

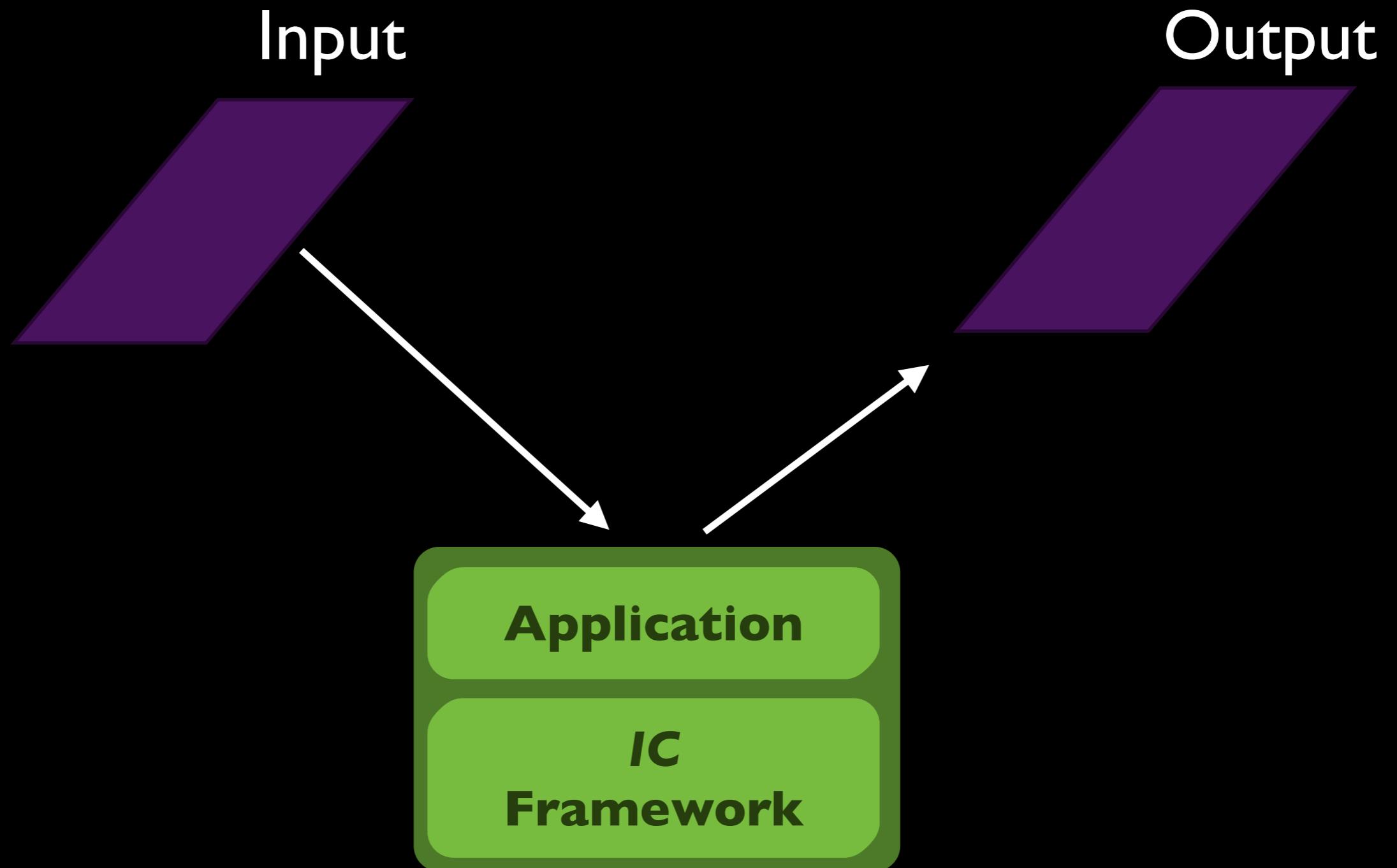
Adafton: *Composable Demand-Driven Incremental Computation*

Matthew A. Hammer

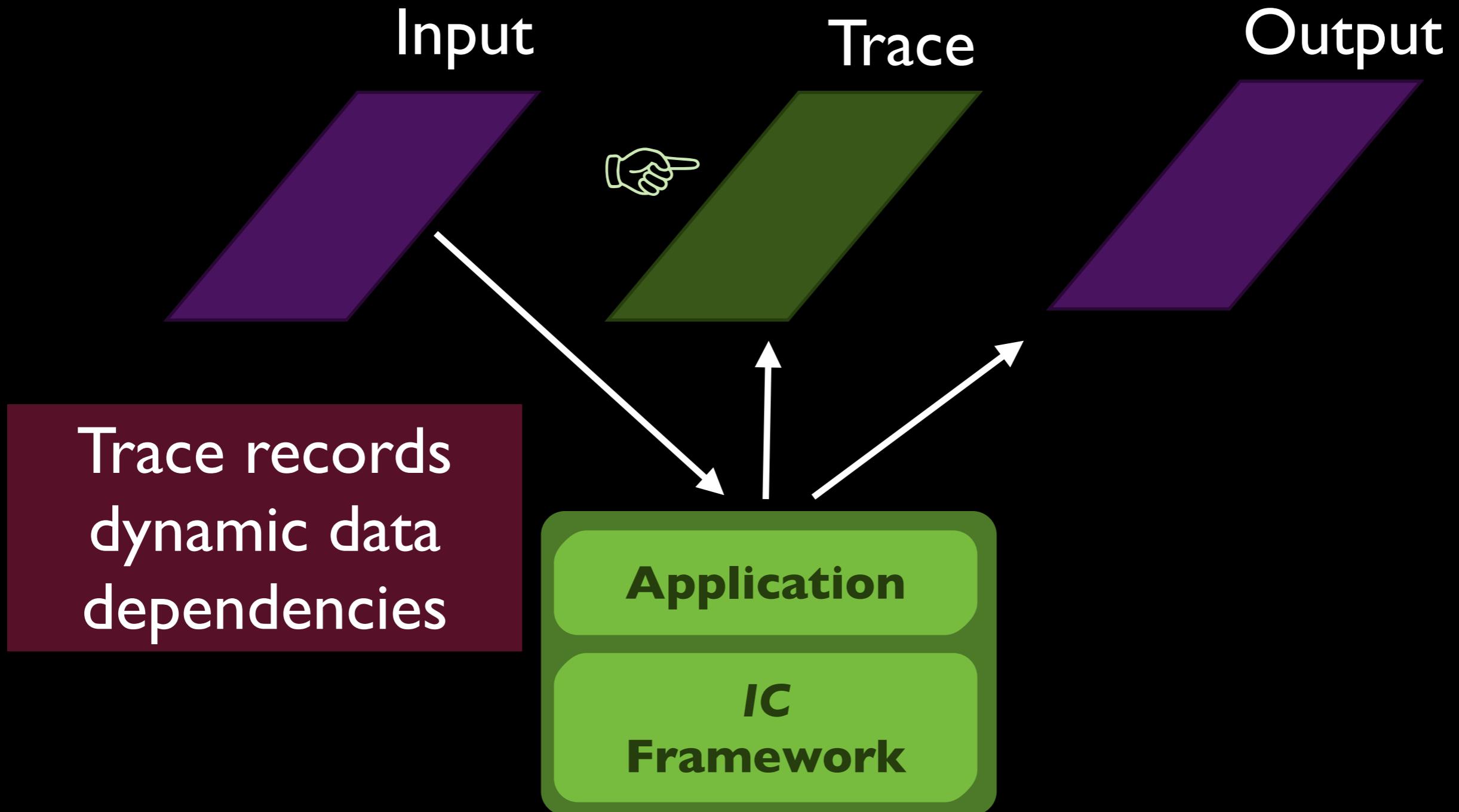
Khoo Yit Phang, Michael Hicks and
Jeffrey S. Foster



