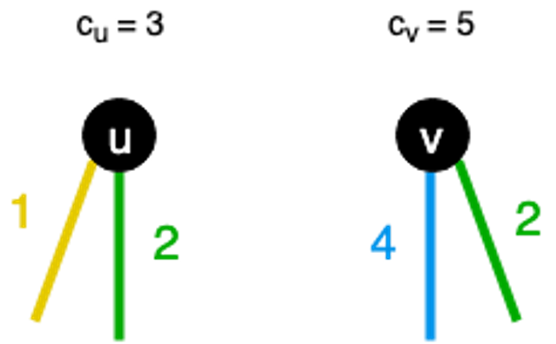
[BDKHS 2019]

- The algorithm is straight-forward.
- Maintain a counter variable c_v for each vertex, initially set to 1.

Random Streaming Edge Coloring

- Maintain a **counter** variable c_v for each vertex, initially set to 1.
- Upon arrival of (u, v) color it $\max(c_v, c_u)$.



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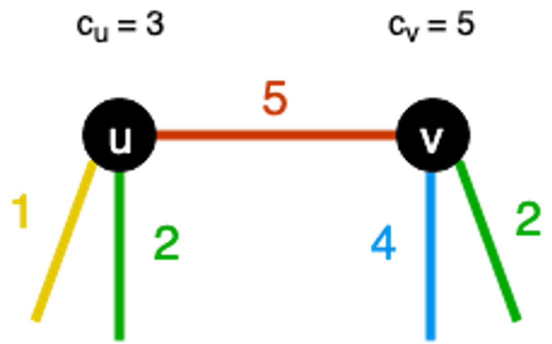
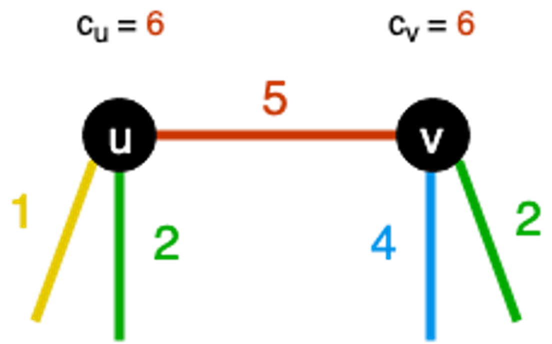


Random Streaming Edge Coloring

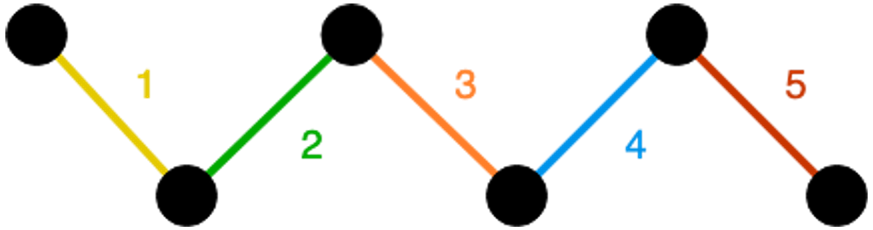
- Maintain a **counter** variable c_v for each vertex, initially set to 1.
- Upon arrival of (u, v) color it $\max(c_v, c_u)$.
- Set both c_v and c_u to $\max(c_v, c_u) + 1$.



Longest **Monotone** Path

- **Lemma:** Ψ is equal to the length of the longest **monotone** path in the **line graph** after the algorithm finishes.

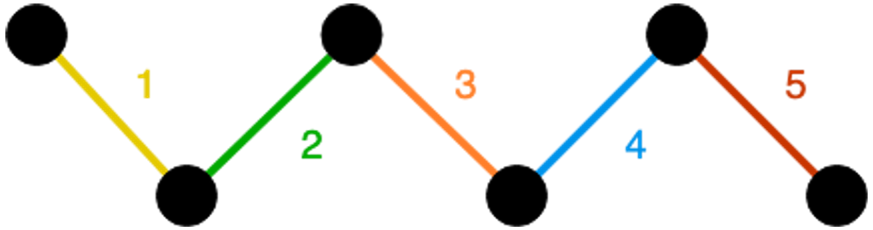
- We can construct a **monotone** path of length Ψ starting from an edge colored Ψ.



Longest **Monotone** Path

- **Lemma:** **Ψ** is equal to the length of the longest **monotone** path in the **line graph** after the algorithm finishes.

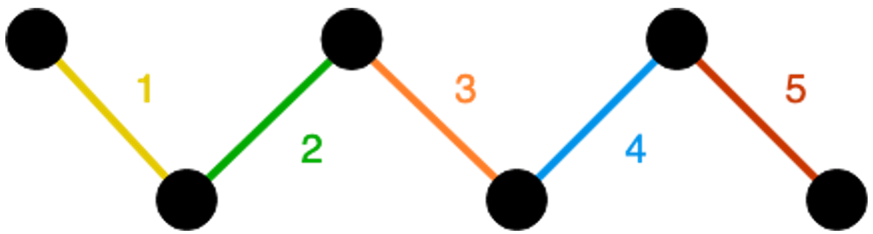
$$\Pr[\Psi \geq \alpha\Delta] \leq \frac{(2\Delta)^{\alpha\Delta}}{(\alpha\Delta)!}$$



Longest **Monotone** Path

- **Lemma:** Ψ is equal to the length of the longest **monotone** path in the **line graph** after the algorithm finishes.

$$\Pr[\Psi \geq (2e + \epsilon)\Delta] \leq n^{-c}$$



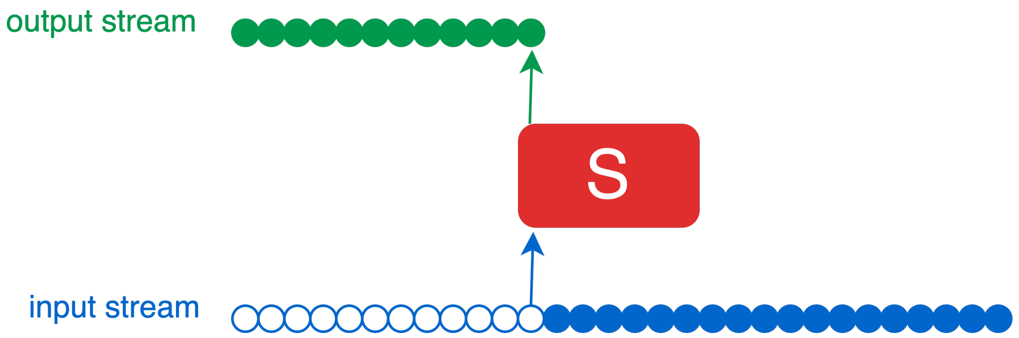
Edge Coloring

Adversarial Streaming

Adversarial Streaming Edge Coloring

- A graph $G = (V, E)$ is given where $|V| = n$ and $|E| = m$, i.e. $N = O(n+m)$.
- There is a **one**-pass W-streaming algorithm, with $S = O(n)$, which streams a valid edge-coloring so that $\Psi = O(\Delta^2)$.