Incremental Computation



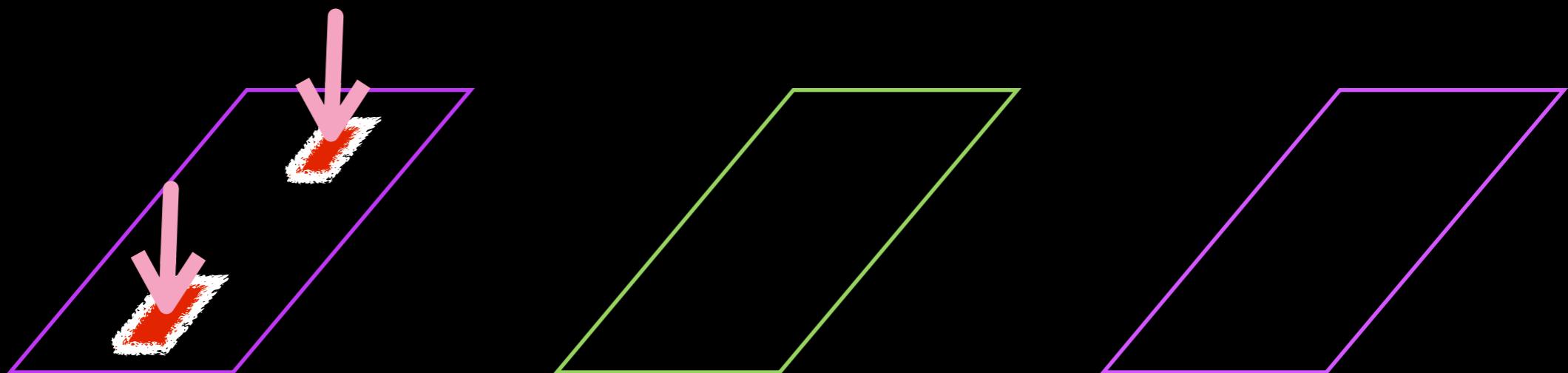
Incremental Computation



Incremental Computation



👉 ***Mutations***



Input

Trace

Output

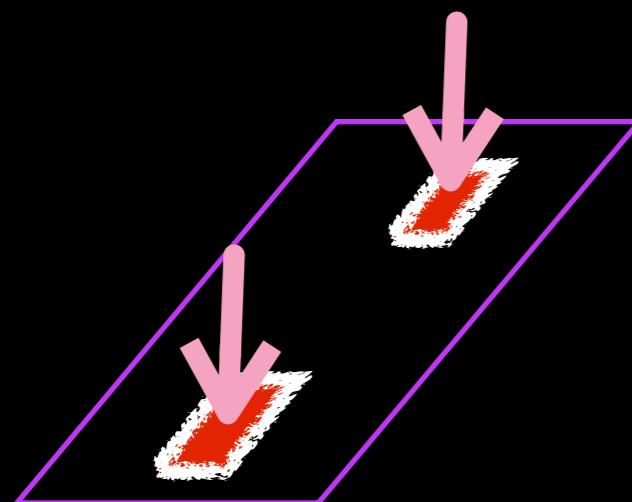
Incremental Computation



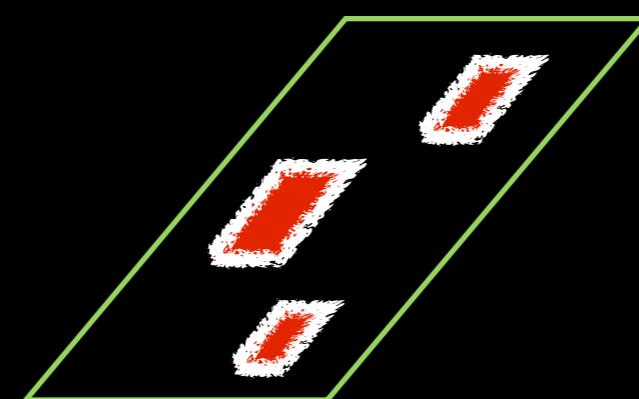
Mutations



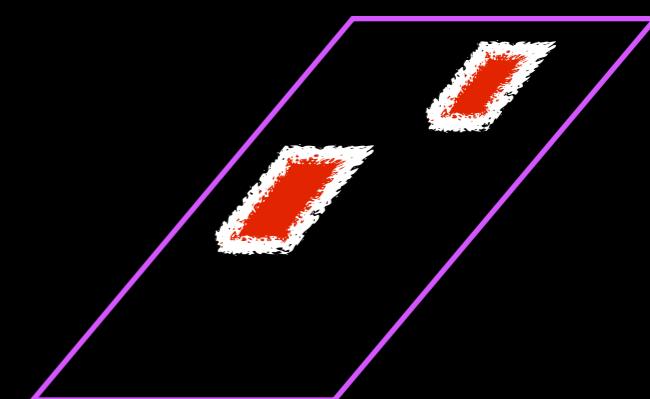
Inconsistencies



Input

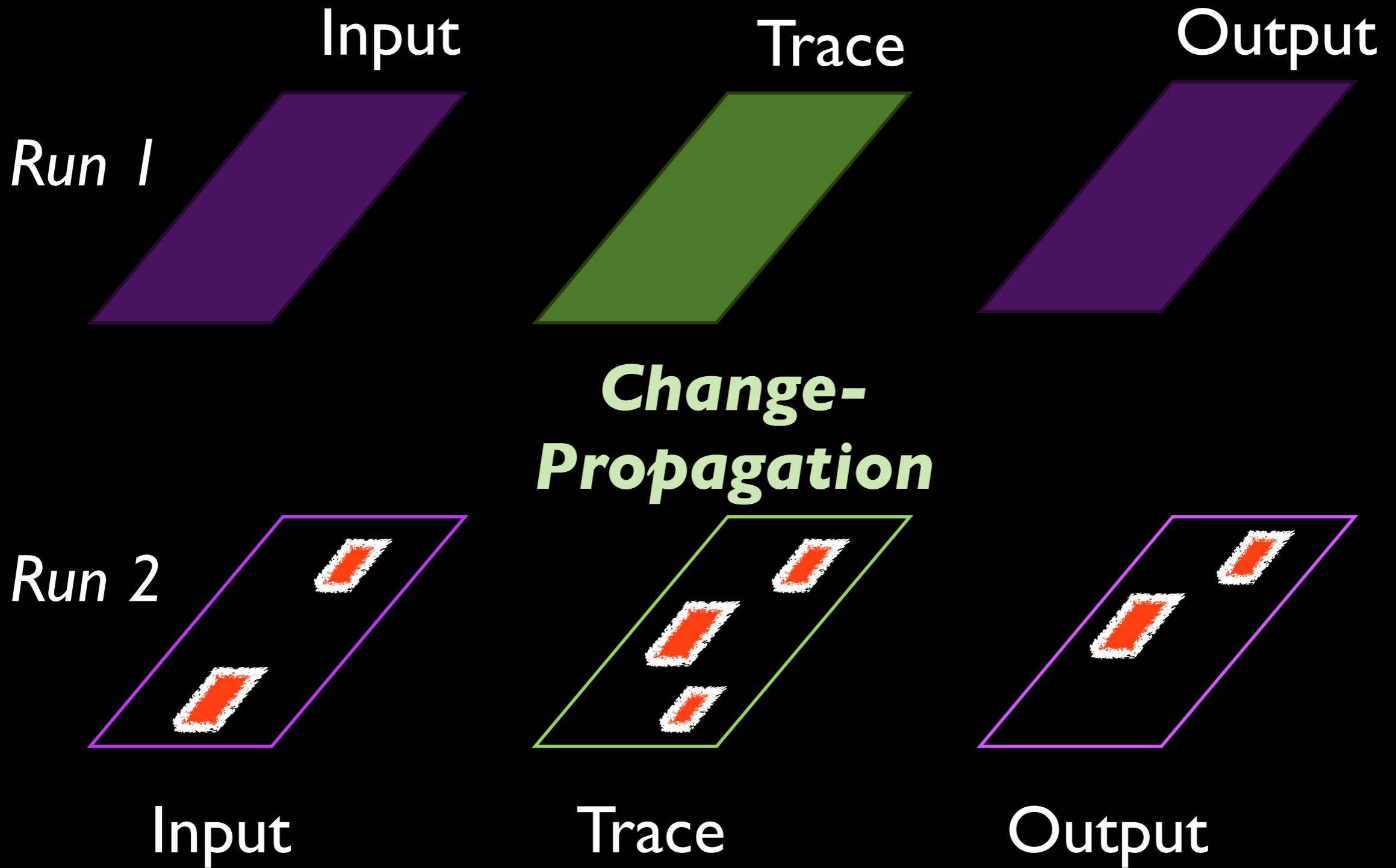


Trace

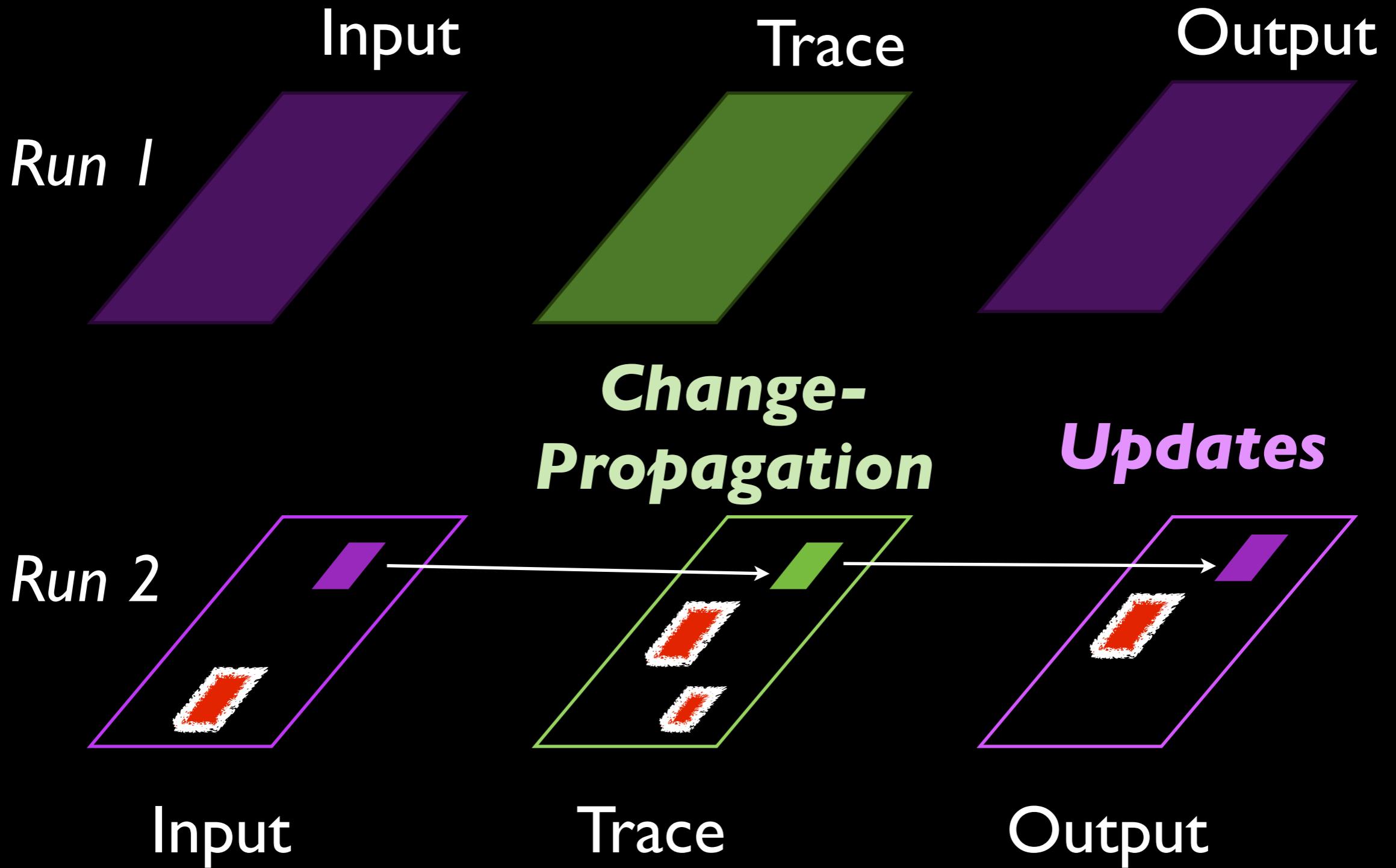


Output

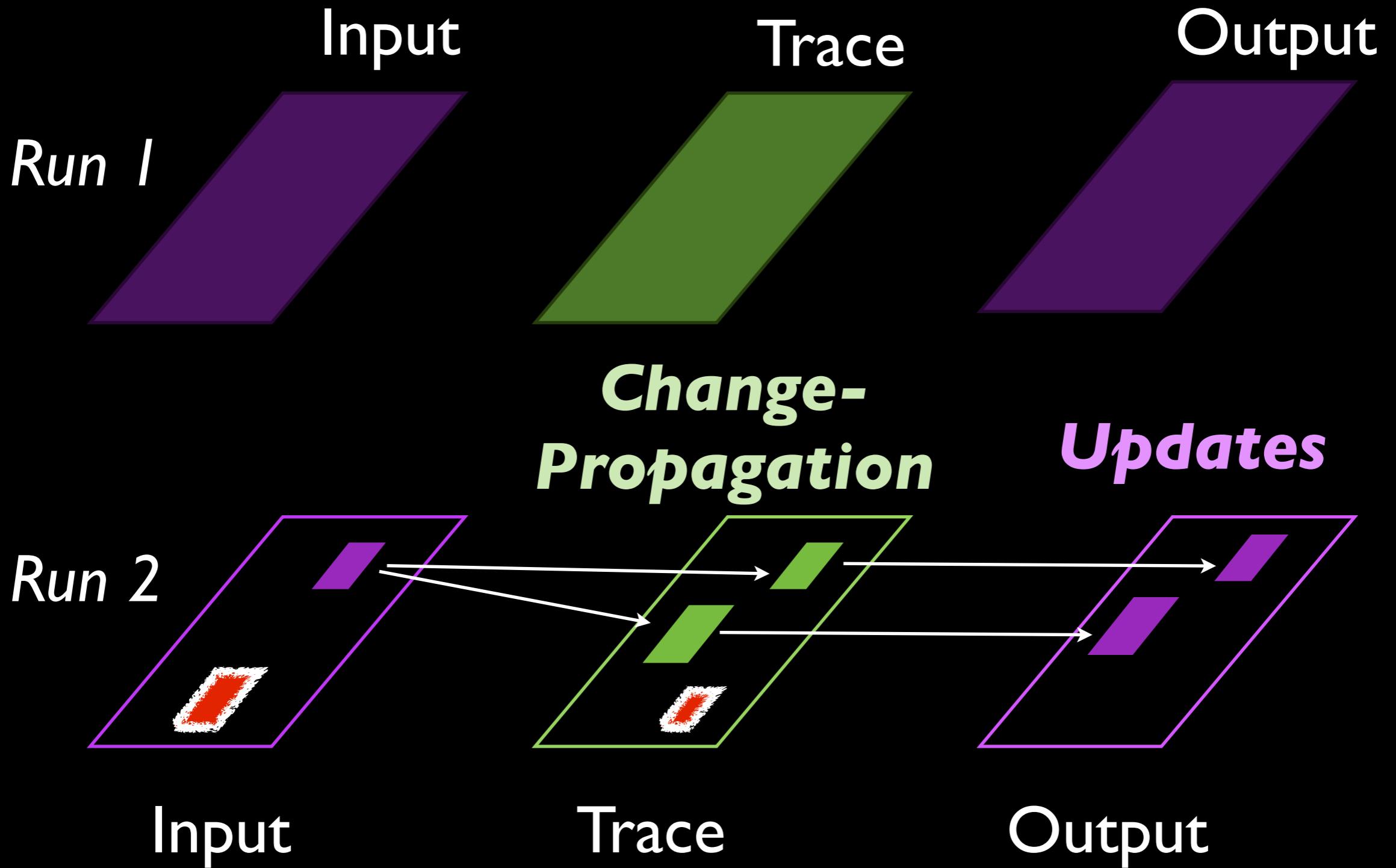
Incremental Computation



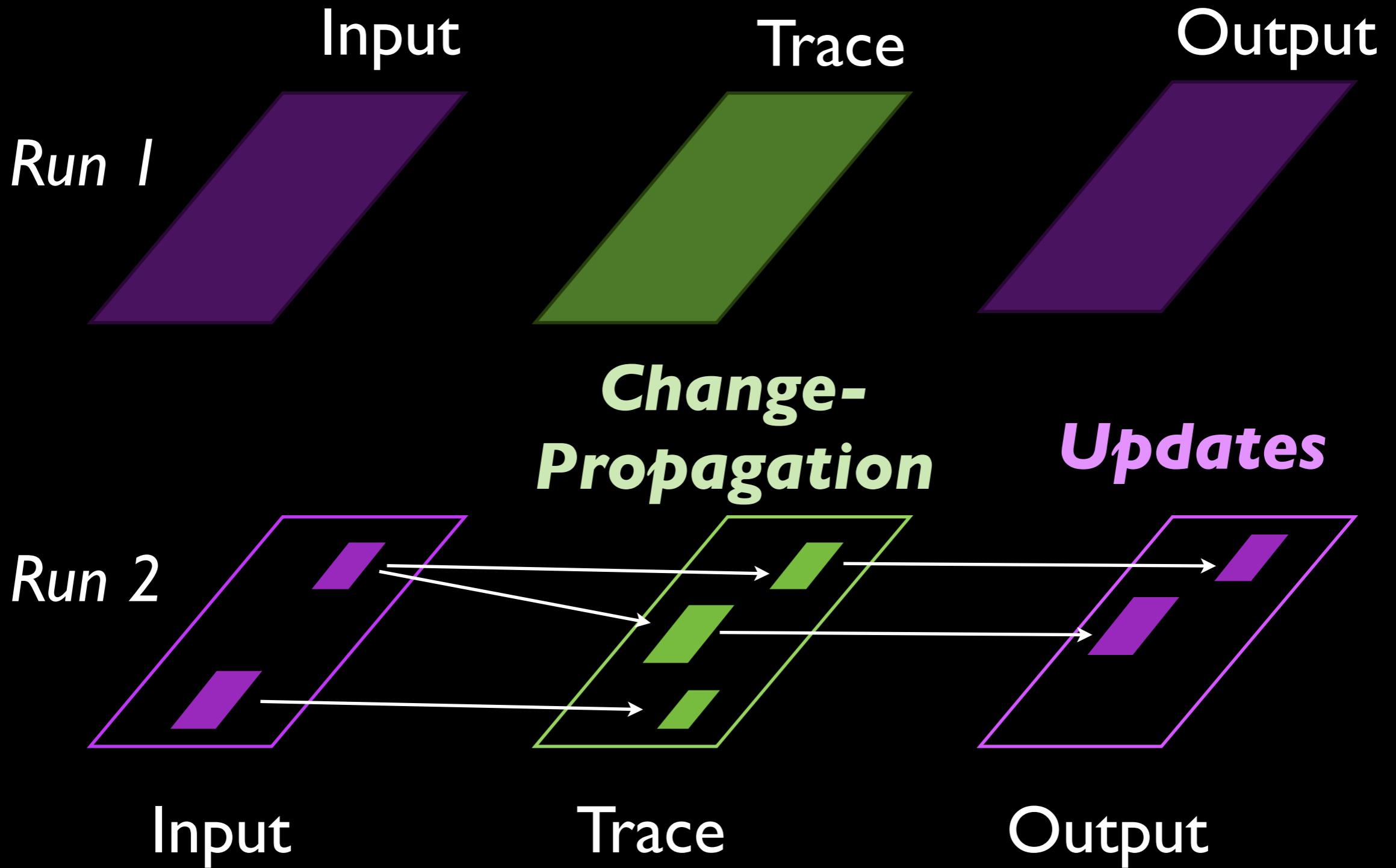
Incremental Computation



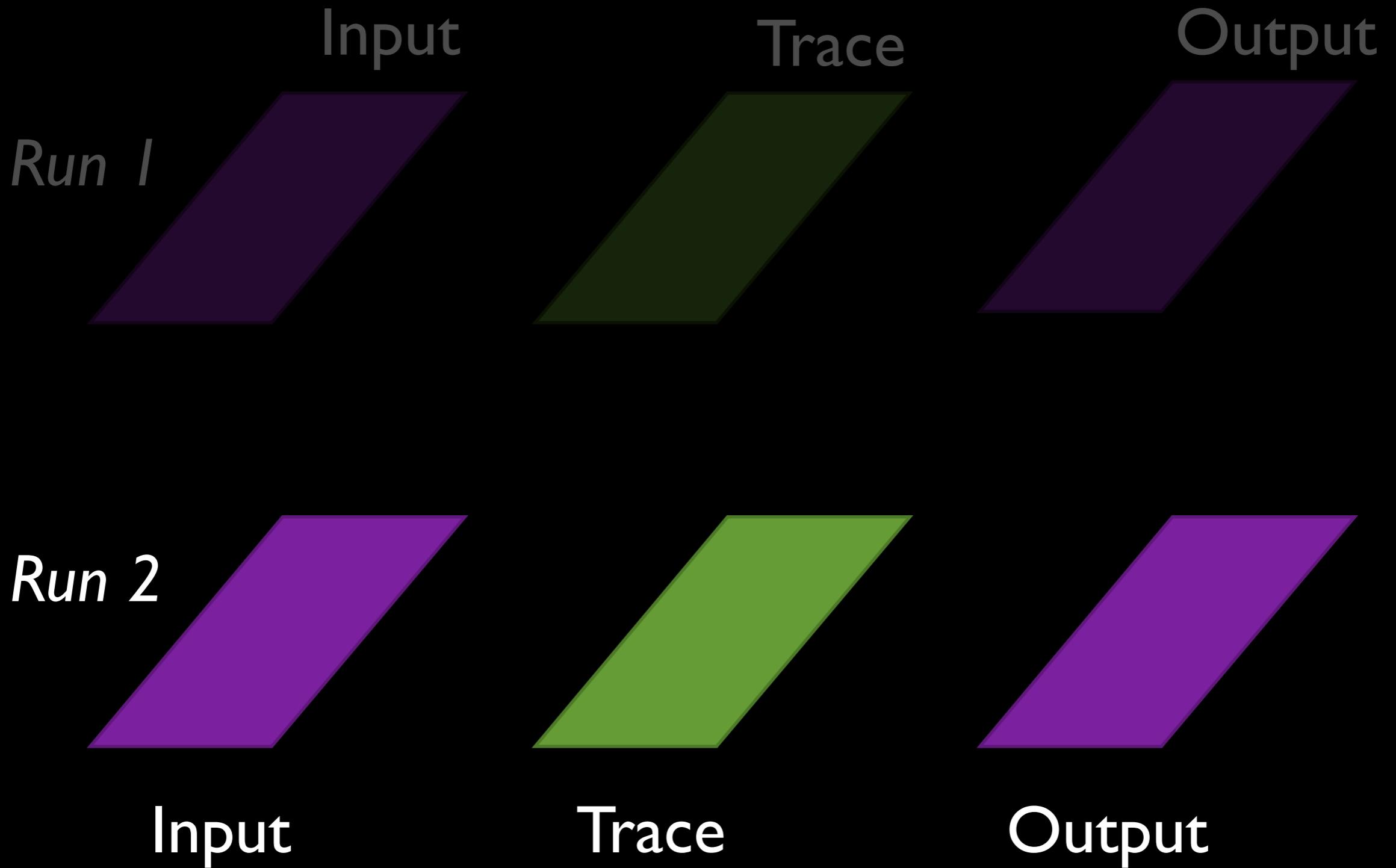
Incremental Computation



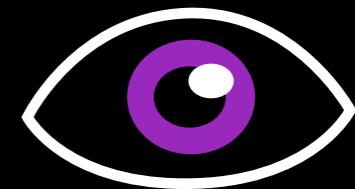
Incremental Computation



Incremental Computation



Incremental Computation



Observations

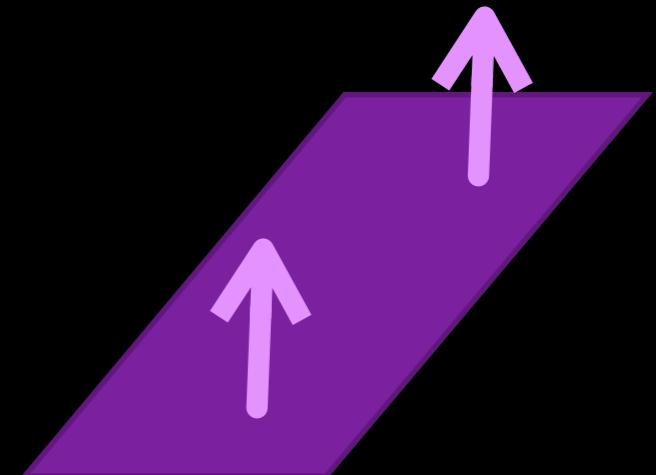
Run 2



Input



Trace



Output

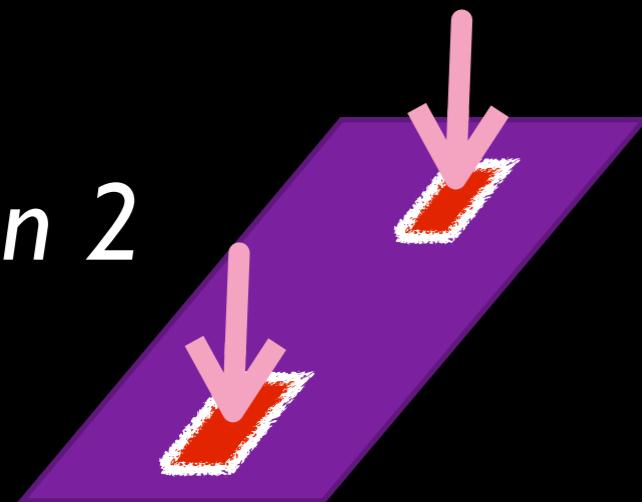
Incremental Computation

loop..



Mutations

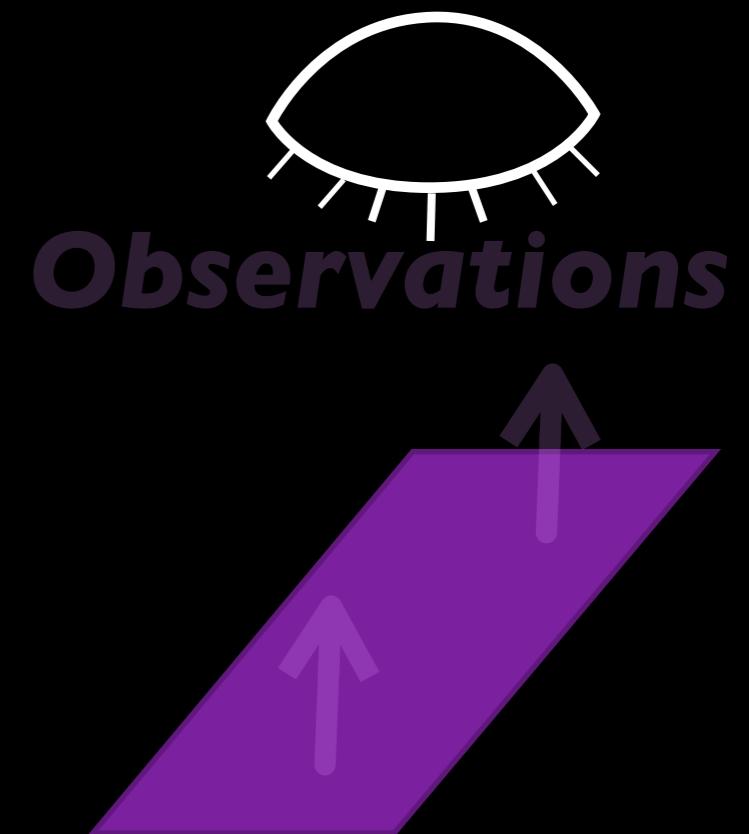
Run 2



Input



Trace



Output

Observations

Incremental Computation

Propagation respects program semantics:

Theorem:
Trace and output are
**“from-scratch”-
consistent**

Equivalently:
Change propagation is
**History
independent**

Run 2

Input

Trace

Output

Existing Limitations (self-adjusting computation)

- ▶ Change propagation is **eager**

Not driven by output observations



- ▶ Trace representation
= **Total ordering**

Limits reuse, excluding certain patterns

Interactive settings suffer in particular

Adapton: Composable, Demand-Driven IC

- Key concepts:

Lazy thunks: programming interface

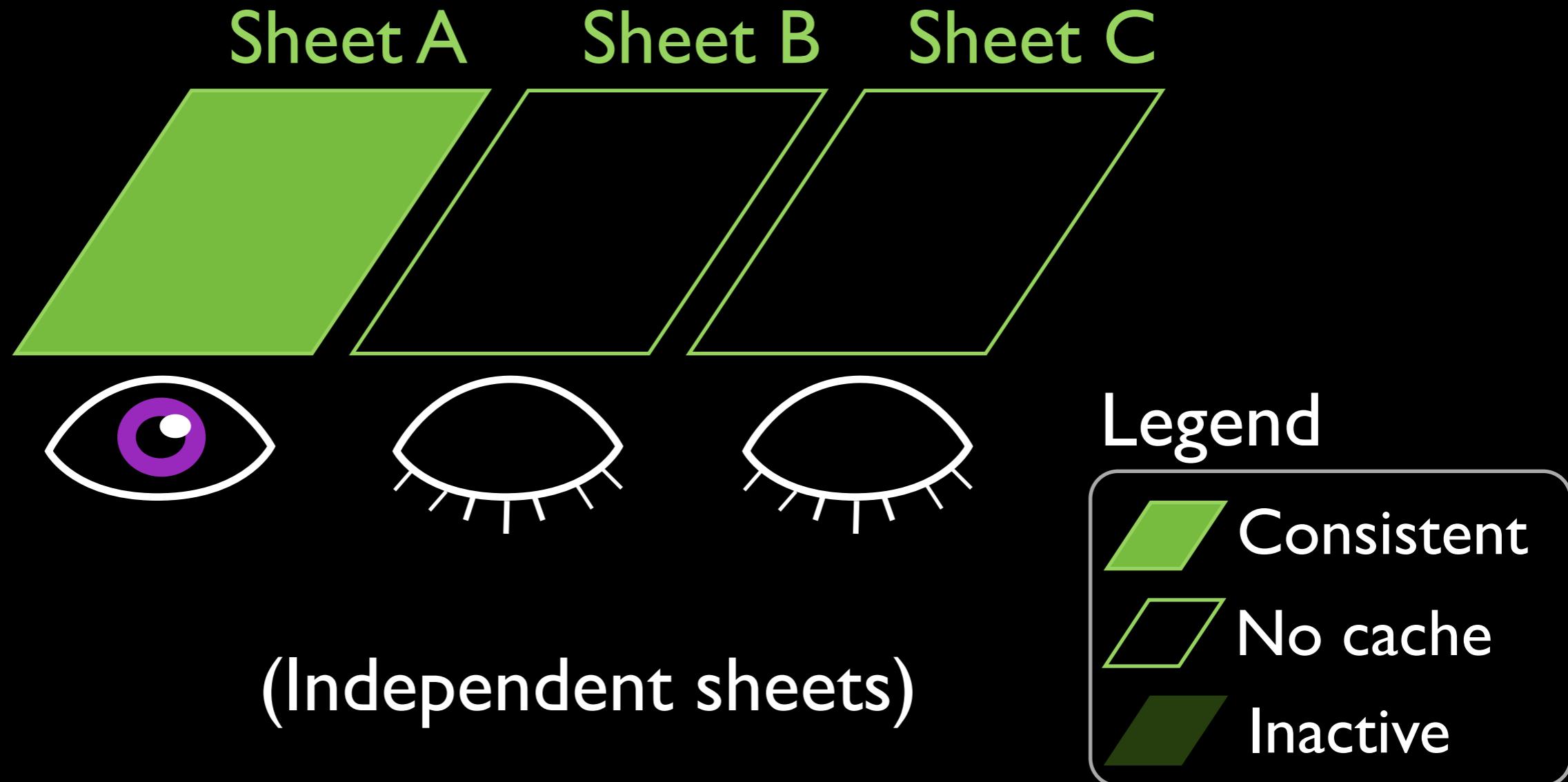
Demanded Computation Graph

(DCG): represents execution trace

- Formal semantics, proven sound
- Implemented in OCaml (and Python)
- Speedups for all patterns (unlike SAC)
- Freely available at <http://ter.ps/adapton>

Interaction Pattern: Laziness

Do not (re)compute obscured sheets

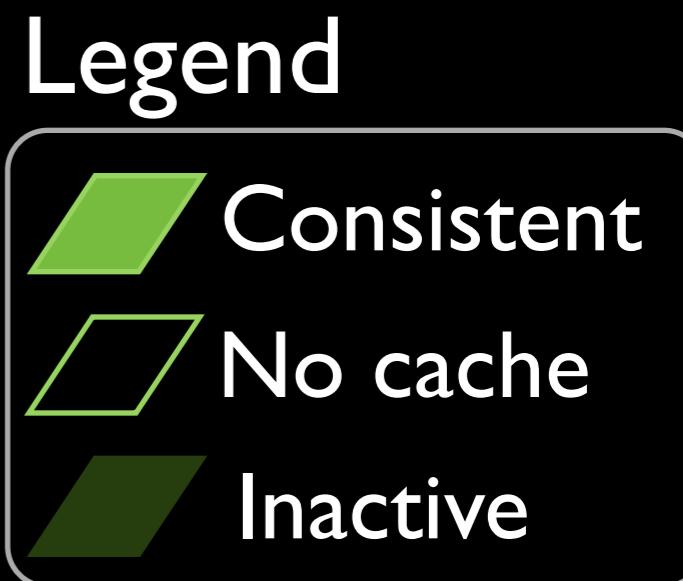


Interaction Pattern: Laziness

Do not (re)compute obscured sheets



(Independent sheets)

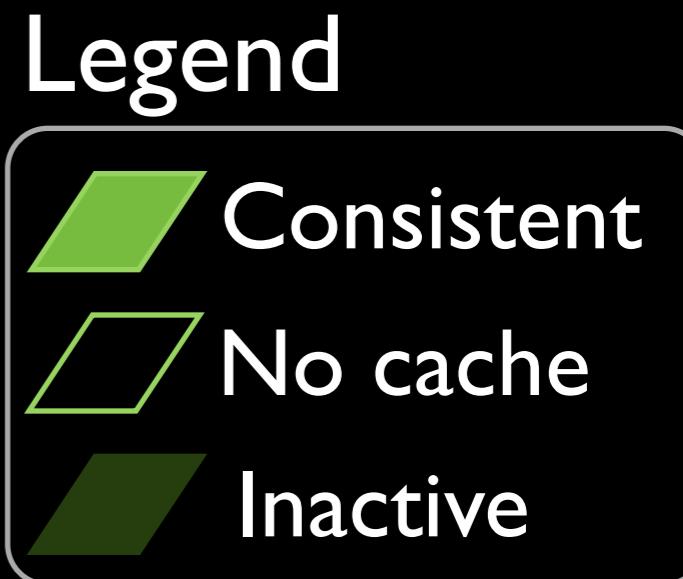


Interaction Pattern: Laziness

Do not (re)compute obscured sheets

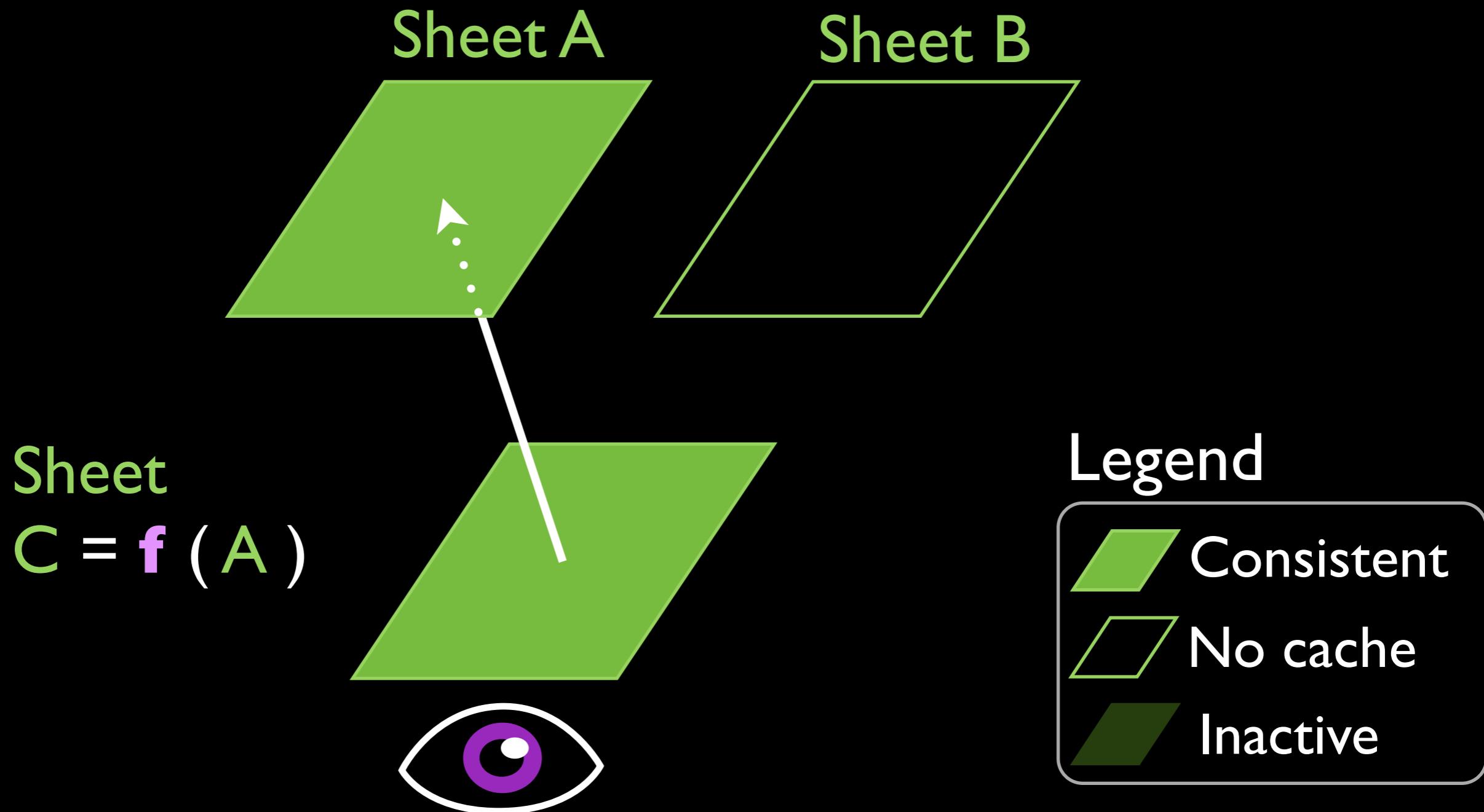


(Independent sheets)