[BDKHS 2019]



Adversarial Streaming Edge Coloring

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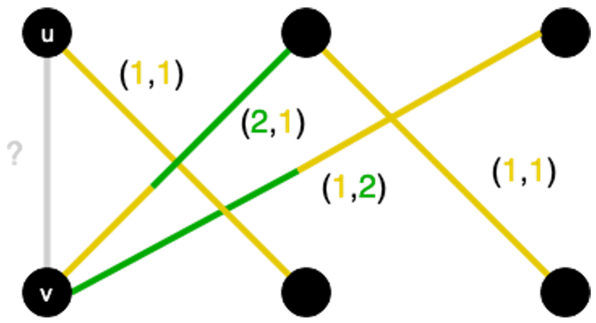
- There is a **one**-pass **W**-streaming algorithm, with $S = O(n)$, which streams a valid edge-coloring so that $\Psi = O(\Delta^2)$.

[BDKHS 2019]

- The algorithm is very similar to the random stream algorithm.
- Maintain a **counter** variable c_v for each vertex, initially set to 1.

Bipartite Edge Coloring

- Maintain a **counter** variable c_v for each vertex, initially set to 1. Also assume the graph is **bipartite**.
- Upon arrival of (u, v) color it (c_u, c_v).



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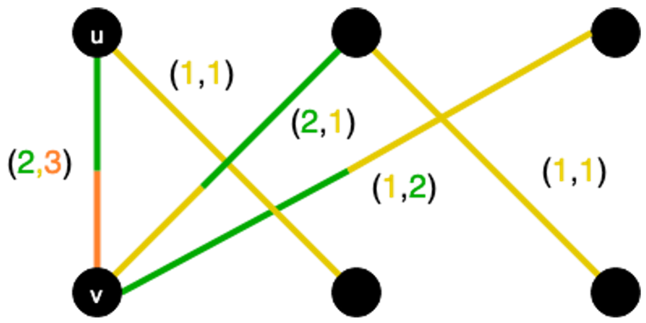
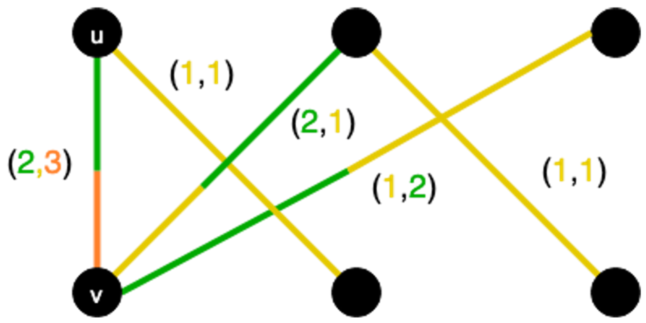


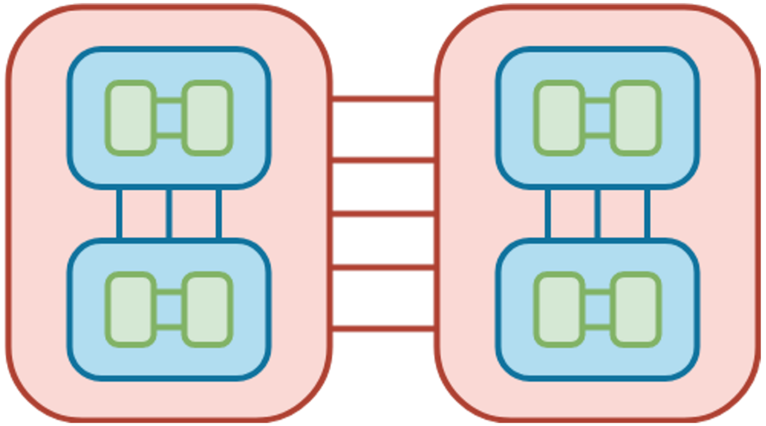
Bipartite Edge Coloring

- Maintain a **counter** variable c_v for each vertex, initially set to 1. Also assume the graph is **bipartite**.
- Upon arrival of (u, v) color it (c_u, c_v).
- Increase both c_u and c_v by 1.



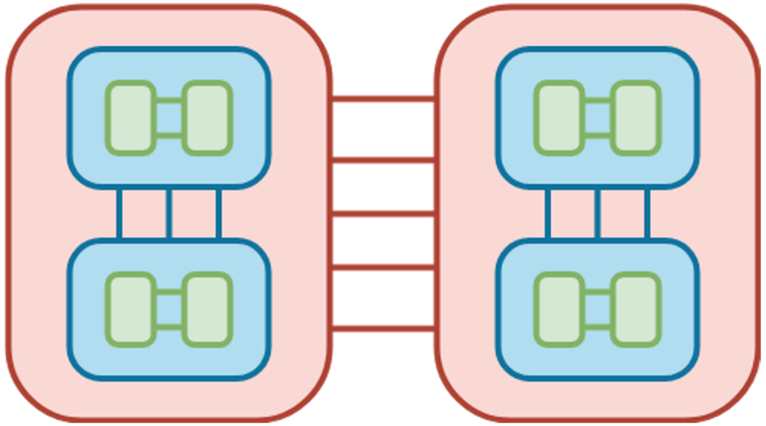
General Edge Coloring

- A **bipartite** graph is colored with Δ^2 colors using this method.
- Decompose the graph into $O(\log n)$ random bipartite graphs, and color each with a different **palette**.