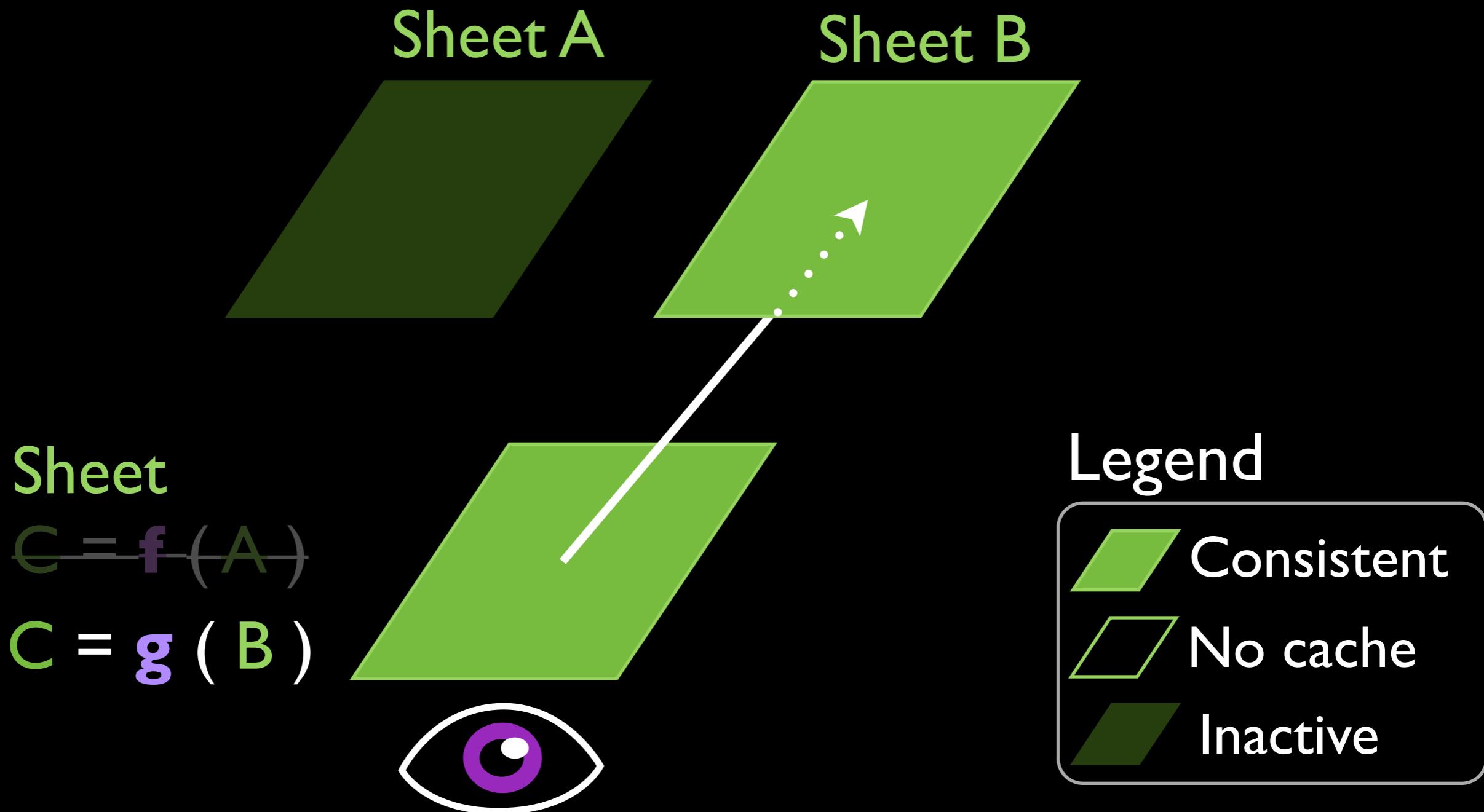
Interactive Pattern: Switching

Demand / control-flow change



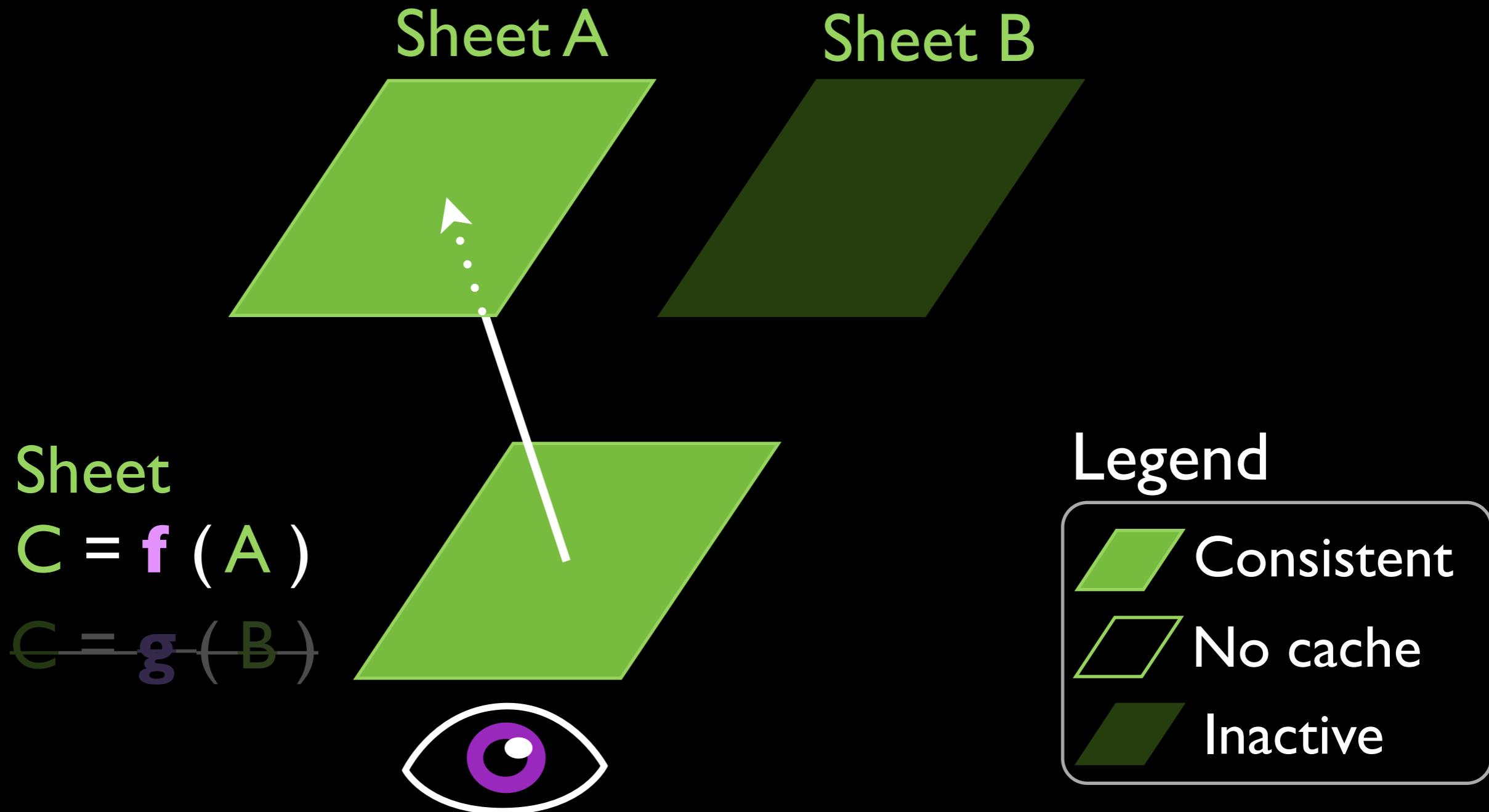
Interactive Pattern: Switching

Demand / control-flow change



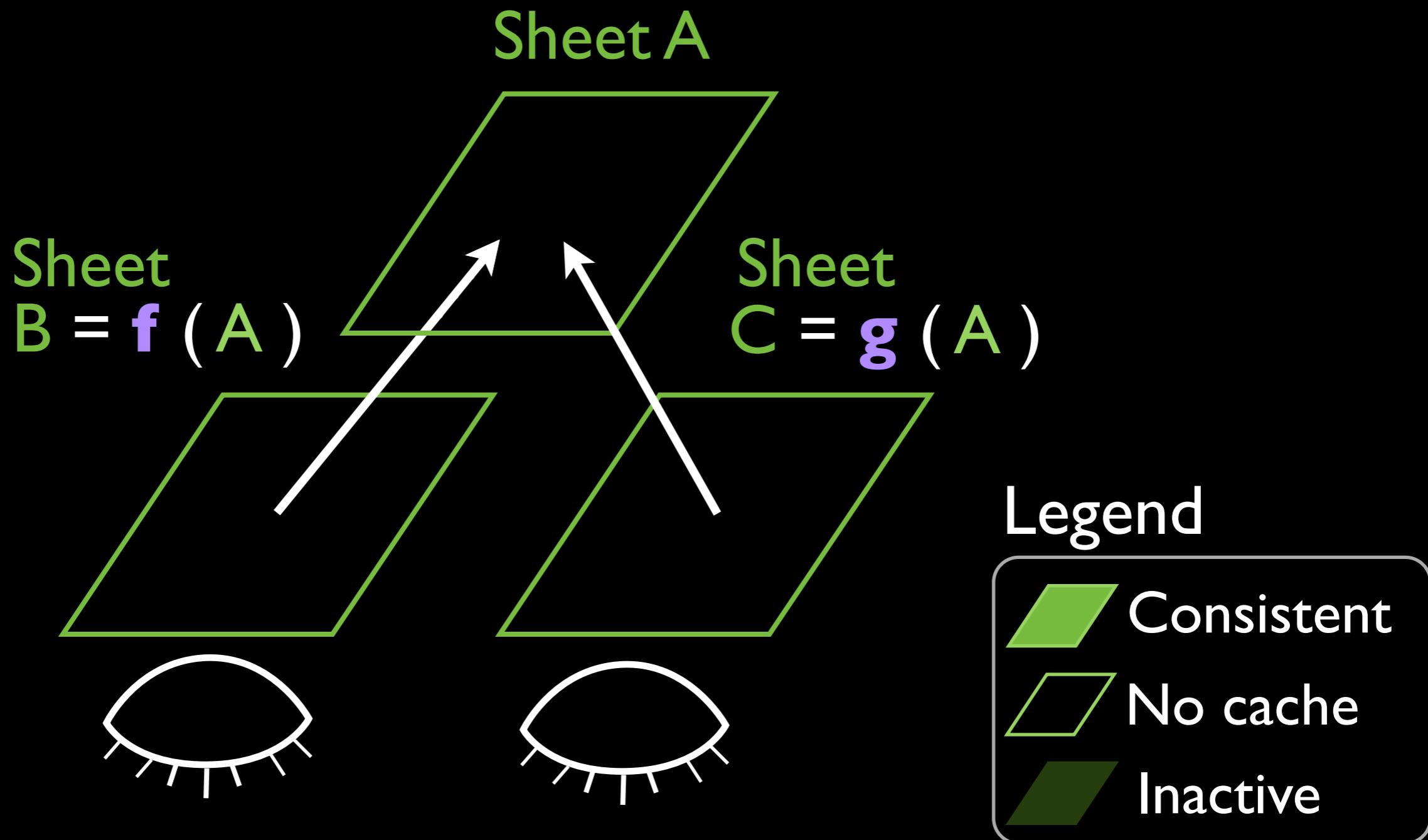
Interactive Pattern: Switching

Demand / control-flow change



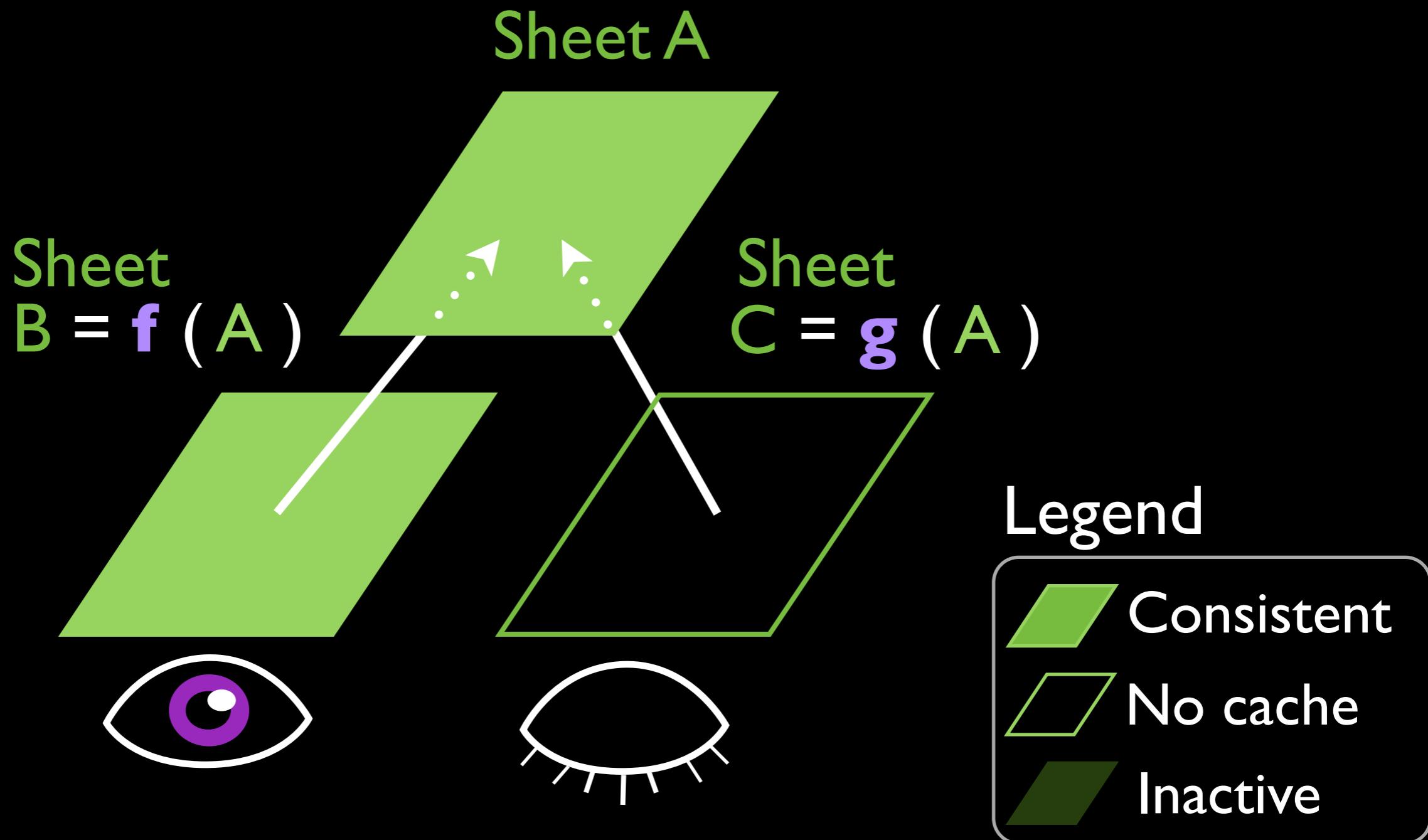
Interaction Pattern: Sharing

B and C share work for A



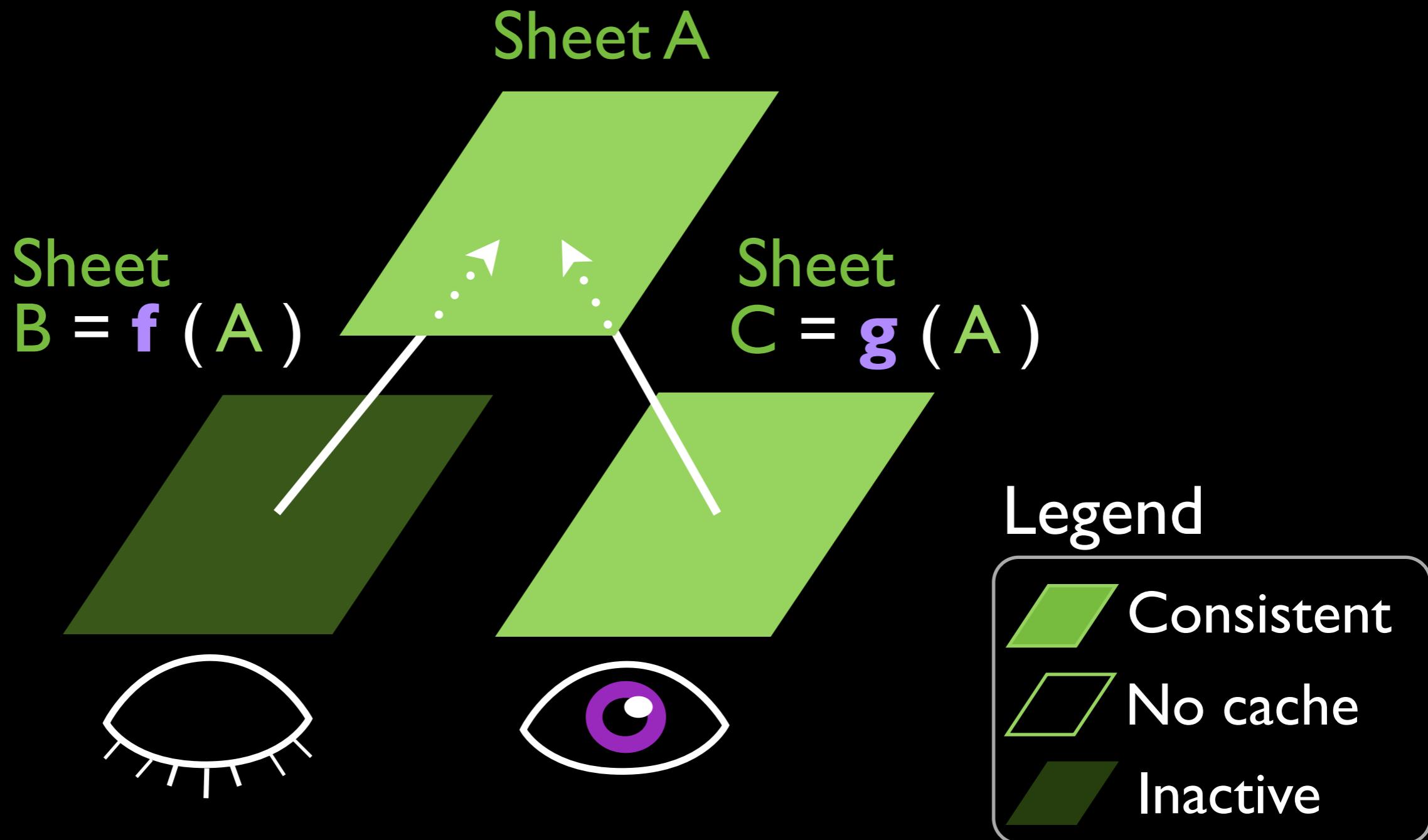
Interaction Pattern: Sharing

B and C share work for A



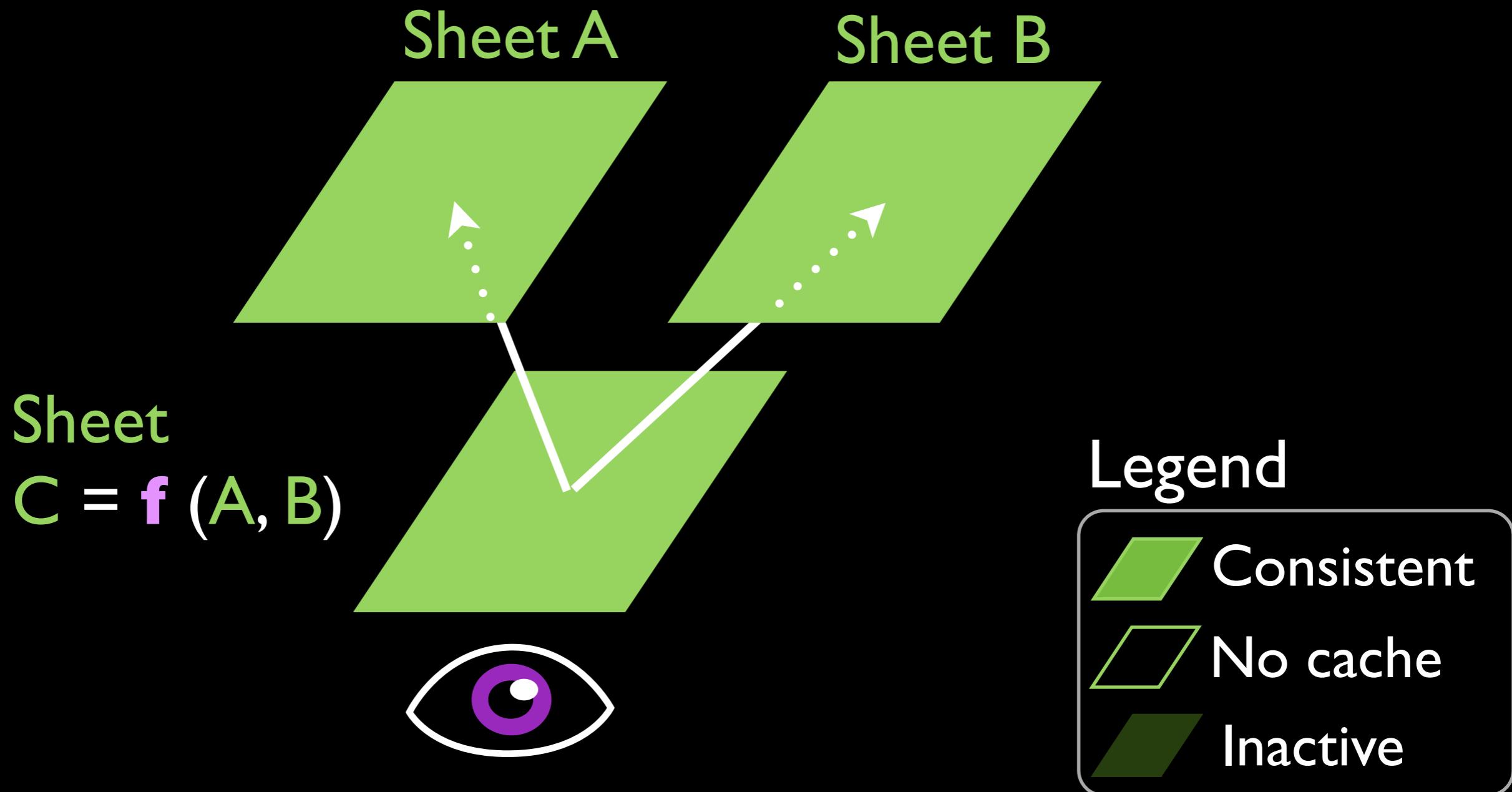
Interaction Pattern: Sharing

B and C share work for A



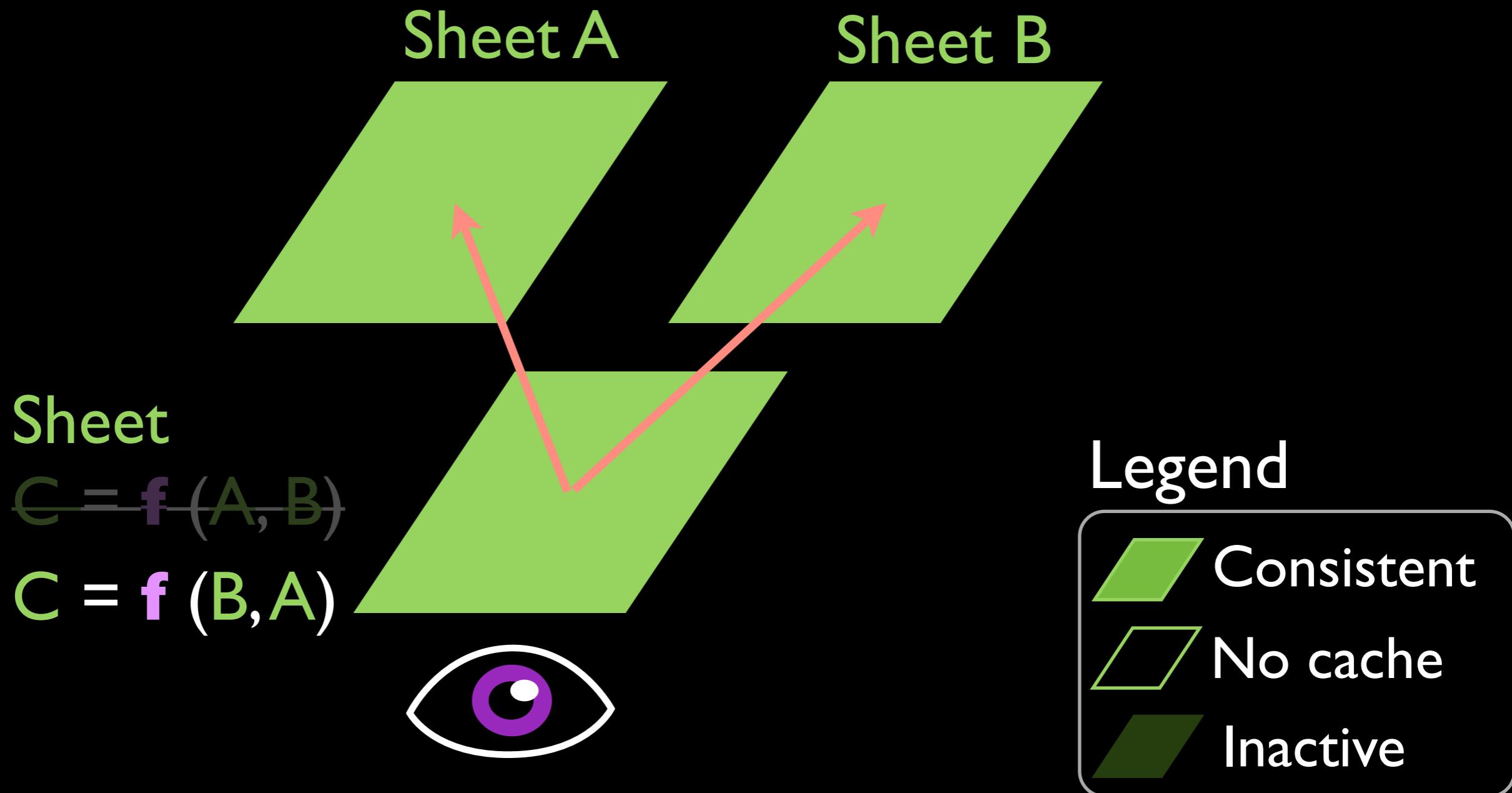
Interactive Pattern: Swapping

Swaps input / evaluation order



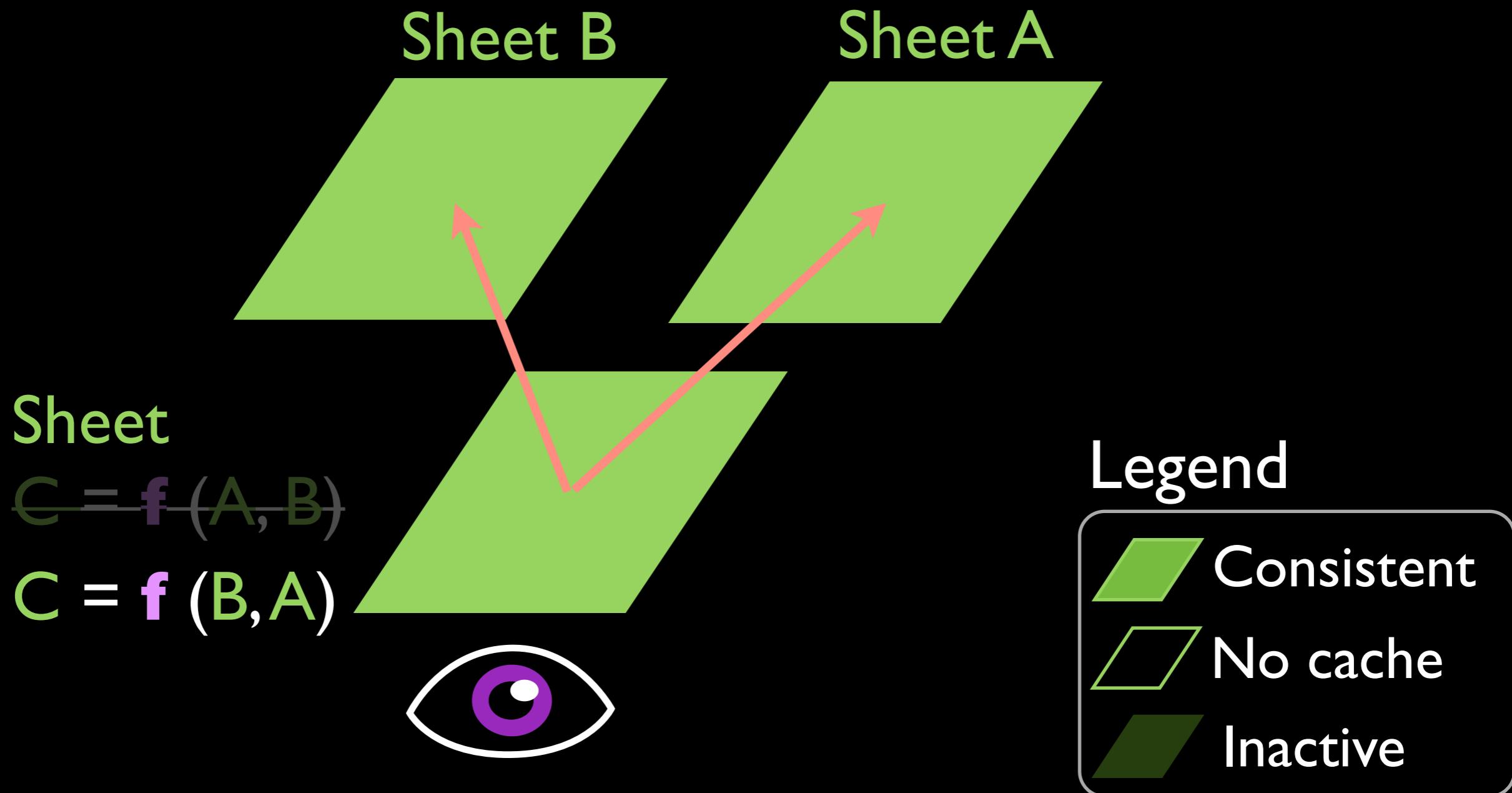
Interactive Pattern: Swapping

Swaps input / evaluation order



Interactive Pattern: Swapping

Swaps input / evaluation order



Adapton's Approach

- When we **mutate** an **input**, we mark dependent computations as **dirty**
- When we **demand a thunk**:
 - **Memo-match** equivalent thunks
 - **Change-propagation** repairs inconsistencies, **on demand**

Spread Sheet Evaluator

```
type cell = formula ref  
and formula =  
| Leaf of int  
| Plus of cell * cell
```

Spread Sheet Evaluator

Mutable

```
type cell = formula ref
```

```
and formula =
```

```
| Leaf of int  
| Plus of cell * cell
```

Depends
on cells

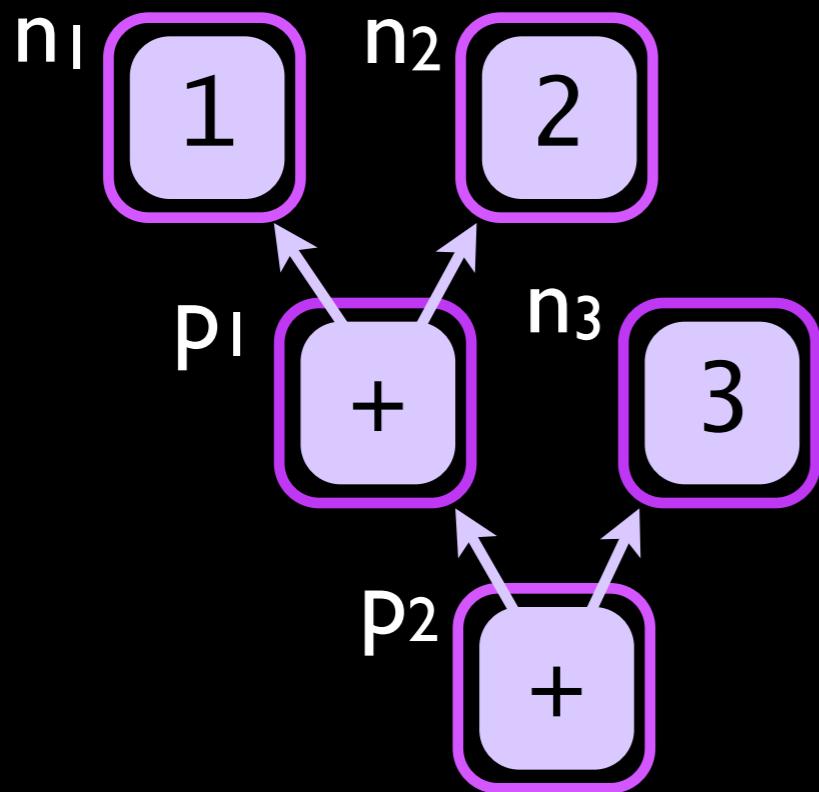
Spread Sheet Evaluator

Example

```
type cell = formula ref  
and formula =  
| Leaf of int  
| Plus of cell * cell
```

```
let n1 = ref (Leaf 1)  
  
let n2 = ref (Leaf 2)  
  
let n3 = ref (Leaf 3)  
  
let p1 = ref (Plus (n1, n2))  
  
let p2 = ref (Plus (p1, n3))
```

Spread Sheet Evaluator



```
type cell = formula ref
```

```
and formula =
```

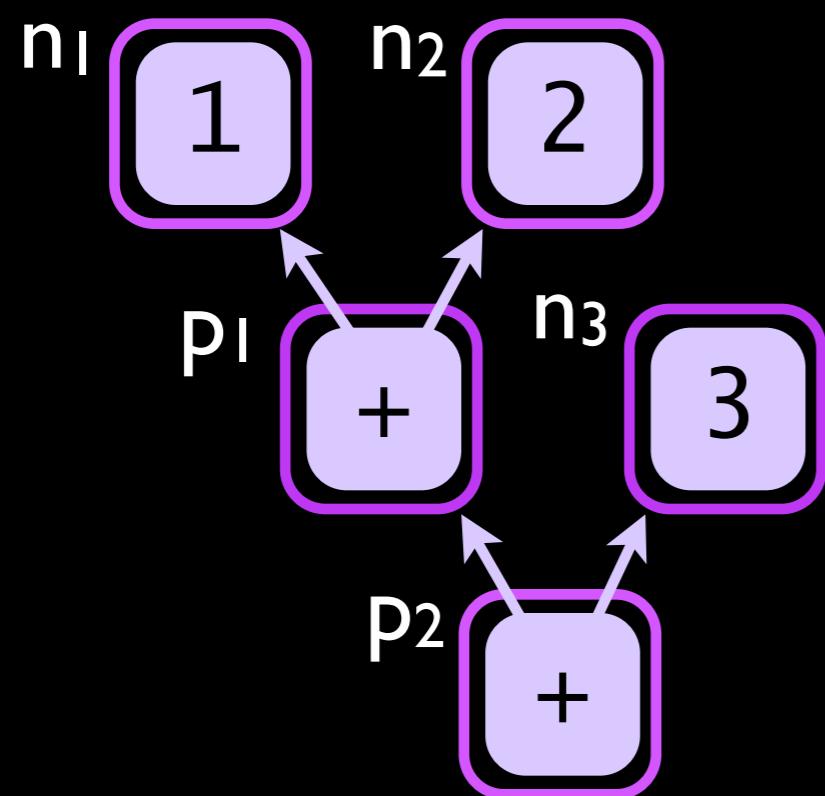
```
| Leaf of int  
| Plus of cell * cell
```

Example

```
let n1 = ref (Leaf 1)  
  
let n2 = ref (Leaf 2)  
  
let n3 = ref (Leaf 3)  
  
let p1 = ref (Plus (n1, n2))  
  
let p2 = ref (Plus (p1, n3))
```

“User interface” (REPL)

Spread Sheet Evaluator

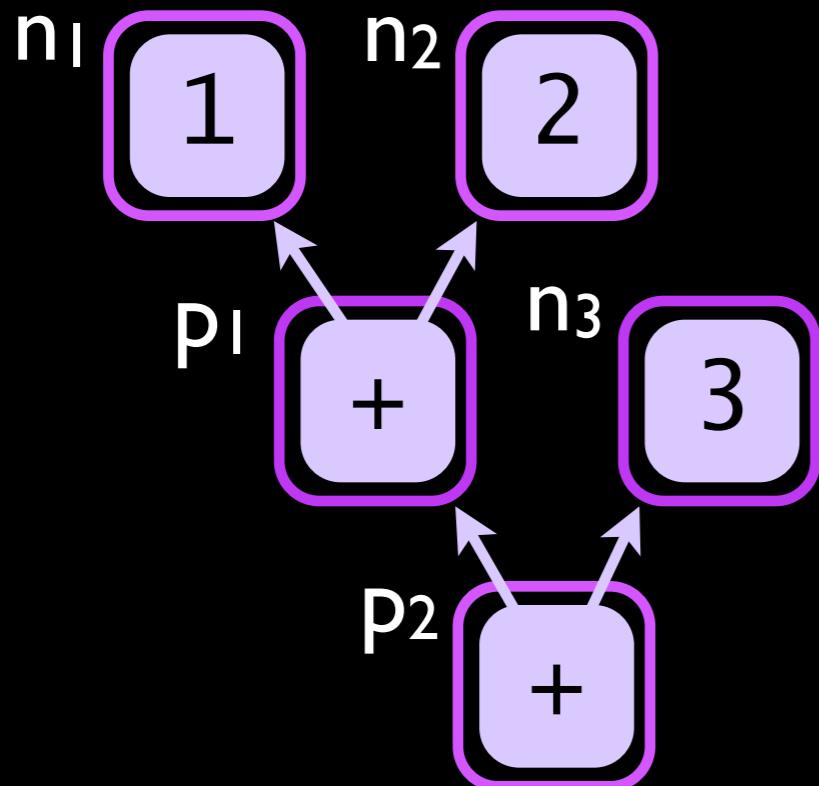


```
type cell = formula ref  
and formula =  
| Leaf of int  
| Plus of cell * cell
```

Evaluator logic

```
eval : cell → (int thunk)  
eval c = thunk ((  
  case (get c) of  
  | Leaf n ⇒ n  
  | Plus(c1, c2) ⇒  
    force (eval c1) +  
    force (eval c2)  
  ))
```

Spread Sheet Evaluator



```
type cell = formula ref
```

```
and formula =
```

```
| Leaf of int  
| Plus of cell * cell
```

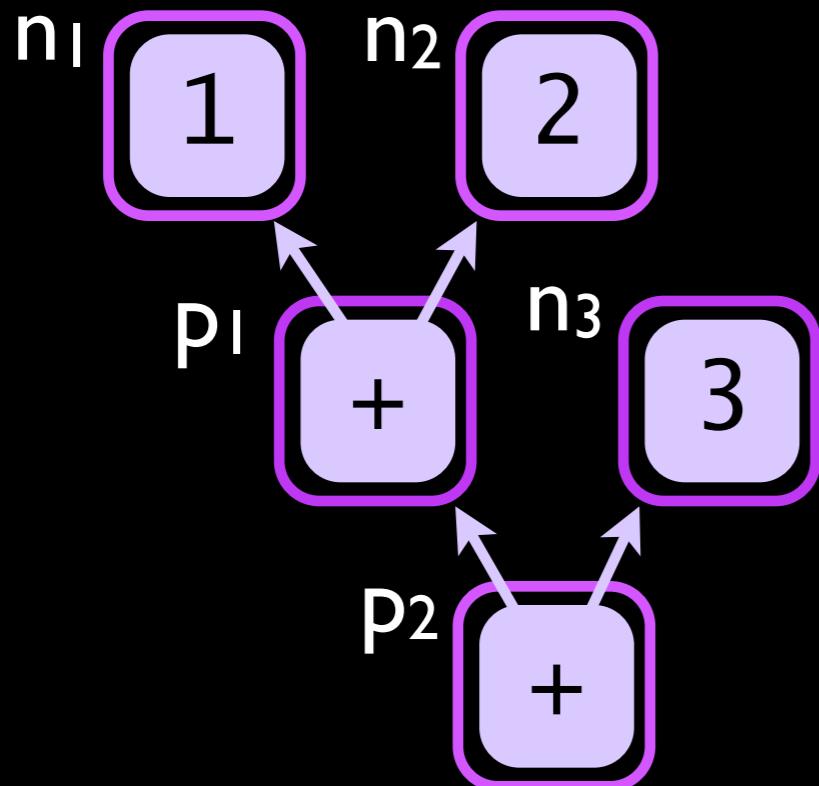
```
set : cell x formula → unit
```

```
eval : cell → (int thunk)
```

```
display : (int thunk) → unit
```

“User interface” (REPL)

Spread Sheet Evaluator



type cell = formula ref

and formula =

- | Leaf of int
- | Plus of cell * cell

set : cell x formula → unit

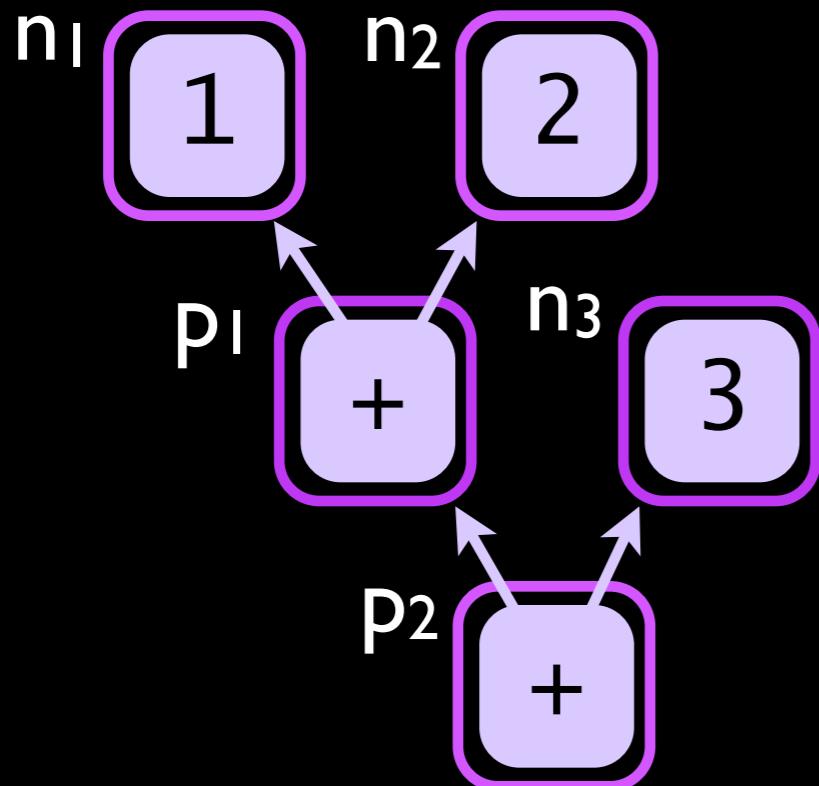
eval : cell → (int thunk)

display : (int thunk) → unit

“User interface” (REPL)

Demands evaluation

Spread Sheet Evaluator



```
type cell = formula ref
```

and formula =

- | Leaf of int
- | Plus of cell * cell

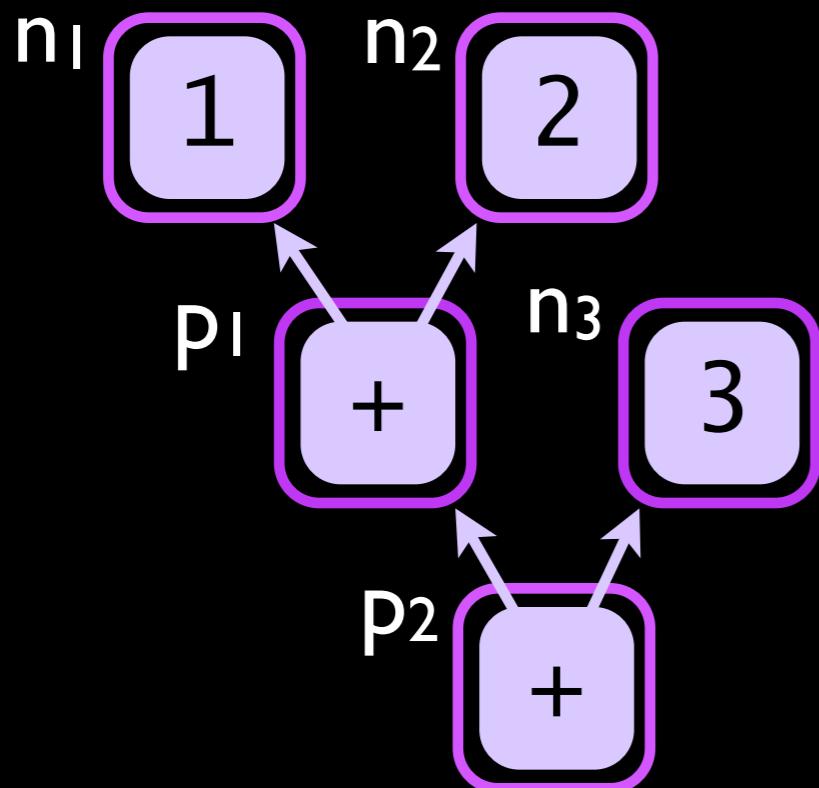
set : cell x formula → unit

eval : cell → (int thunk)

display : (int thunk) → unit

“User interface” (REPL)

Spread Sheet Evaluator



```
type cell = formula ref
```

and formula =

- | Leaf of int
- | Plus of cell * cell

set : cell x formula → unit

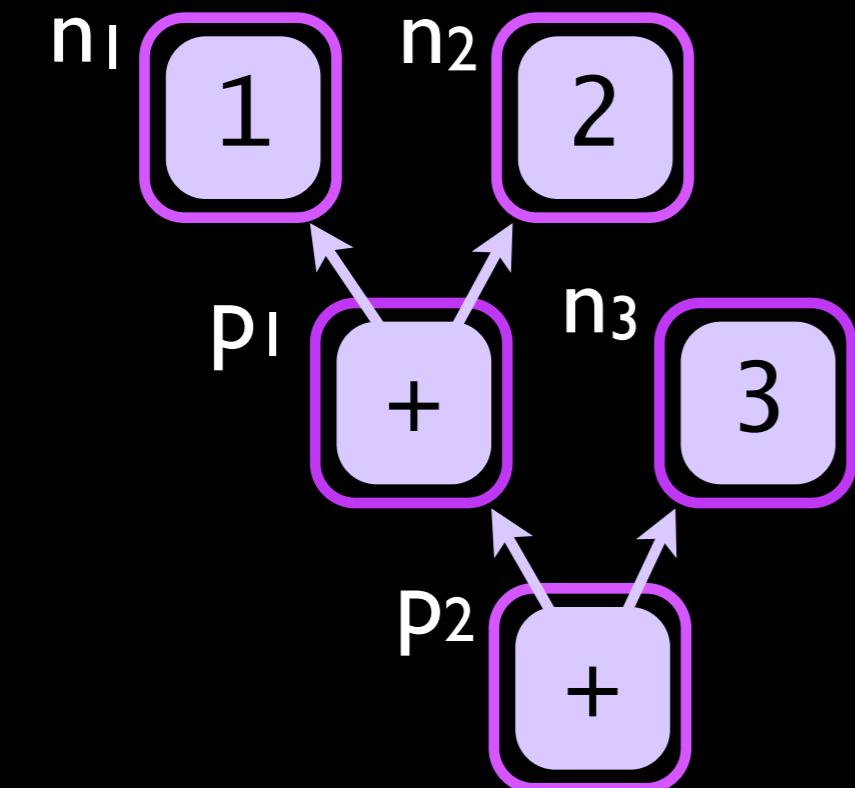
eval : cell → (int thunk)

display : (int thunk) → unit

“User interface” (REPL)