General Edge Coloring

- A **bipartite** graph is colored with Δ^2 colors using this method.
- Decompose the graph into $O(\log n)$ random bipartite graphs, and color each with a different **palette**.
- In total, $\Psi = O(\Delta^2)$.



String Matching

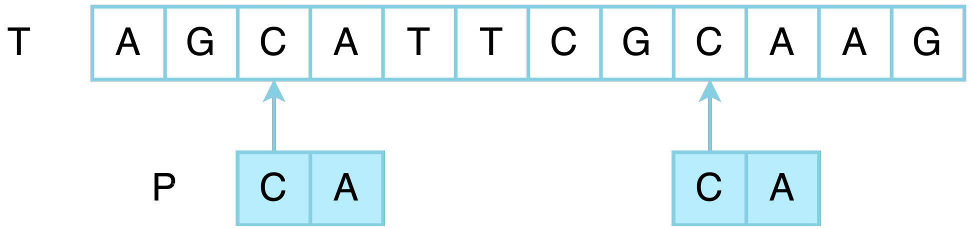
Massively Parallel Algorithms for String Matching with Wildcards **[arXiv]**

Hajiaghayi, **me**, Seddighin, Sun

The **String Matching** problem

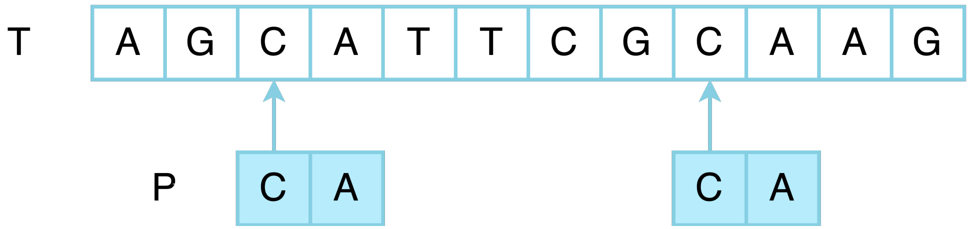
- An essential problem in bio-informatics and many other areas.

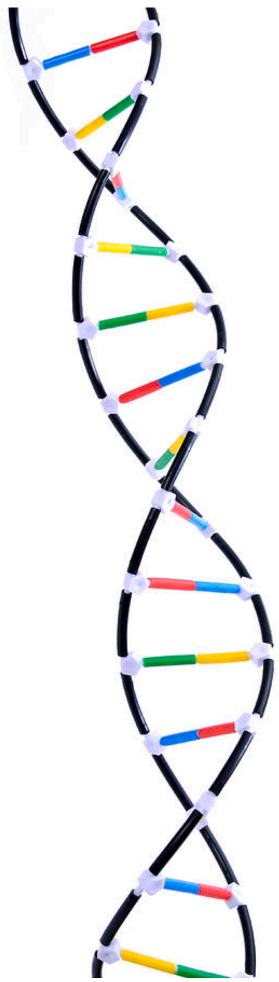
- Given a **text** T and a **pattern** P, we wish to find all substrings of T that match P.



The **String Matching** problem

- Given a **text** T and a **pattern** P, we wish to find all substrings of T that match P.
- In the simplest form both T and P are using the same alphabet Σ, and there is no special character.

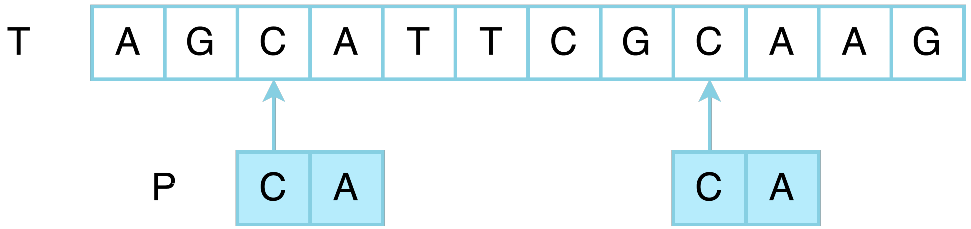




The **String Matching** problem

- Given a **text** T and a **pattern** P, we wish to find all substrings of T that match P.

- We also study the case when P can also have special characters known as **wildcards**. We are interested in {**'?'**, **'+'**, **'*'**}.

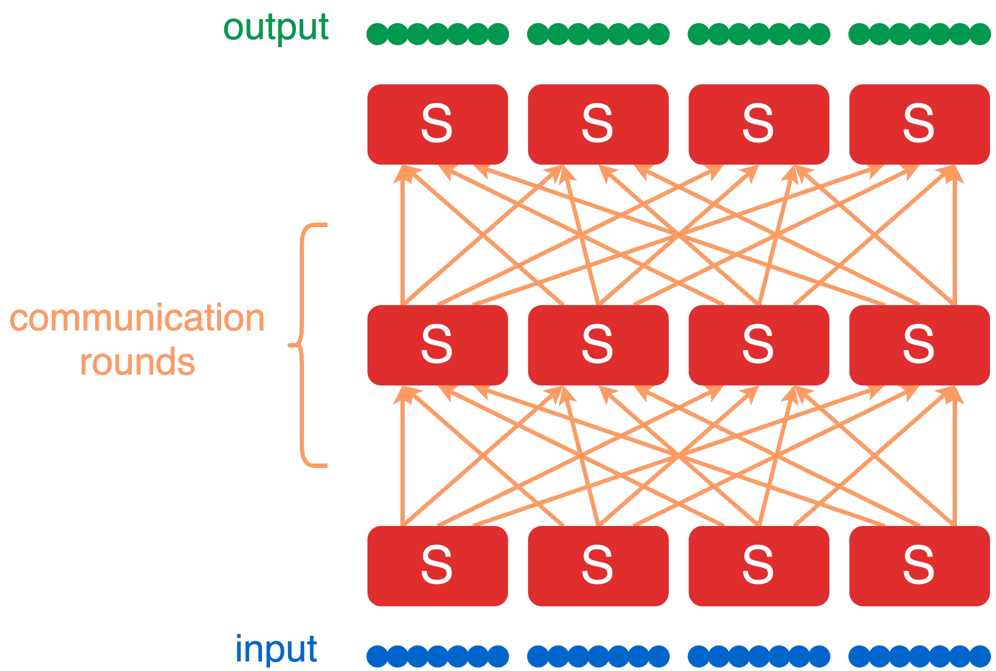


String Matching in MPC

- A **text** $T \subseteq \Sigma^n$ and a **pattern** $P \subseteq \Sigma^m$ are given.