👉 let t₁ = eval p₁

Spread Sheet Evaluator



t_1
p1

set : cell x formula → unit

eval : cell → (int thunk)

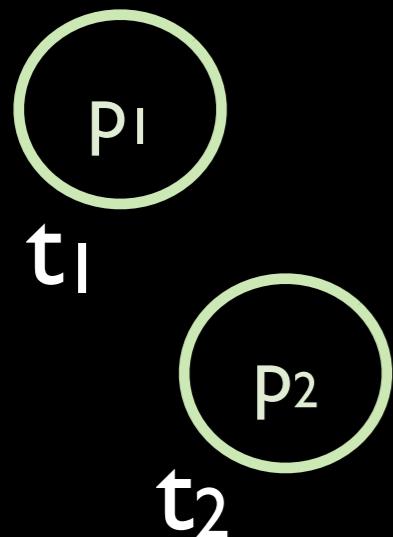
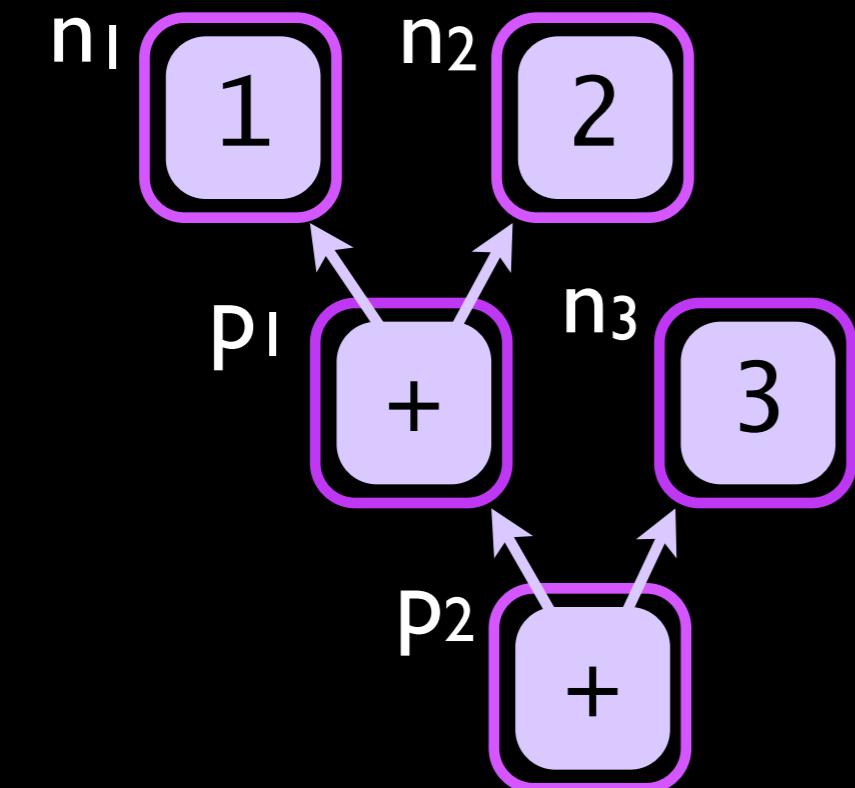
display : (int thunk) → unit

“User interface” (REPL)

☞ let $t_1 = \text{eval } p_1$



Spread Sheet Evaluator



`set : cell × formula → unit`

`eval : cell → (int thunk)`

`display : (int thunk) → unit`

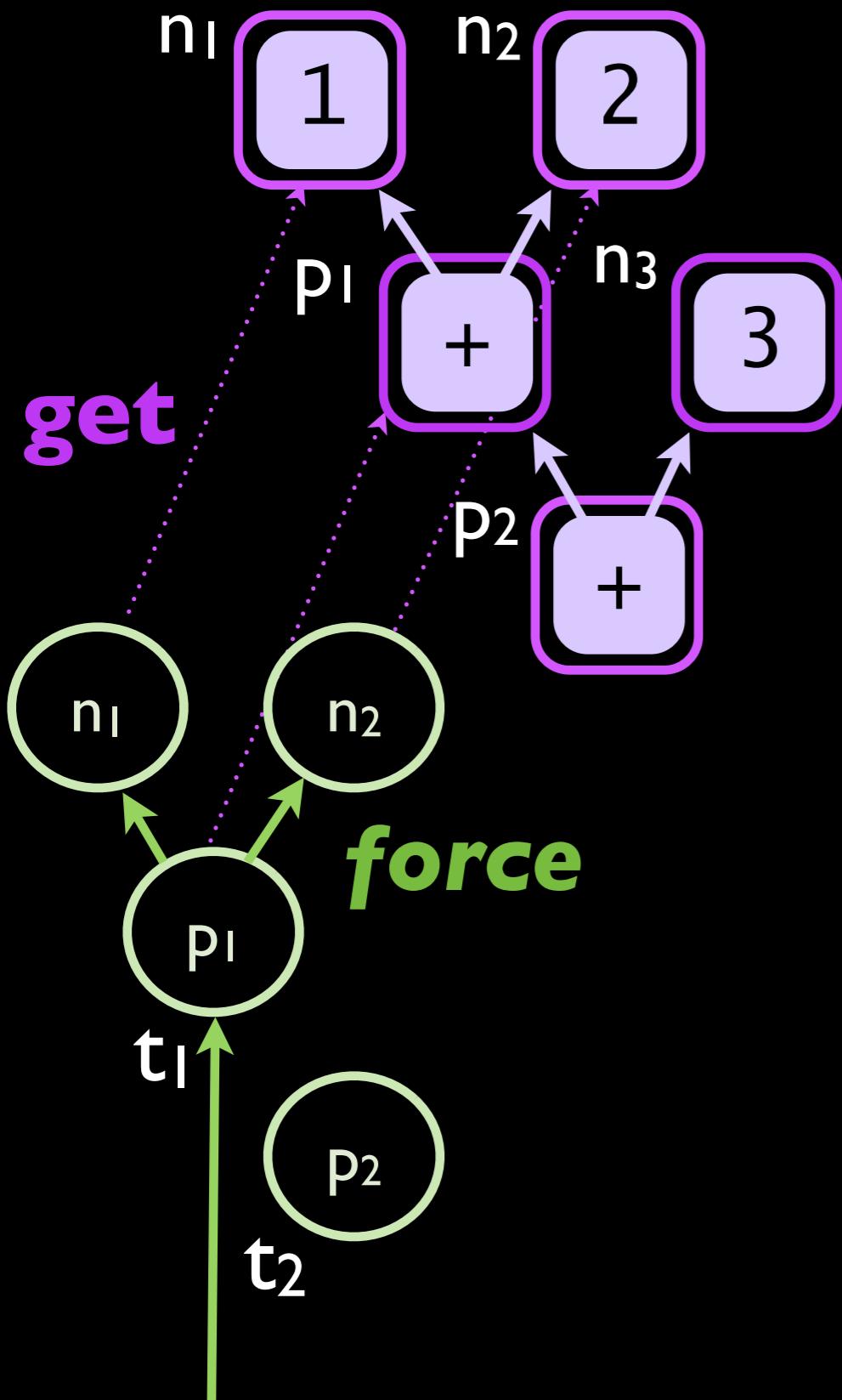
“User interface” (REPL)

👉 `let t1 = eval p1`

👉 `let t2 = eval p2`



Spread Sheet Evaluator



set : cell × formula → unit

eval : cell → (int thunk)

display : (int thunk) → unit

“User interface” (REPL)

👉 let $t_1 = \text{eval } p_1$

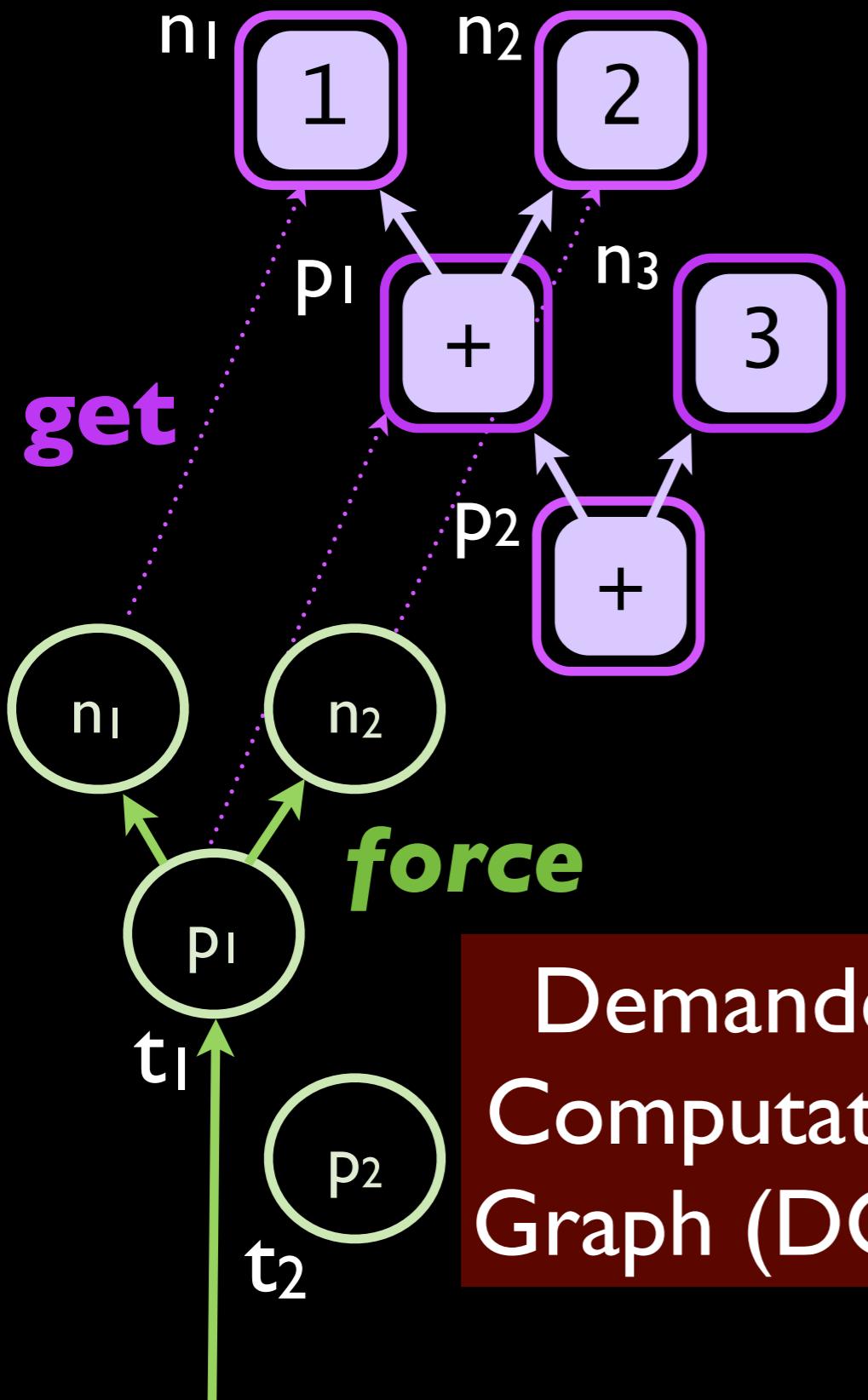
👉 let $t_2 = \text{eval } p_2$

👉 display t_1

demand!

3

Spread Sheet Evaluator



set : cell \times formula \rightarrow unit

eval : cell \rightarrow (int thunk)

display : (int thunk) \rightarrow unit

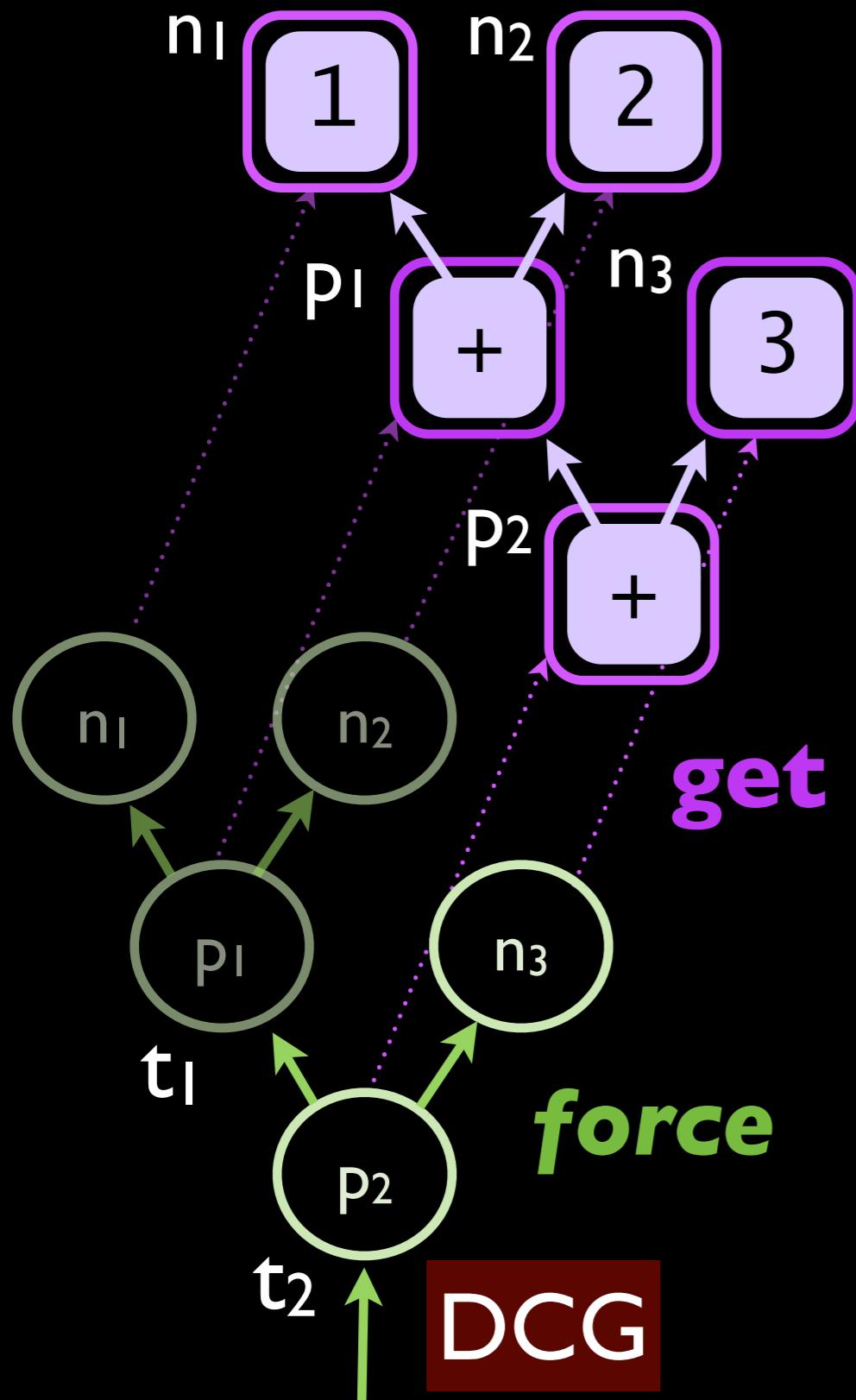
“User interface” (REPL)

👉 let $t_1 = \text{eval } p_1$

👉 let $t_2 = \text{eval } p_2$

👉 display t_1

Spread Sheet Evaluator



set : cell × formula → unit

eval : cell → (int thunk)

display : (int thunk) → unit

“User interface” (REPL)

☞ let $t_1 = \text{eval } p_1$

☞ let $t_2 = \text{eval } p_2$

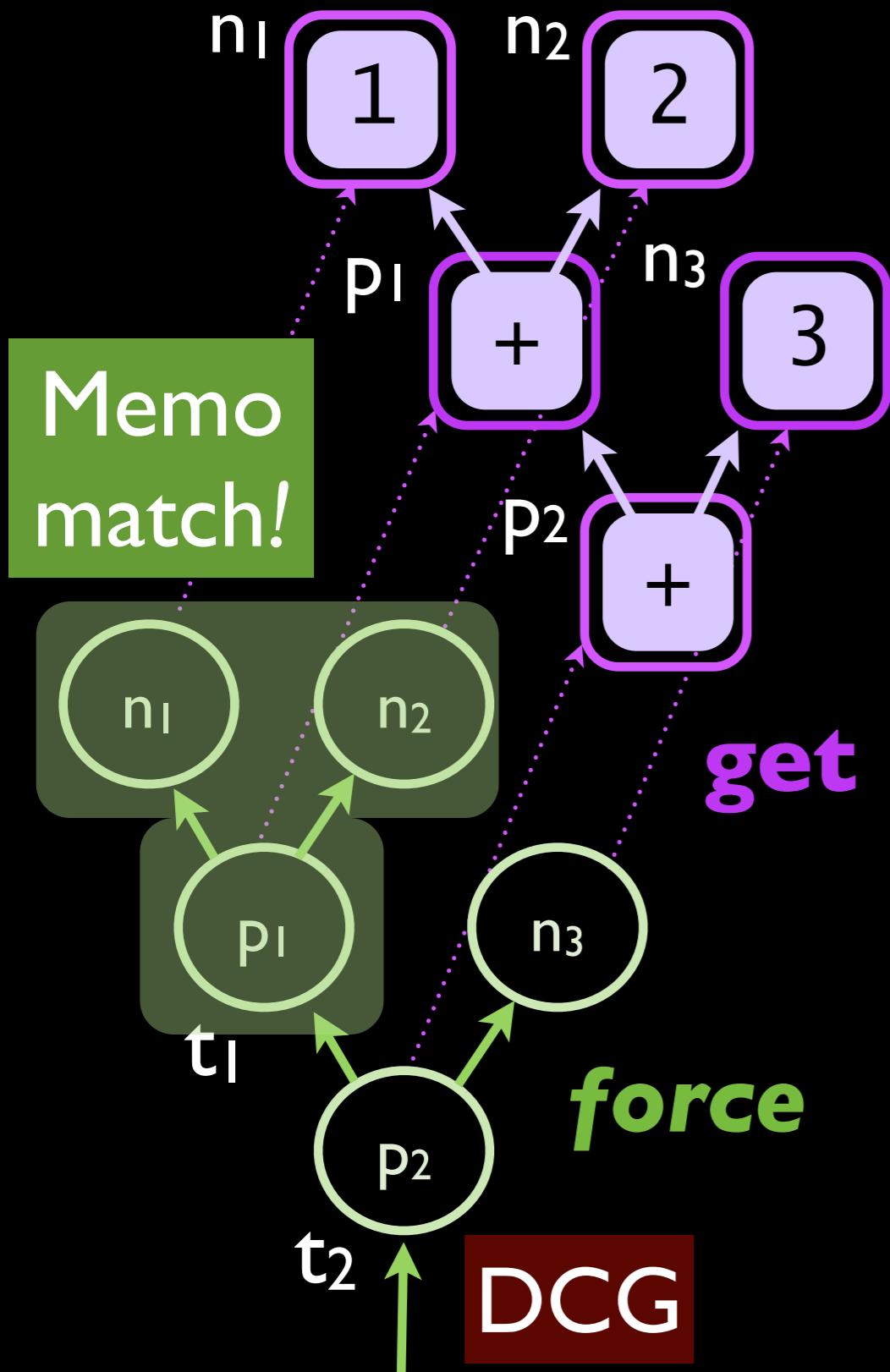
☞ display t_1

☞ display t_2

6

demand!

Spread Sheet Evaluator



set : cell × formula → unit

eval : cell → (int thunk)

display : (int thunk) → unit

“User interface” (REPL)

let $t_1 = \text{eval } p_1$

let $t_2 = \text{eval } p_2$

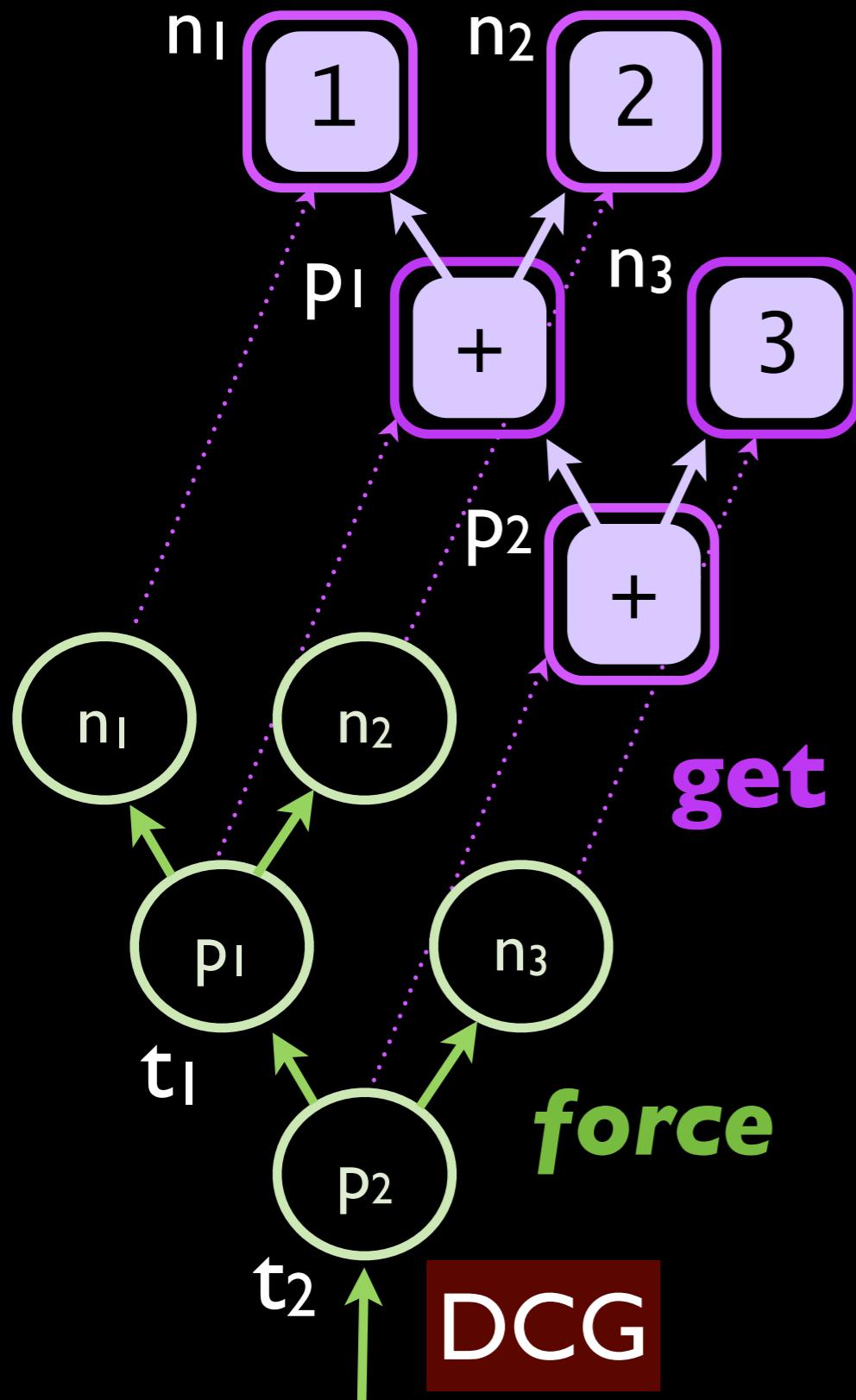
display t_1

display t_2

Memo match!
Sharing

6

Spread Sheet Evaluator



set : cell × formula → unit

eval : cell → (int thunk)

display : (int thunk) → unit

“User interface” (REPL)

☞ let $t_1 = \text{eval } p_1$

☞ let $t_2 = \text{eval } p_2$

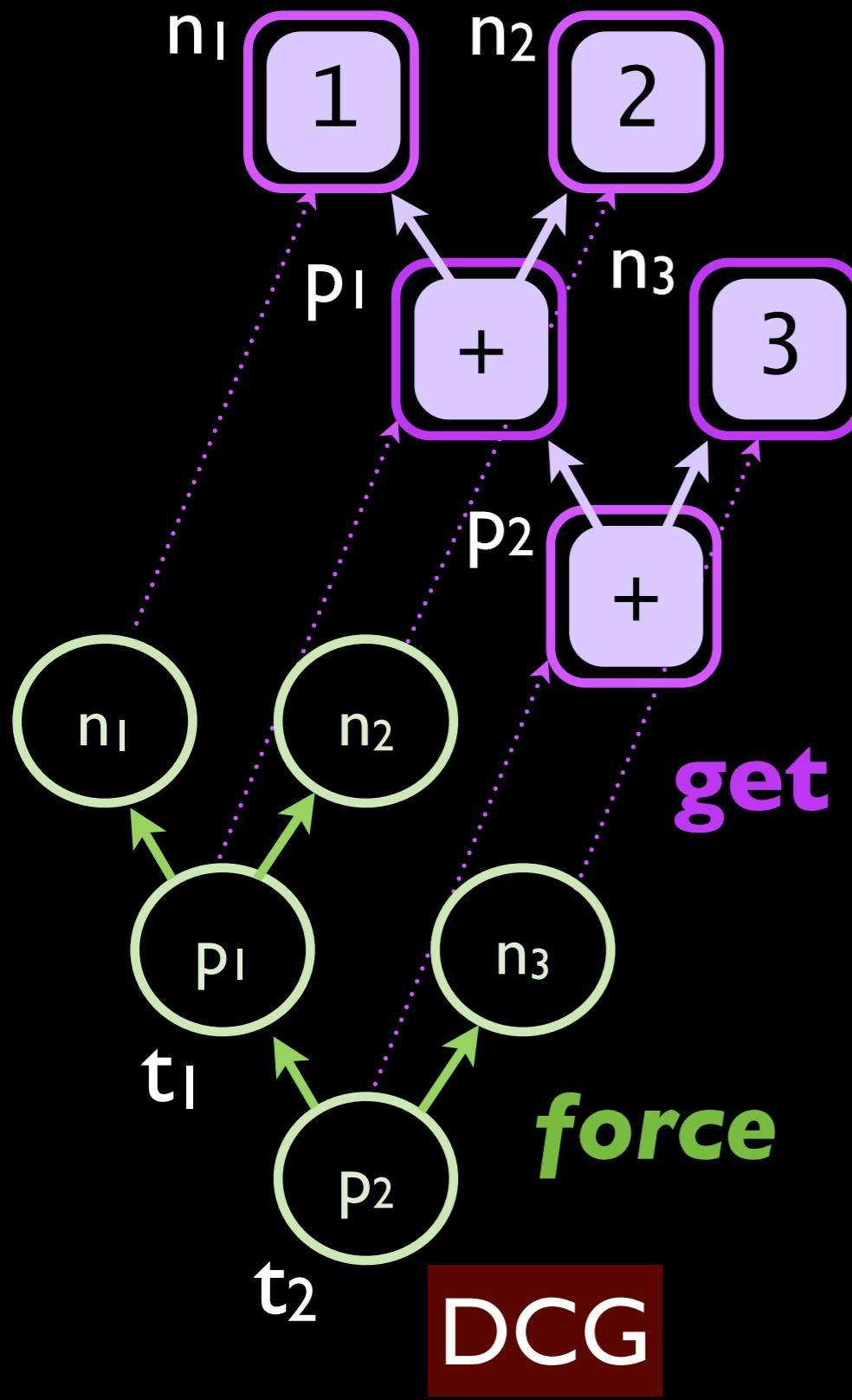
☞ display t_1

☞ display t_2

☞ clear

6

Spread Sheet Evaluator



set : cell × formula → unit

eval : cell → (int thunk)

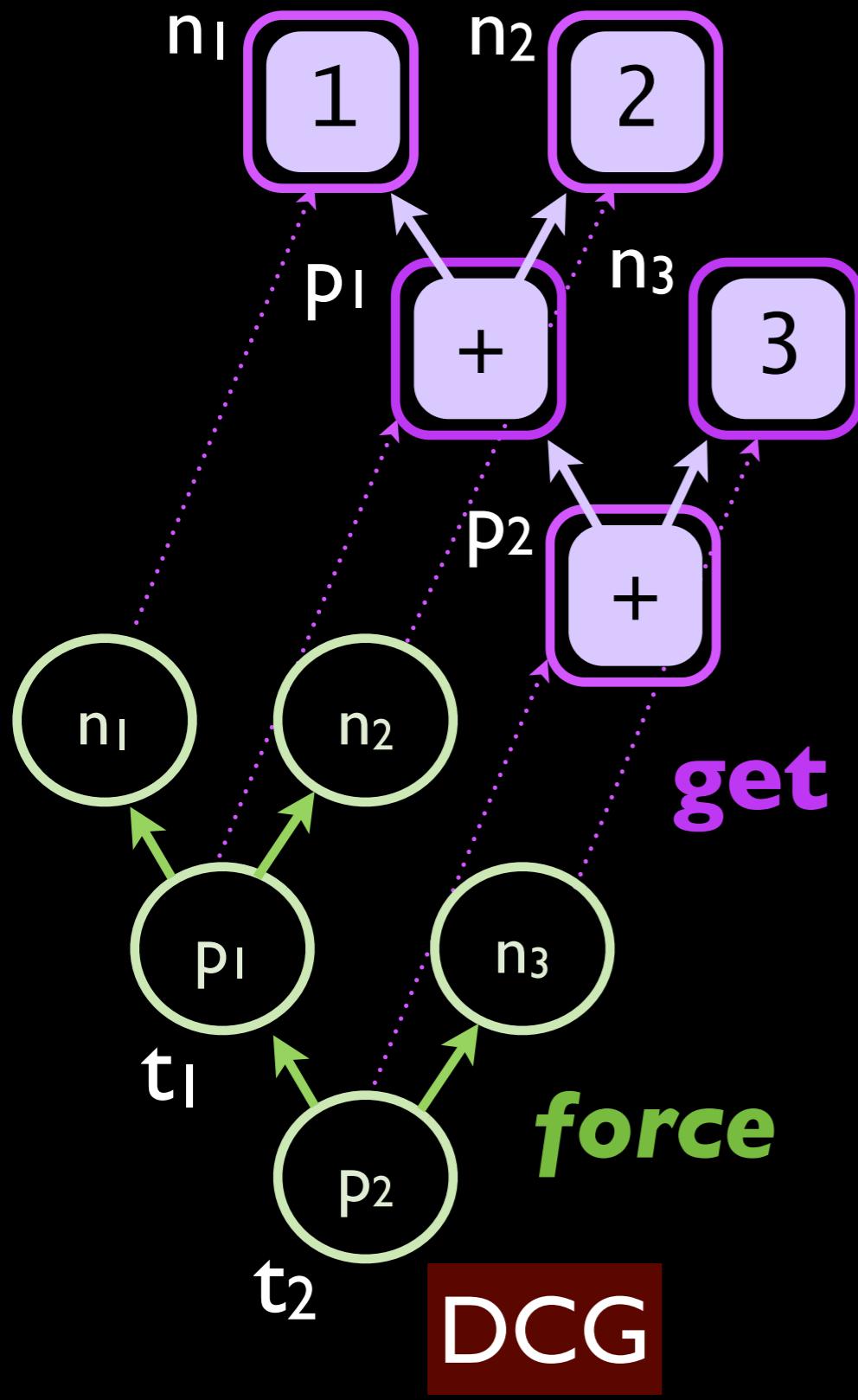
display : (int thunk) → unit

“User interface” (REPL)



DCG

Spread Sheet Evaluator



set : cell × formula → unit

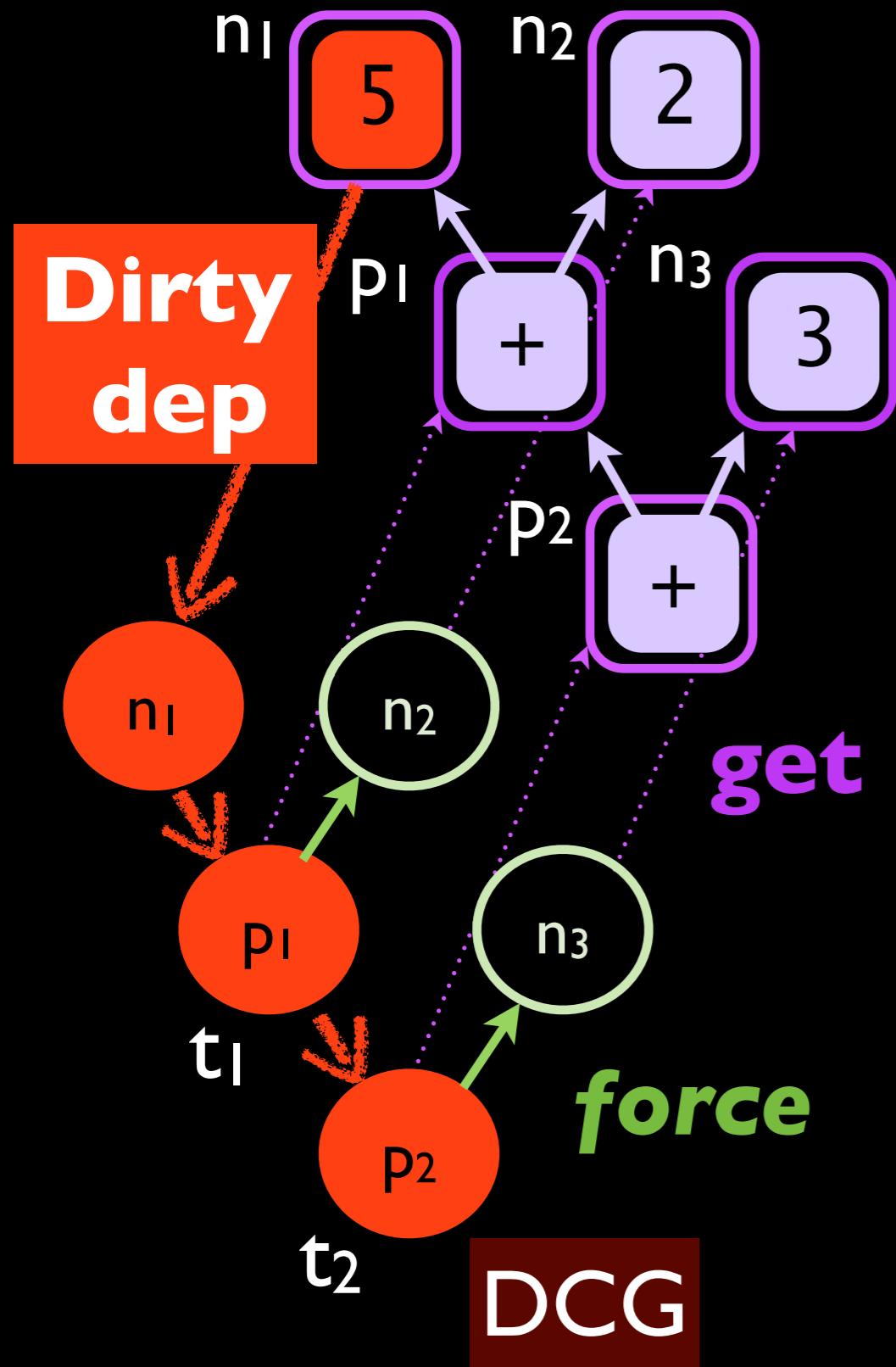
eval : cell → (int thunk)

display : (int thunk) → unit

“User interface” (REPL)

👉 set $n_1 \leftarrow$ Leaf 5

Spread Sheet Evaluator



set : cell × formula → unit

eval : cell → (int thunk)

display : (int thunk) → unit

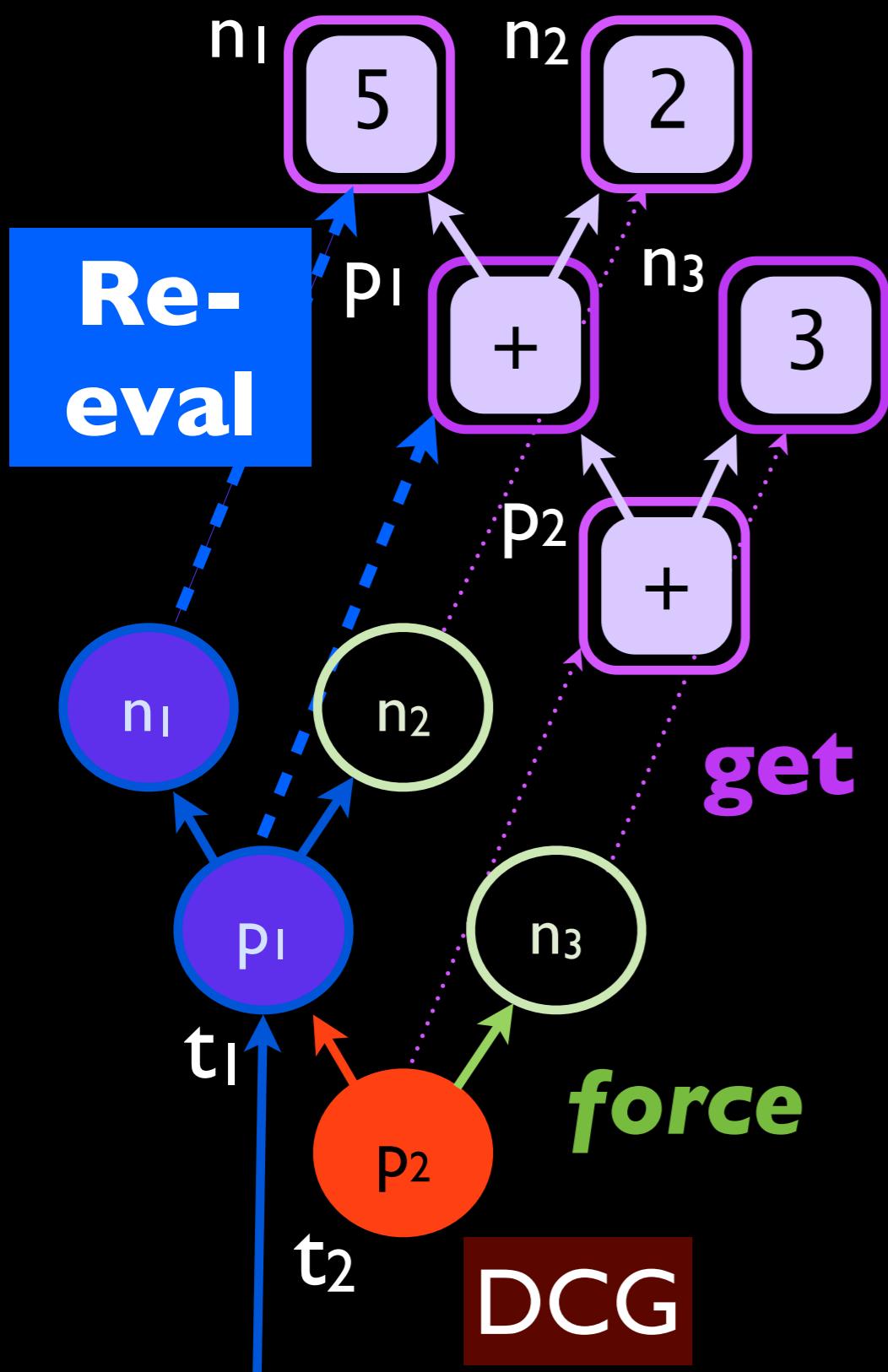
“User interface” (REPL)

👉 set $n_1 \leftarrow$ Leaf 5



Dirty phase

Spread Sheet Evaluator



set : cell × formula → unit

eval : cell → (int thunk)

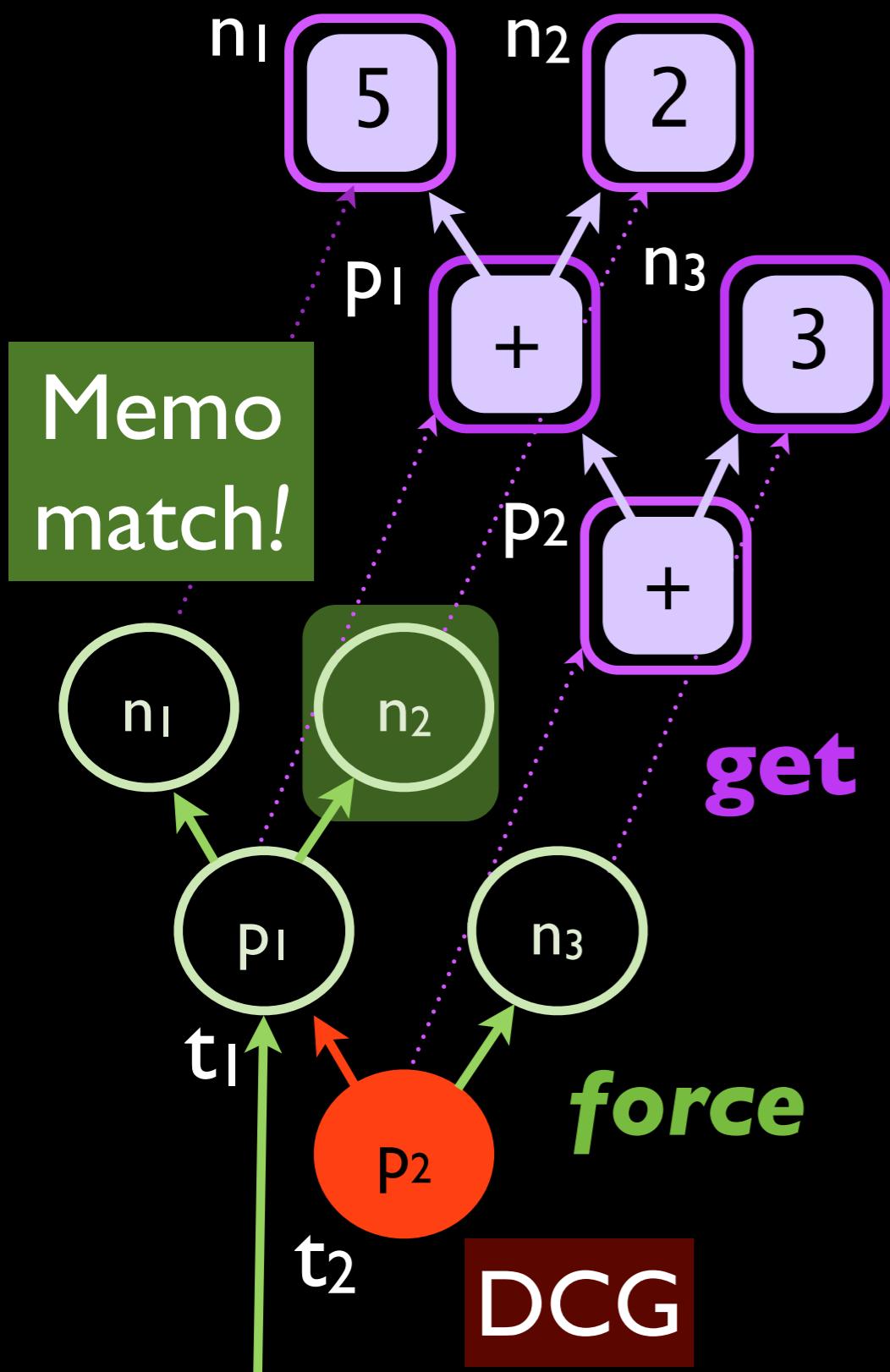
display : (int thunk) → unit

“User interface” (REPL)

set $n_1 \leftarrow$ Leaf 5

display t_1

Spread Sheet Evaluator



set : cell × formula → unit

eval : cell → (int thunk)

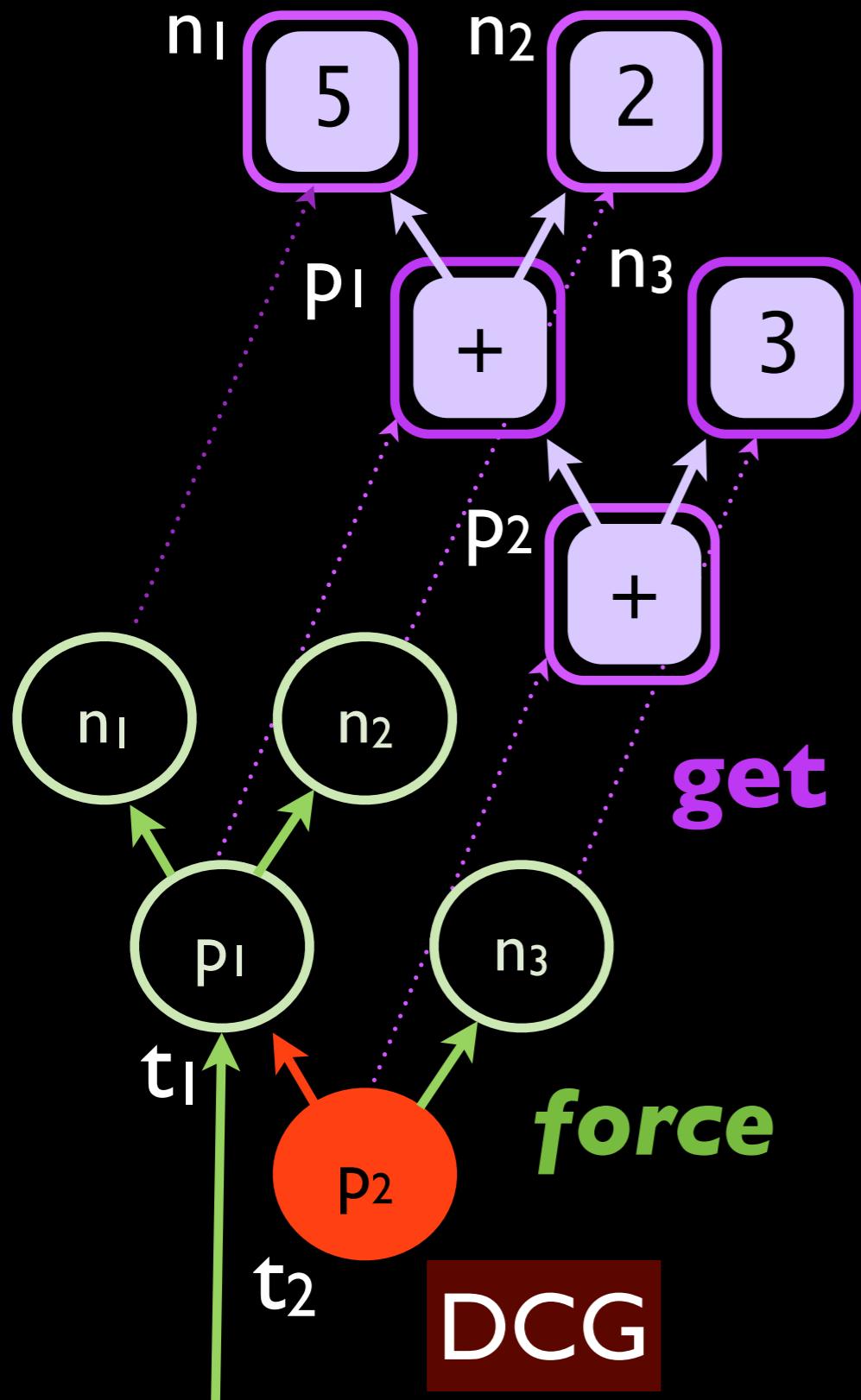
display : (int thunk) → unit

“User interface” (REPL)

set $n_1 \leftarrow$ Leaf 5

display t_1

Spread Sheet Evaluator



set : cell \times formula \rightarrow unit

eval : cell \rightarrow (int thunk)