- There is a **constant**-round MPC algorithm, with $S = O(n^{1-x})$ and $M = O(n^x)$ for any $x < 0.5$, which finds all occurrences of P in T.

[HSSS 2019]



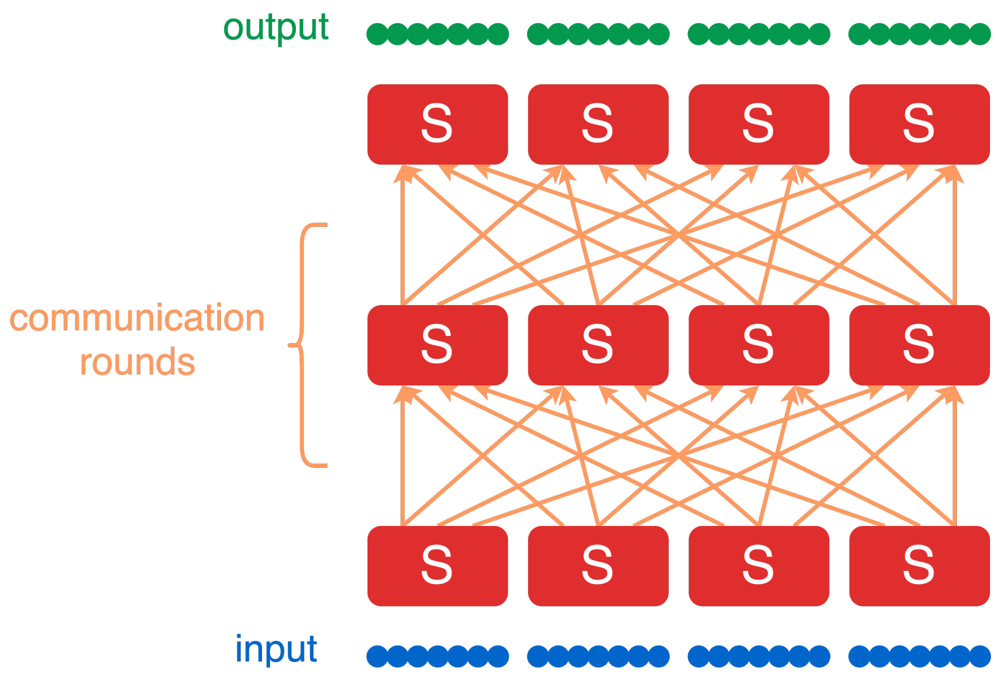
String Matching in MPC

- A **text** $T \subseteq \Sigma^n$ and a **pattern** $P \subseteq \Sigma^m$ are given.

- There is a **constant**-round MPC algorithm, with $\boldsymbol{S} = O(n^{1-x})$ and $\boldsymbol{M} = O(n^x)$ for any $x < 0.5$, which finds all occurrences of P in T.

  [HSSS 2019]

- Easy when $\mathrm{m} = O(n^{1-x})$: **Double**-covering.



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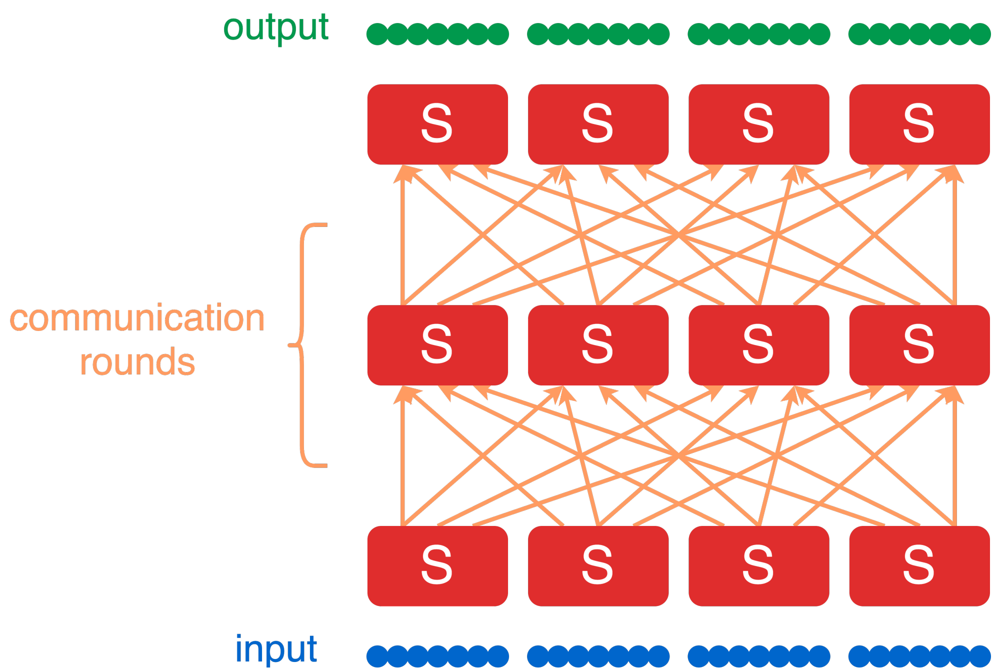
  [HSSS 2019]
- Also easy for general $\mathbf{m}$: Partial **hash**ing.