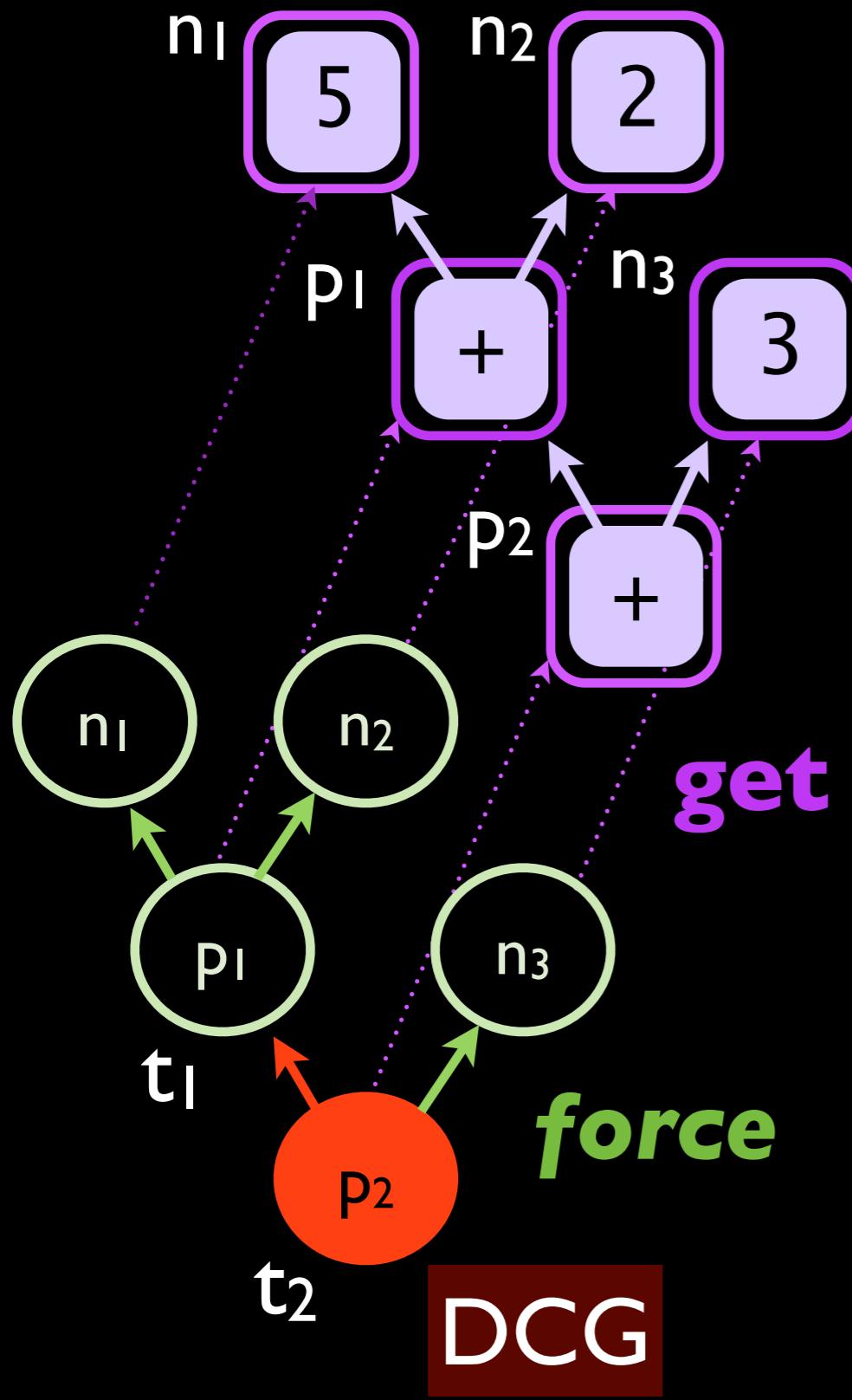
display : (int thunk) \rightarrow unit

“User interface” (REPL)

👉 set $n_1 \leftarrow$ Leaf 5

👉 display t_1

Spread Sheet Evaluator



set : cell × formula → unit

eval : cell → (int thunk)

display : (int thunk) → unit

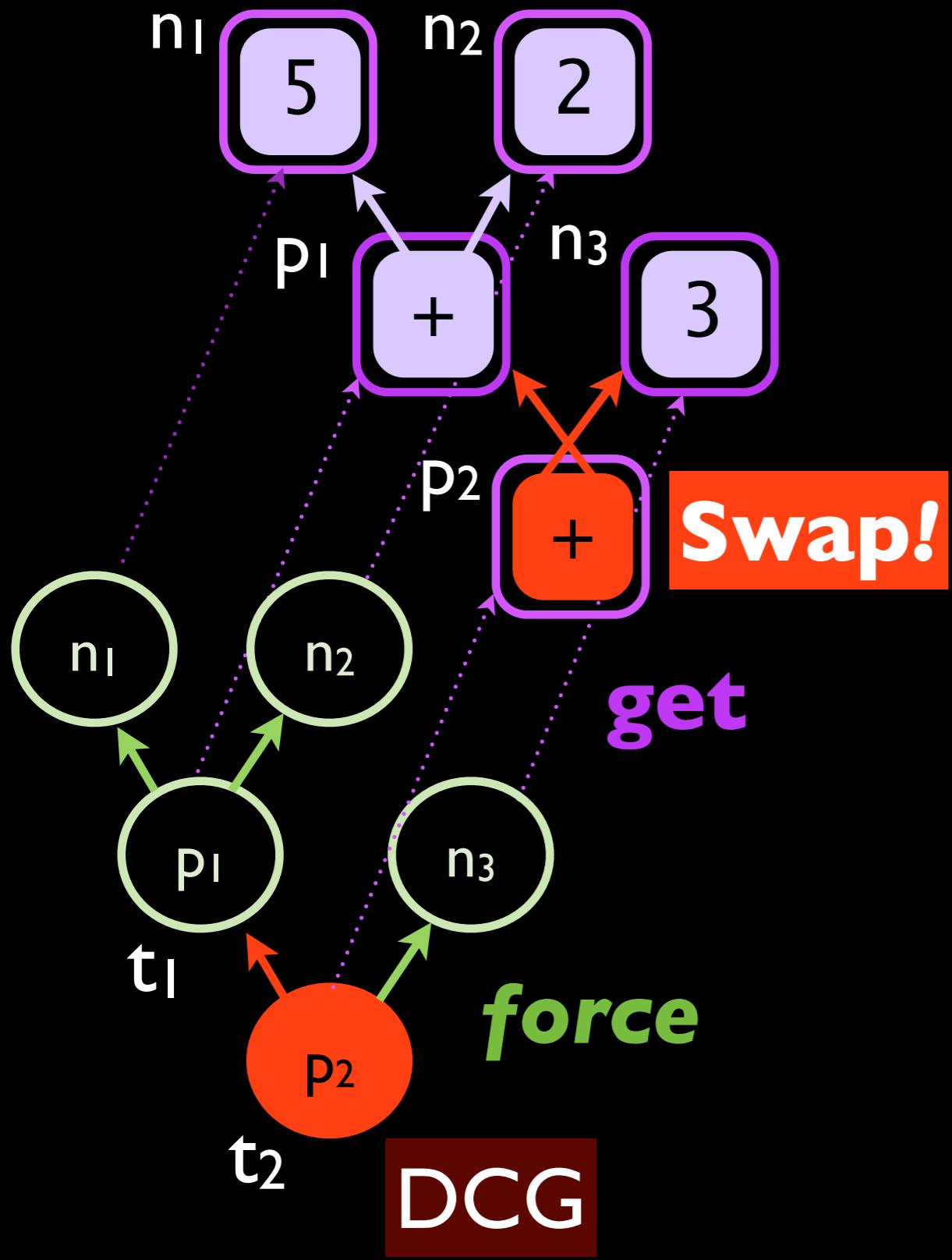
“User interface” (REPL)

👉 set $n_1 \leftarrow$ Leaf 5

👉 display t_1

👉 set $p_2 \leftarrow$ Plus(n_3, p_1)

Spread Sheet Evaluator



`set : cell x formula → unit`

`eval : cell → (int thunk)`

`display : (int thunk) → unit`

“User interface” (REPL)

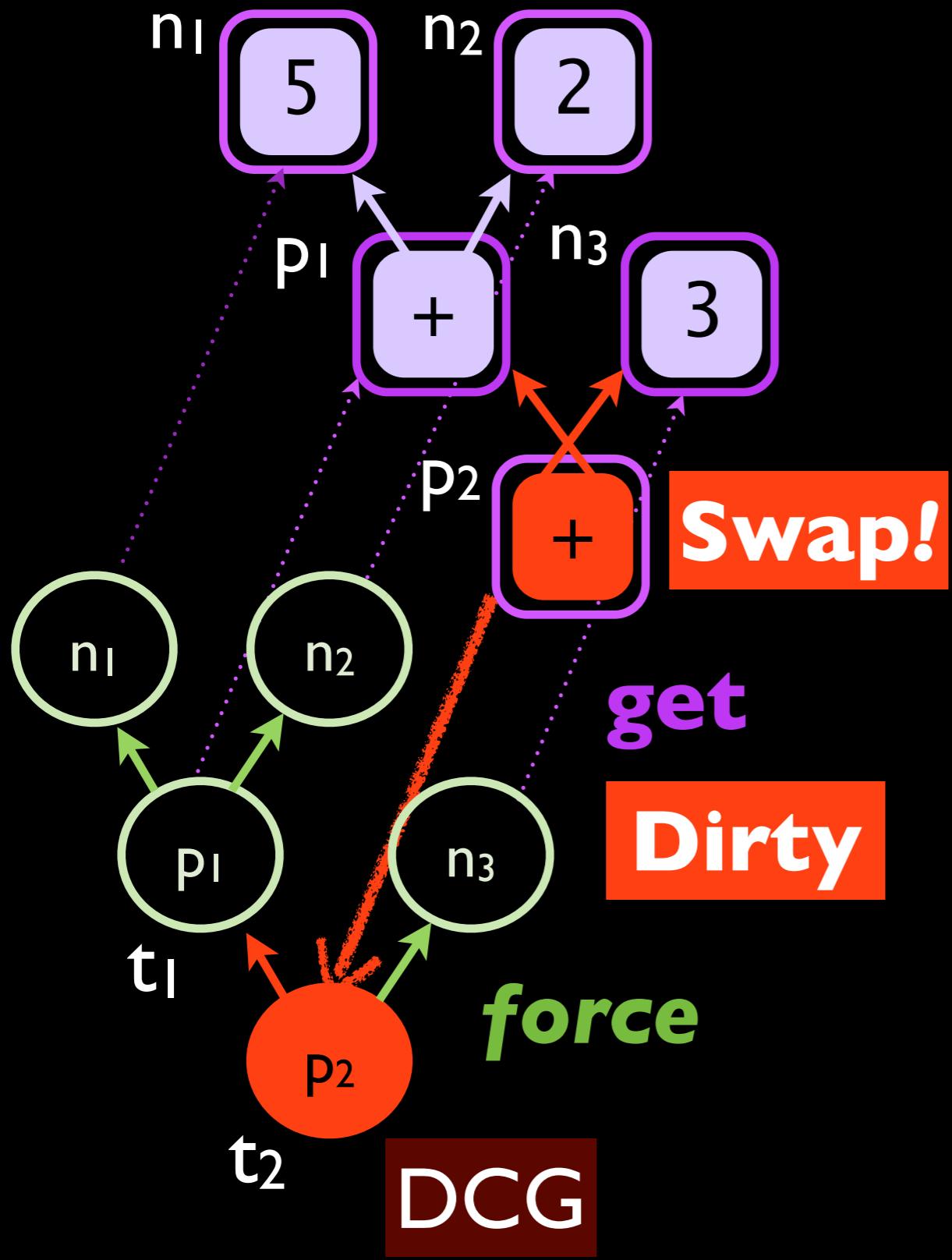
👉 `set n1 ← Leaf 5`

👉 `display t1`

👉 `set p2 ← Plus(n3, p1)`



Spread Sheet Evaluator



`set : cell x formula → unit`

`eval : cell → (int thunk)`

`display : (int thunk) → unit`

“User interface” (REPL)

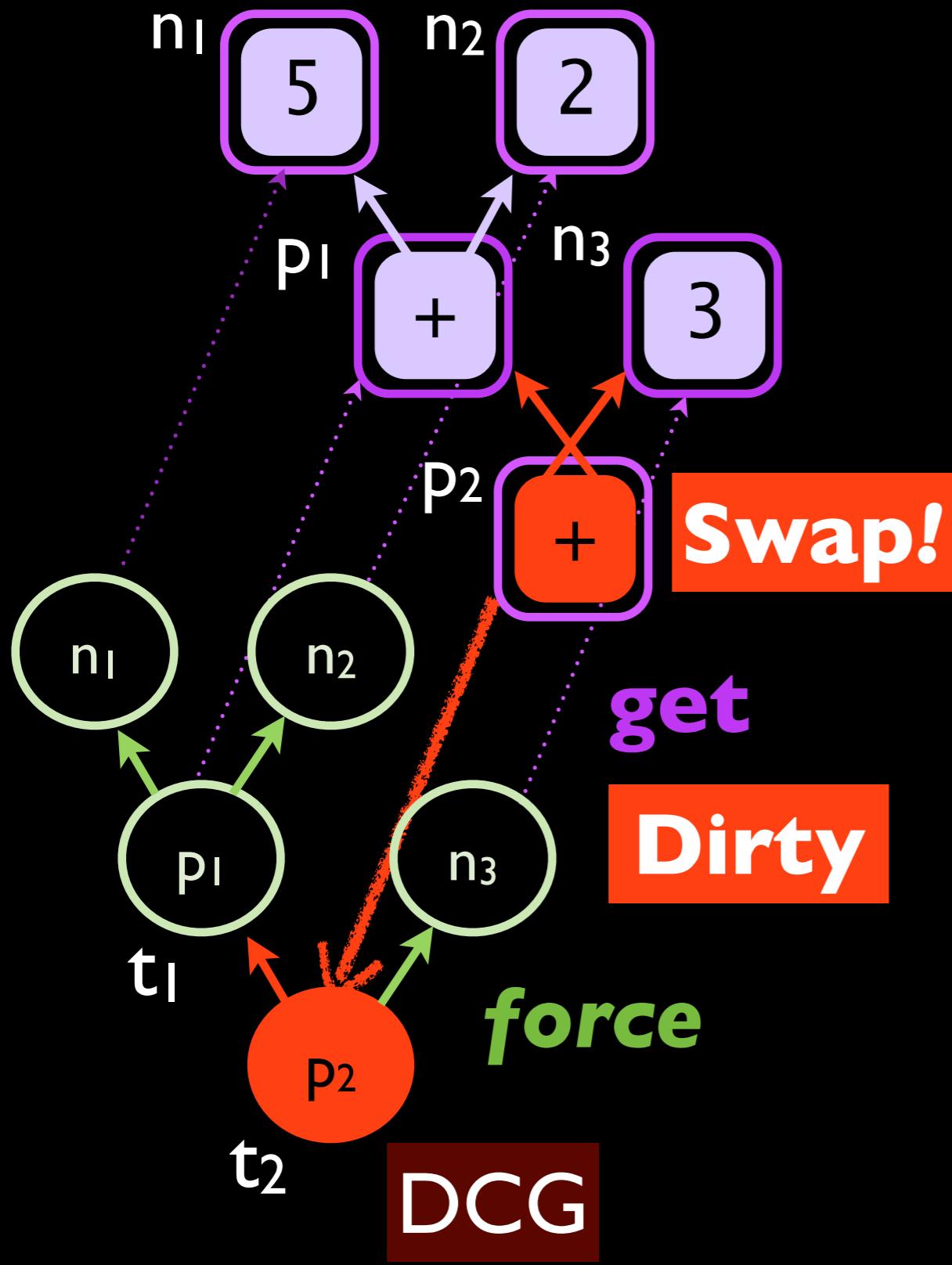
👉 `set n1 ← Leaf 5`

👉 `display t1`

👉 `set p2 ← Plus(n3, p1)`



Spread Sheet Evaluator



set : cell × formula → unit

eval : cell → (int thunk)

display : (int thunk) → unit

“User interface” (REPL)

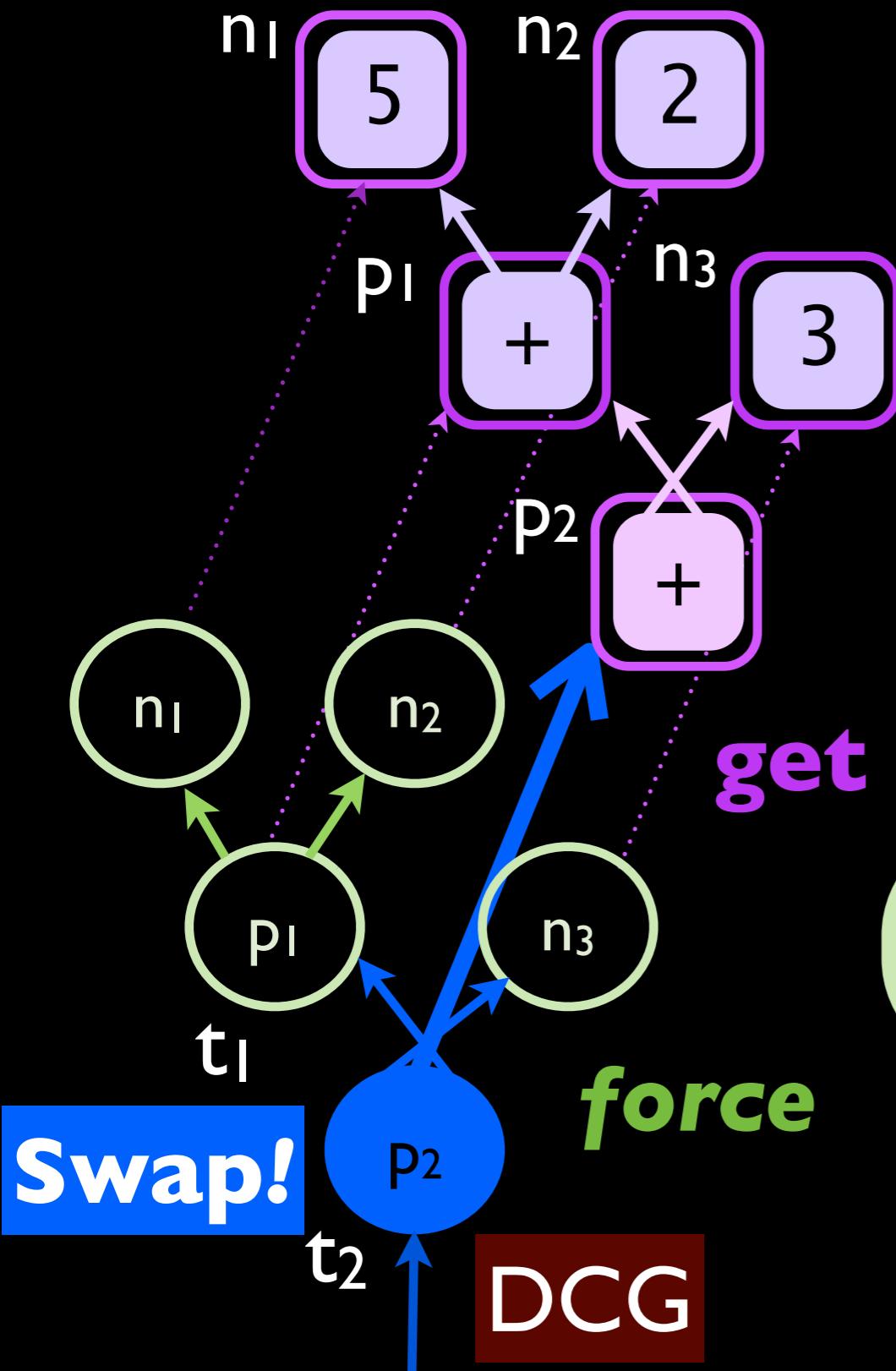
👉 set $n_1 \leftarrow$ Leaf 5

👉 display t_1

👉 set $p_2 \leftarrow$ Plus(n_3 , p_1)

👉 display t_2

Spread Sheet Evaluator



set : cell × formula → unit

eval : cell → (int thunk)

display : (int thunk) → unit

“User interface” (REPL)