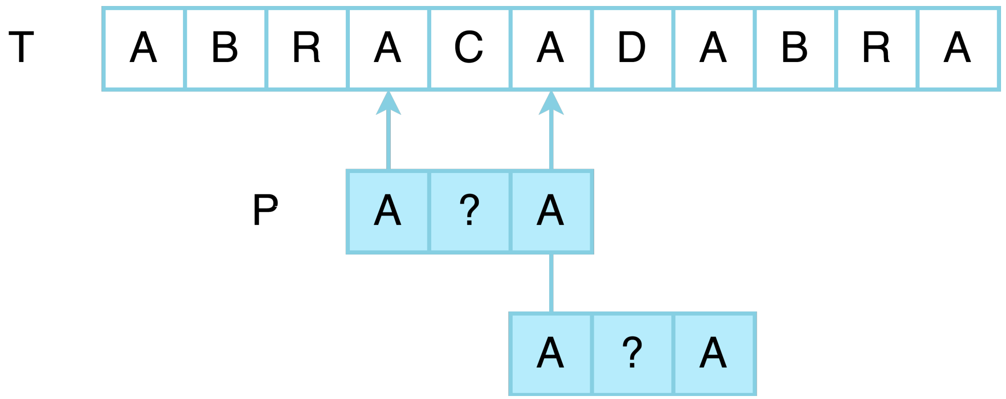


String Matching

with ‘?’ wildcard

String Matching with '?' wildcard

- A **text** $T \subseteq \Sigma^n$ and a **pattern** $P \subseteq (\Sigma \cup \text{'?'})^m$ are given.
- The special character '?' can be replaced with any arbitrary character.



String Matching with '?' wildcard

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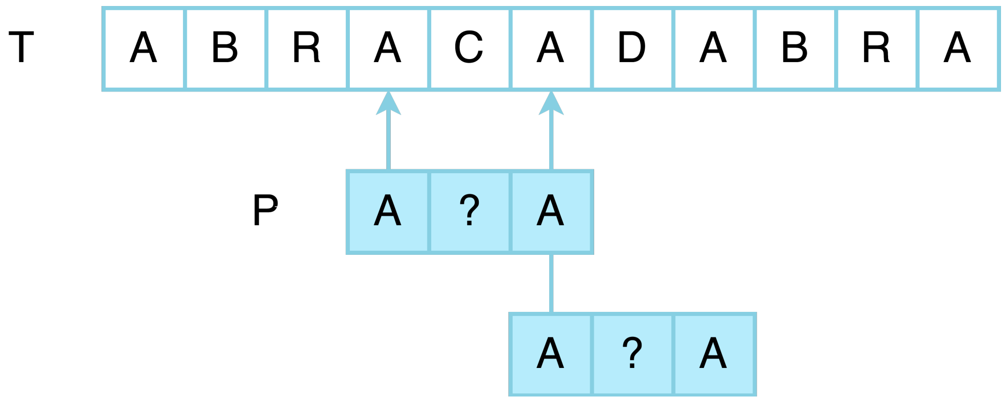
[HSSS 2019]



String Matching with '?' wildcard

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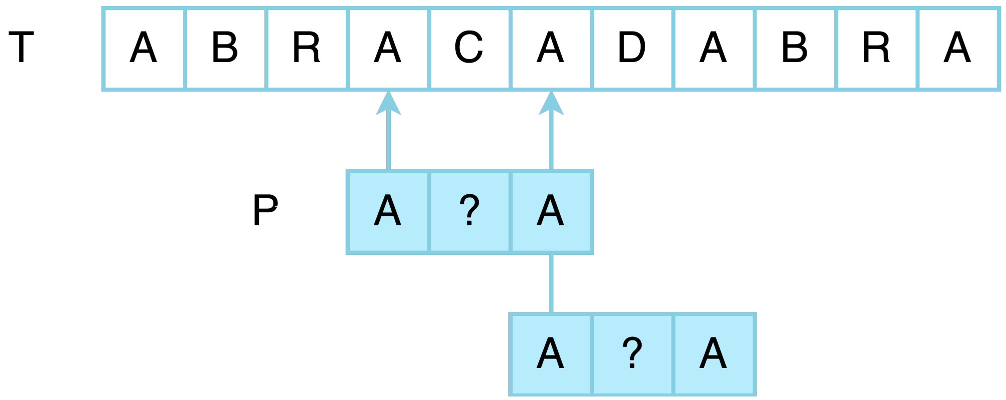
- There is a **constant**-round MPC algorithm, with $S = O(n^{1-x})$ and $M = O(n^x)$ for any $x < 0.5$, which finds all occurrences of P in T.

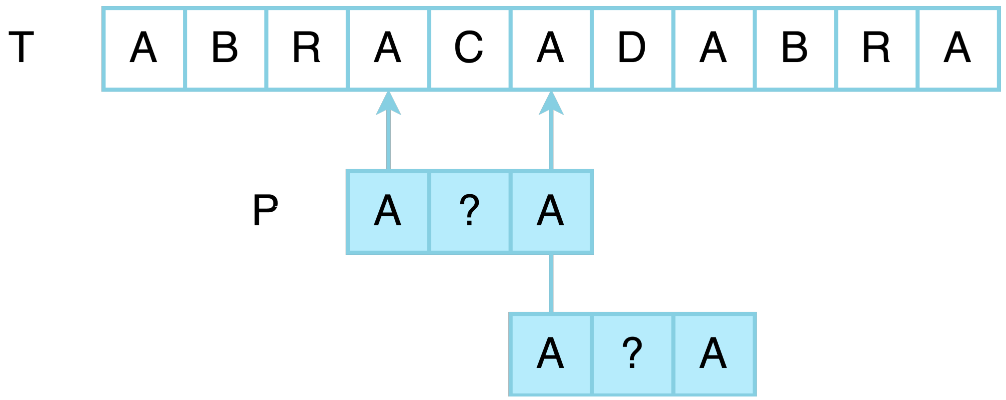
  [HSSS 2019]
- This can be improved to any constant $x < 1$ at the cost of $O(x^{-1})$ rounds.



‘?’ wildcard and convolution

- A **text** $T \subseteq \Sigma^n$ and a **pattern** $P \subseteq (\Sigma \cup \text{‘?’})^m$ are given.
- This variant of the string matching problem can be reduced to the convolution of two arrays.

[Fischer et. al 1974]



$$T^\dagger = \langle \mathrm{mp}_{T_1}, \mathrm{mp}_{T_1}^{-1}, \mathrm{mp}_{T_2}, \mathrm{mp}_{T_2}^{-1}, \ldots, \mathrm{mp}_{T_n}, \mathrm{mp}_{T_n}^{-1} \rangle$$
$$P^\dagger = \langle \mathrm{mp}_{P_1}, \mathrm{mp}_{P_1}^{-1}, \mathrm{mp}_{P_2}, \mathrm{mp}_{P_2}^{-1}, \ldots, \mathrm{mp}_{P_n}, \mathrm{mp}_{P_n}^{-1} \rangle$$

$$\mathrm{mp}_{\text{‘?’}} = \mathrm{mp}_{\text{‘?’}}^{-1} = 0, \mathrm{mp}_{\text{‘C’}} = 3, \mathrm{mp}_{\text{‘C’}}^{-1} = 1/3$$

'?' wildcard and **convolution**

- A **text** $T \subseteq \Sigma^n$ and a **pattern** $P \subseteq (\Sigma \cup \text{'?'})^m$ are given.
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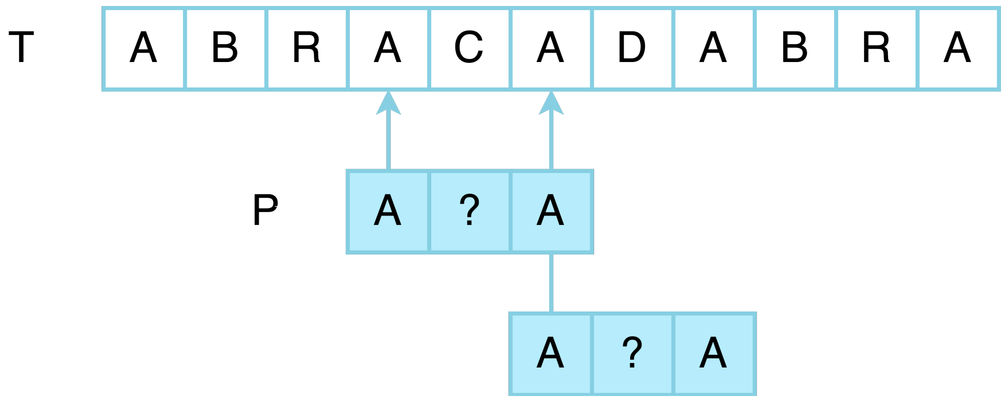
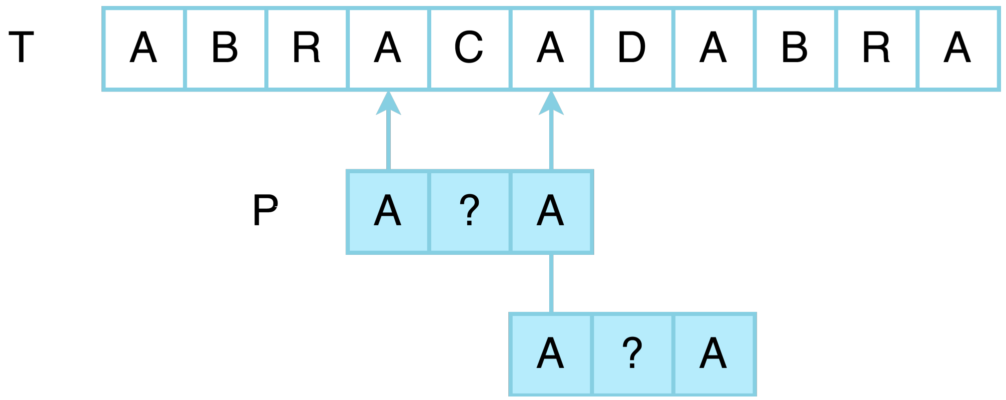


$$T^\dagger = \langle 1, \frac{1}{1}, 2, \frac{1}{2}, 18, \frac{1}{18}, 1, \frac{1}{1}, 3, \frac{1}{3}, 1, \frac{1}{1}, 4, \frac{1}{4}, 1, \frac{1}{1}, 2, \frac{1}{2}, 18, \frac{1}{18}, 1, \frac{1}{1} \rangle$$

$$P^\dagger = \langle 1, \frac{1}{1}, 0, 0, 1, \frac{1}{1} \rangle$$

‘?’ wildcard and convolution

- A **text** $T \subseteq \Sigma^n$ and a **pattern** $P \subseteq (\Sigma \cup \text{‘?’})^m$ are given.

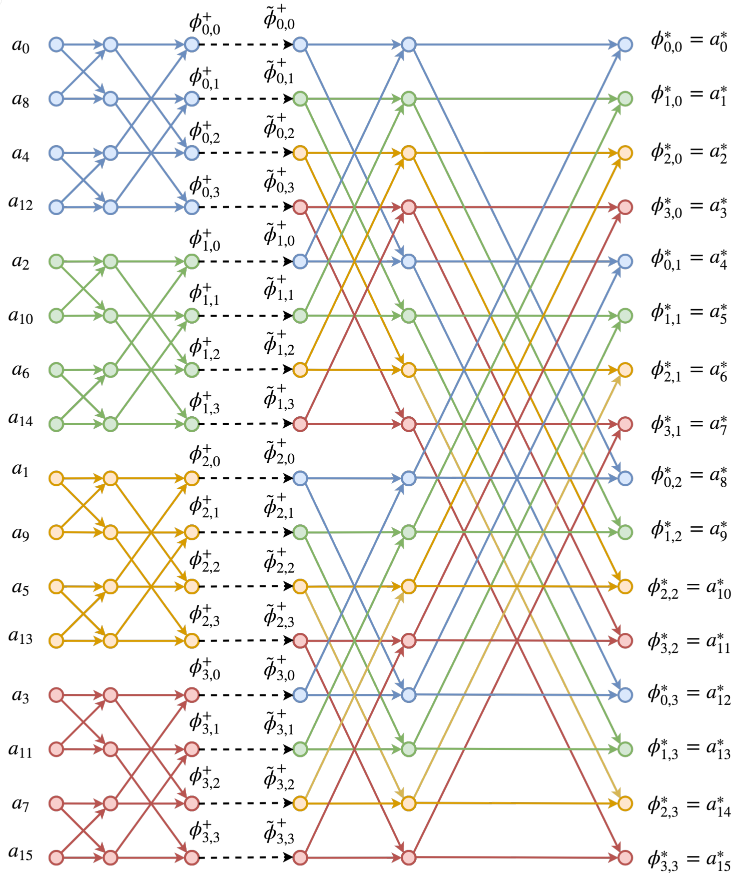


$$T^\dagger = \langle 1, \frac{1}{1}, 2, \frac{1}{2}, 18, \frac{1}{18}, 1, \frac{1}{1}, 3, \frac{1}{3}, 1, \frac{1}{1}, 4, \frac{1}{4}, 1, \frac{1}{1}, 2, \frac{1}{2}, 18, \frac{1}{18}, 1, \frac{1}{1} \rangle$$
$$P^\dagger = \langle 1, \frac{1}{1}, 0, 0, 1, \frac{1}{1} \rangle$$

$$C = T^\dagger * \mathrm{rev}(P^\dagger)$$

FFT in constant rounds

- Performing a **bit-reversal** operation makes the divide and conquer pattern clean.
- It is easy to decompose the **precedence** graph into the **Butterfly** graphs of different sizes.
- Cooley-Tukey with radix $R = n^{1-x}$.
- Further implications such as the **knapsack** problem.

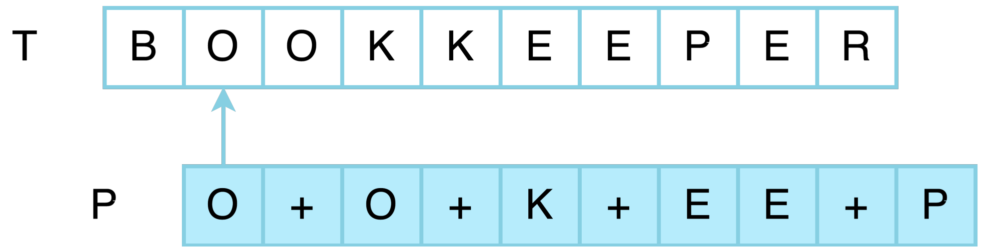


String Matching

with ‘+’ wildcard

String Matching with '+' wildcard

- A **text** $T \subseteq \Sigma^n$ and a **pattern** $P \subseteq (\Sigma \cup \text{'+'})^m$ are given.
- The special character '+' means that the preceding character can be repeated arbitrary times.

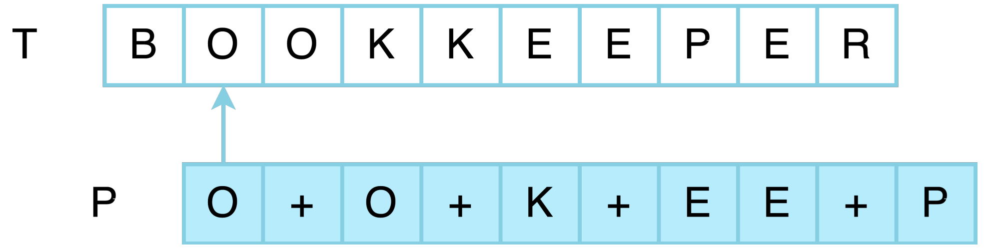


String Matching with '+' wildcard

- A **text** $T \subseteq \Sigma^n$ and a **pattern** $P \subseteq (\Sigma \cup \text{'+'})^m$ are given.

- There is a **constant**-round MPC algorithm, with $S = O(n^{1-x})$ and $M = O(n^x)$ for any $x < 0.5$, which finds all occurrences of P in T.

[HSSS 2019]



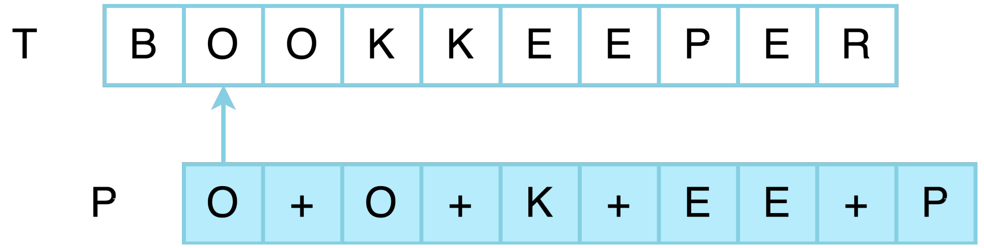
Run Length Encoding

- A **text** $T \subseteq \Sigma^n$ and a **pattern** $P \subseteq (\Sigma \cup \text{'+'})^m$ are given.

- Perform Run Length Encoding on both strings.

$$T^\circ = \langle\langle \mathsf{b}, 1\rangle, \langle \mathsf{o}, 2\rangle, \langle \mathsf{k}, 2\rangle, \langle \mathsf{e}, 2\rangle, \langle \mathsf{p}, 1\rangle, \langle \mathsf{e}, 1\rangle, \langle \mathsf{r}, 1\rangle\rangle$$
$$P^\circ = \langle\langle \mathsf{o}, 2+\rangle, \langle \mathsf{k}, 1+\rangle, \langle \mathsf{e}, 2+\rangle, \langle \mathsf{p}, 1\rangle\rangle$$

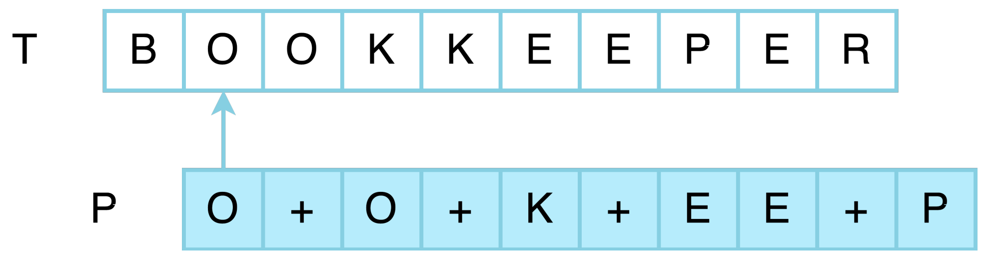
- Reduces to **Greater-than** matching.



Run Length Encoding

- Reduces to **Greater-than** matching.
- A special case of **Subset** matching.
- The **Subset** matching problem can be solved in $O(n \log^2 m)$ by a careful reduction to sparse convolution.

  [Cole et. al 2002]
- It's possible to implement it in $O(1)$ MPC rounds.

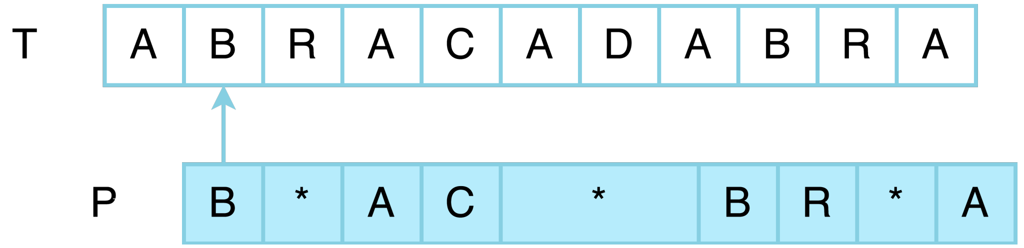


String Matching

with ‘*’ wildcard

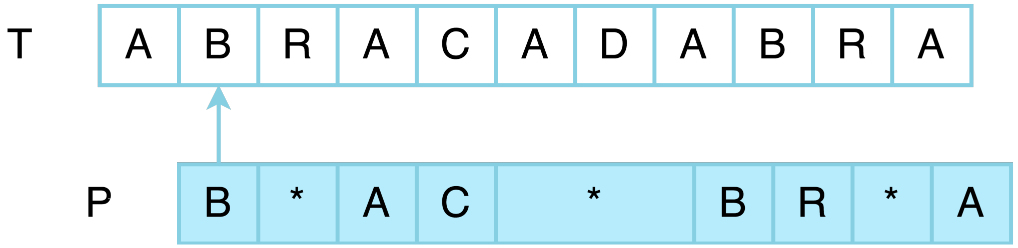
String Matching with '*' wildcard

- A **text** $T \subseteq \Sigma^n$ and a **pattern** $P \subseteq (\Sigma \cup \text{'*'})^m$ are given.
- The special character '*' can be replaced with any arbitrary string.



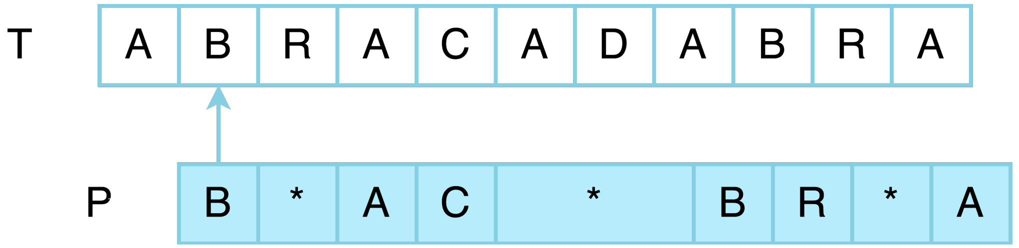
String Matching with ‘*’ wildcard

- A **text** $T \subseteq \Sigma^n$ and a **pattern** $P \subseteq (\Sigma \cup \text{‘*’})^m$ are given.
- The special character ‘*’ can be replaced with any arbitrary string.
- Unlike ‘$*$’ and ‘$*$’, we have no positive result for this wildcard even in $O(\log n)$ rounds…
- There is a conjecture that we can’t solve graph **connectivity** in $o(\log n)$ rounds.



String Matching with ‘*’ wildcard

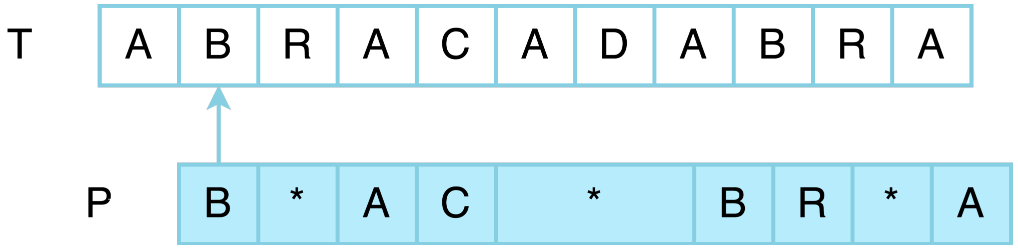
- A **text** $T \subseteq \Sigma^n$ and a **pattern** $P \subseteq (\Sigma \cup \text{‘*’})^m$ are given.
- The special character ‘*’ can be replaced with any arbitrary string.
- Unlike ‘$*$’ and ‘$*$’, we have no positive result for this wildcard even in $O(\log n)$ rounds…
- But we can solve it in special cases.



‘*’ wildcard in **small** patterns

- A **text** $T \subseteq \Sigma^n$ and a **pattern** $P \subseteq (\Sigma \cup \text{‘*’})^m$ are given such that $m = O(n^{1-x})$.
- There is a $O(\log n)$-round MPC algorithm, with $S = O(n^{1-x})$ and $M = O(n^x)$ for any $x < 0.5$, which finds all occurrences of P in T.

[HSSS 2019]

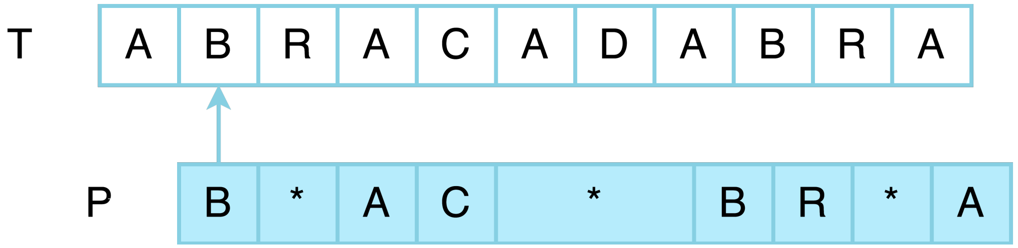


‘*’ wildcard in no **common prefix** case

- A **text** $T \subseteq \Sigma^n$ and a **pattern** $P \subseteq (\Sigma \cup \text{‘*’})^m$ are given such that no two **sub-patterns** share a common **prefix**.

- There is a $O(\log n)$-round MPC algorithm, with $\boldsymbol{S} = O(n^{1-x})$ and $\boldsymbol{M} = O(n^x)$ for any $x < 0.5$, which finds all occurrences of P in T.

[HSSS 2019]



Future work

- The Hypergraph matching problem in **MPC**.
- Improving the **edge-coloring** bound in adversarial streams.
- A constant round MPC algorithm for weighted exact **knapsack**.

Acknowledgements

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Thanks for watching!

Any questions?