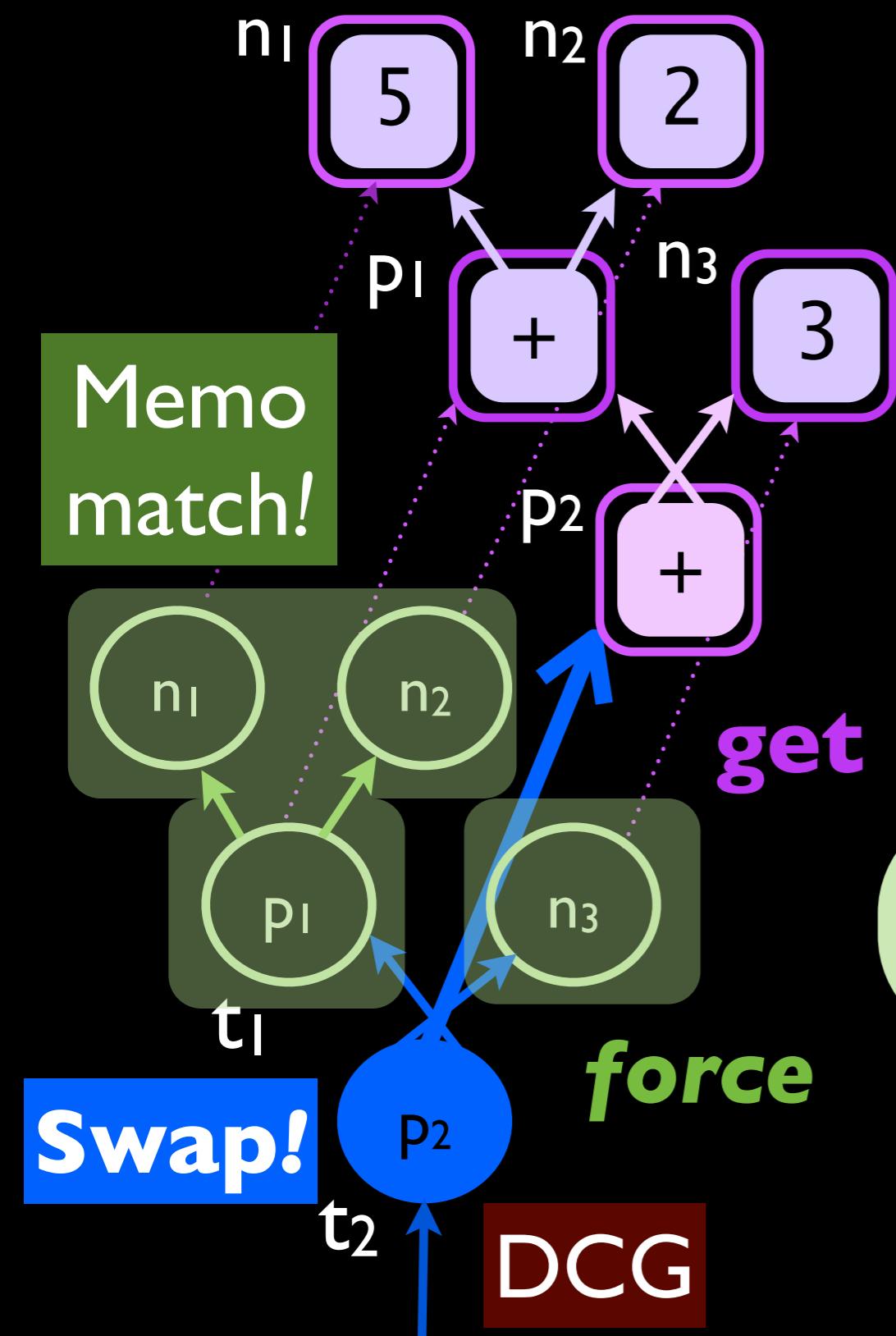
👉 set $n_1 \leftarrow$ Leaf 5

👉 display t_1

👉 set $p_2 \leftarrow$ Plus(n_3 , p_1)

👉 display t_2

Spread Sheet Evaluator



set : cell × formula → unit

eval : cell → (int thunk)

display : (int thunk) → unit

“User interface” (REPL)

👉 set $n_1 \leftarrow$ Leaf 5

👉 display t_1

👉 set $p_2 \leftarrow$ Plus(n_3, p_1)

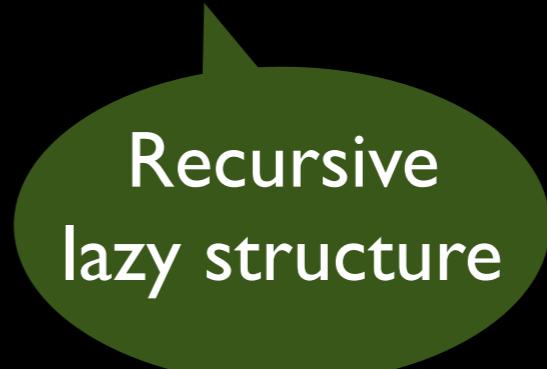
👉 display t_2

Lazy Structures

Laziness generalizes ***beyond scalars***

Recursive structures: **lists, trees and graphs**

```
type 'a lzlist =  
| Nil  
| Cons of 'a * ('a lzlist) thunk
```



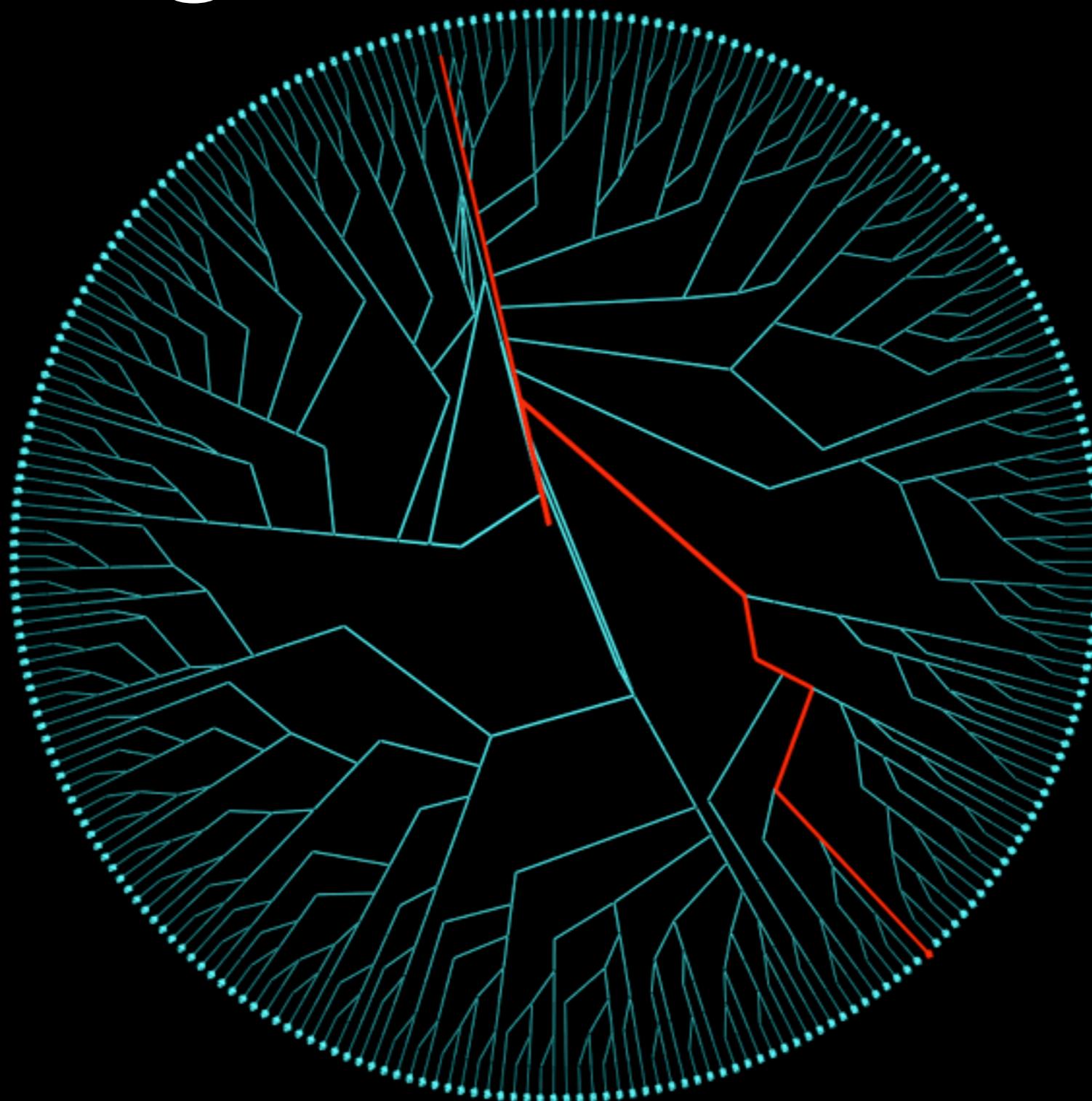
Recursive
lazy structure

Merging Lazy Lists

As in conventional lazy programming

```
let rec merge l1 l2 = function
| l1, Nil ⇒ l1
| Nil, l2 ⇒ l2
| Cons(h1,t1), Cons(h2,t2) ⇒
  if h1 <= h2 then
    Cons(h1, thunk(merge (force t1) l2))
  else
    Cons(h2, thunk(merge l1 (force t2)))
```

Mergesort DCG Viz.



Graphics by **Piotr Mardziel**

Micro Benchmarks

List and tree applications:

filter, map

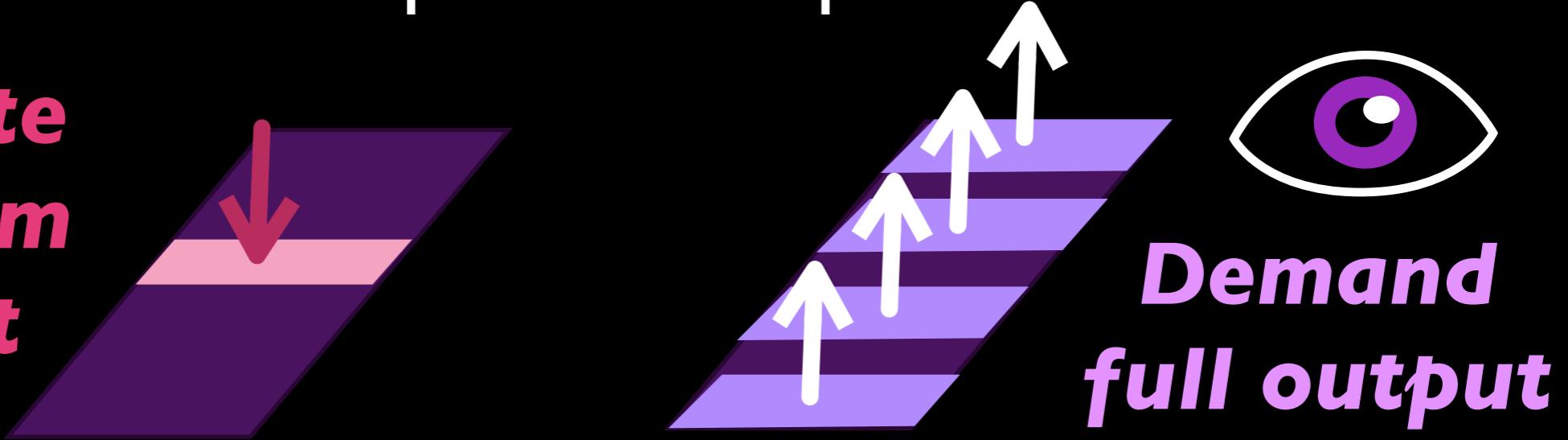
fold{min,sum}

quicksort, mergesort

expression tree evaluation

Batch Pattern: Experimental procedure:

Mutate
random
input

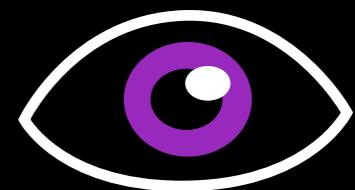
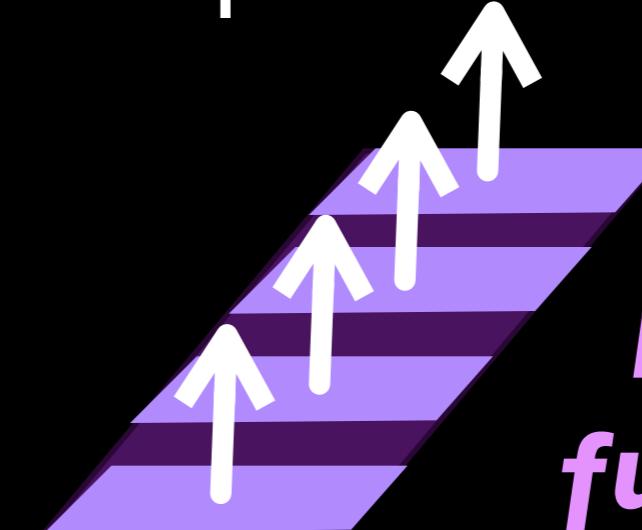
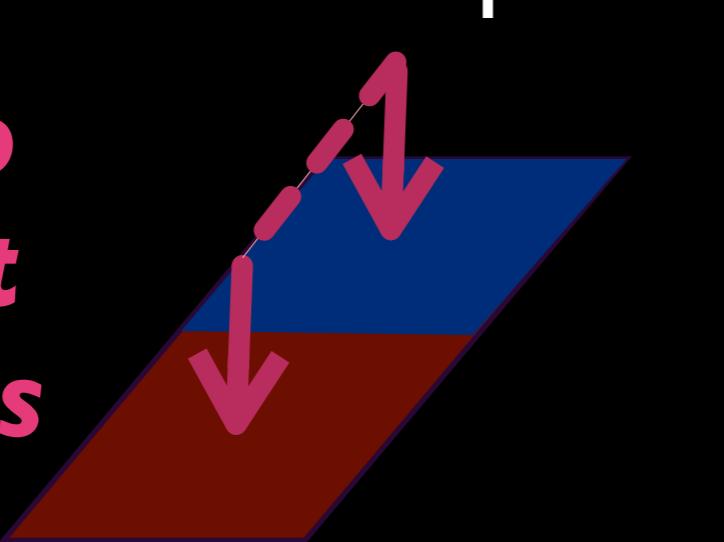


Demand
full output

Batch	Baseline time (s)	Adaption speedup	SAC speedup
filter	0.6	2.0	4.11
map	1.2	2.2	3.32
fold min	1.4	4350	3090
fold sum	1.5	1640	4220
exptree	0.3	497	1490

Swap Pattern: Experimental procedure:

**Swap
input
halves**

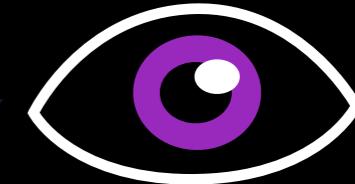


**Demand
full output**

Swap	Baseline time (s)	Adaption speedup	SAC speedup
filter	0.5	2.0	0.14
map	0.9	2.4	0.25
fold min	1.0	472	0.12
fold sum	1.1	501	0.13
exptree	0.3	667	10

Lazy Pattern: Experimental procedure:

Mutate
random
input

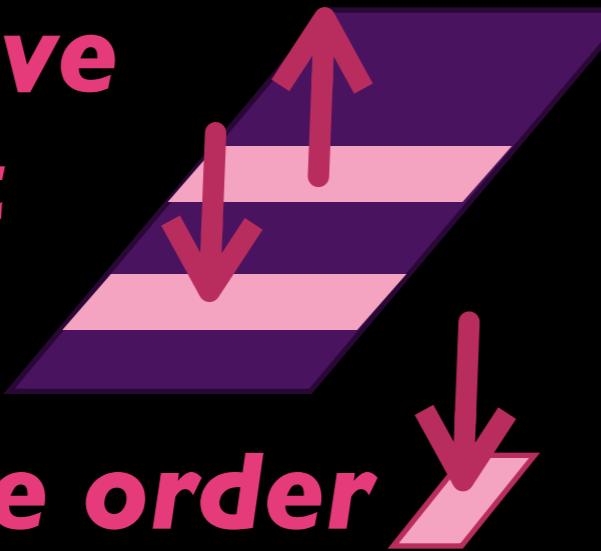


Demand
first output

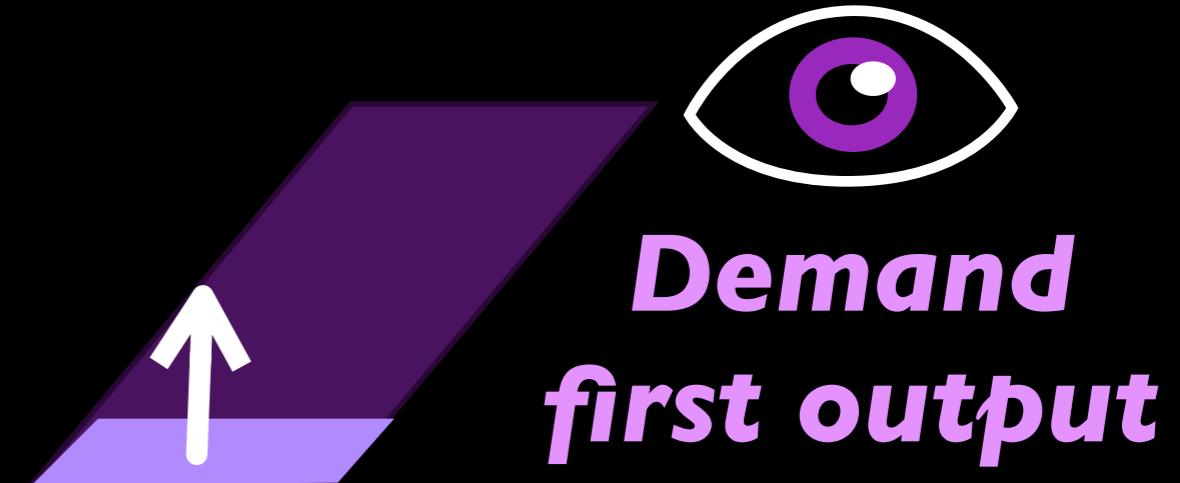
Lazy	Baseline time (s)	Adapton speedup	SAC speedup
filter	1.16E-05	12.8	2.2
map	6.86E-06	7.8	1.5
quicksort	7.41E-02	2020	22.9
mergesort	3.46E-01	336	0.148

Switch Pattern: Experimental procedure:

1. Remove
2. Insert



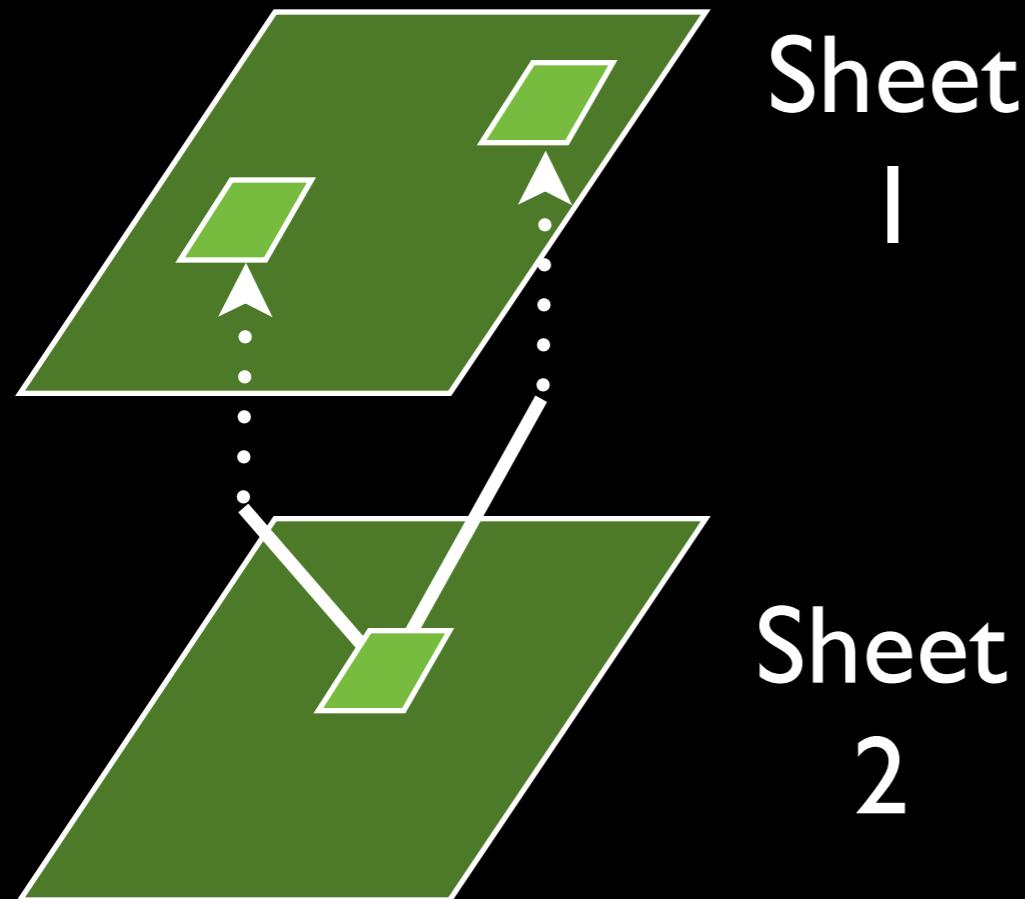
3. Toggle order



Demand
first output

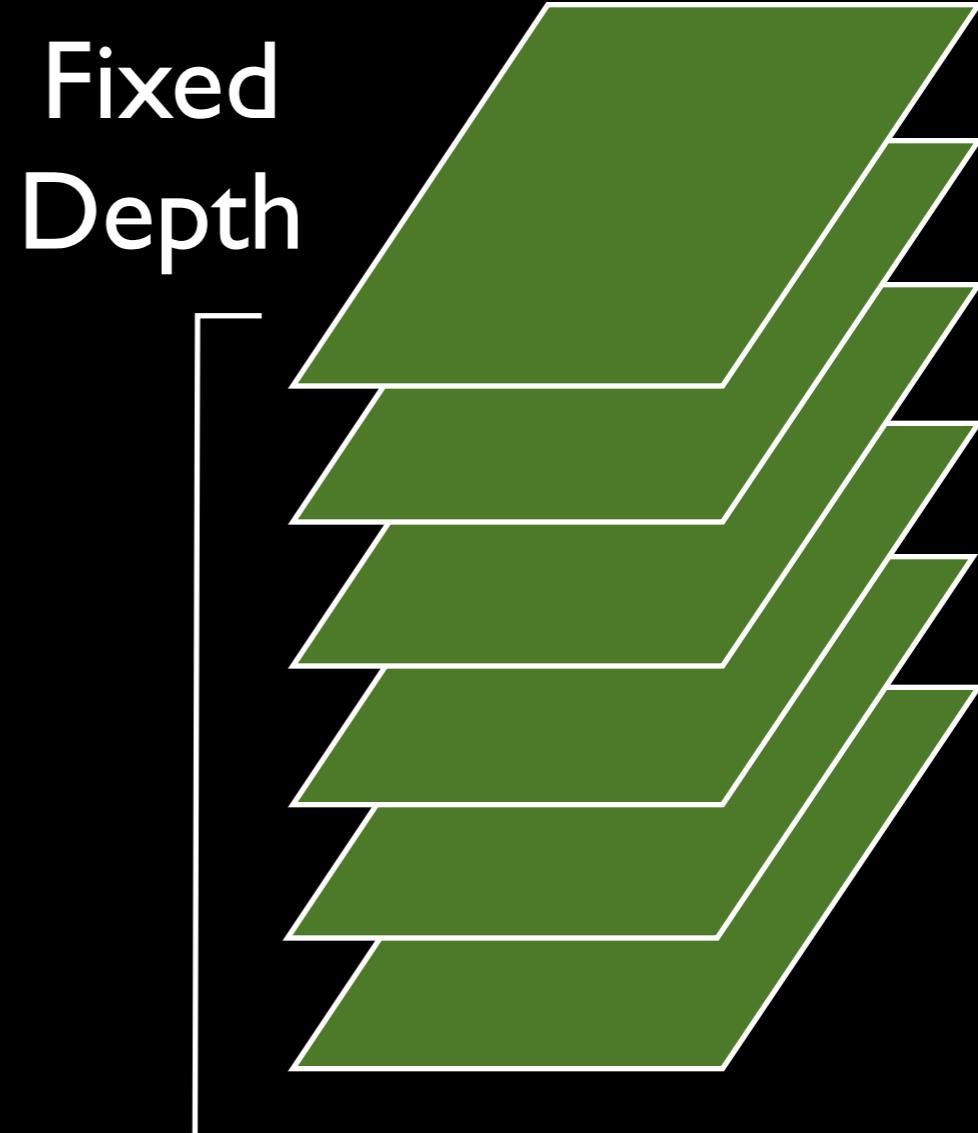
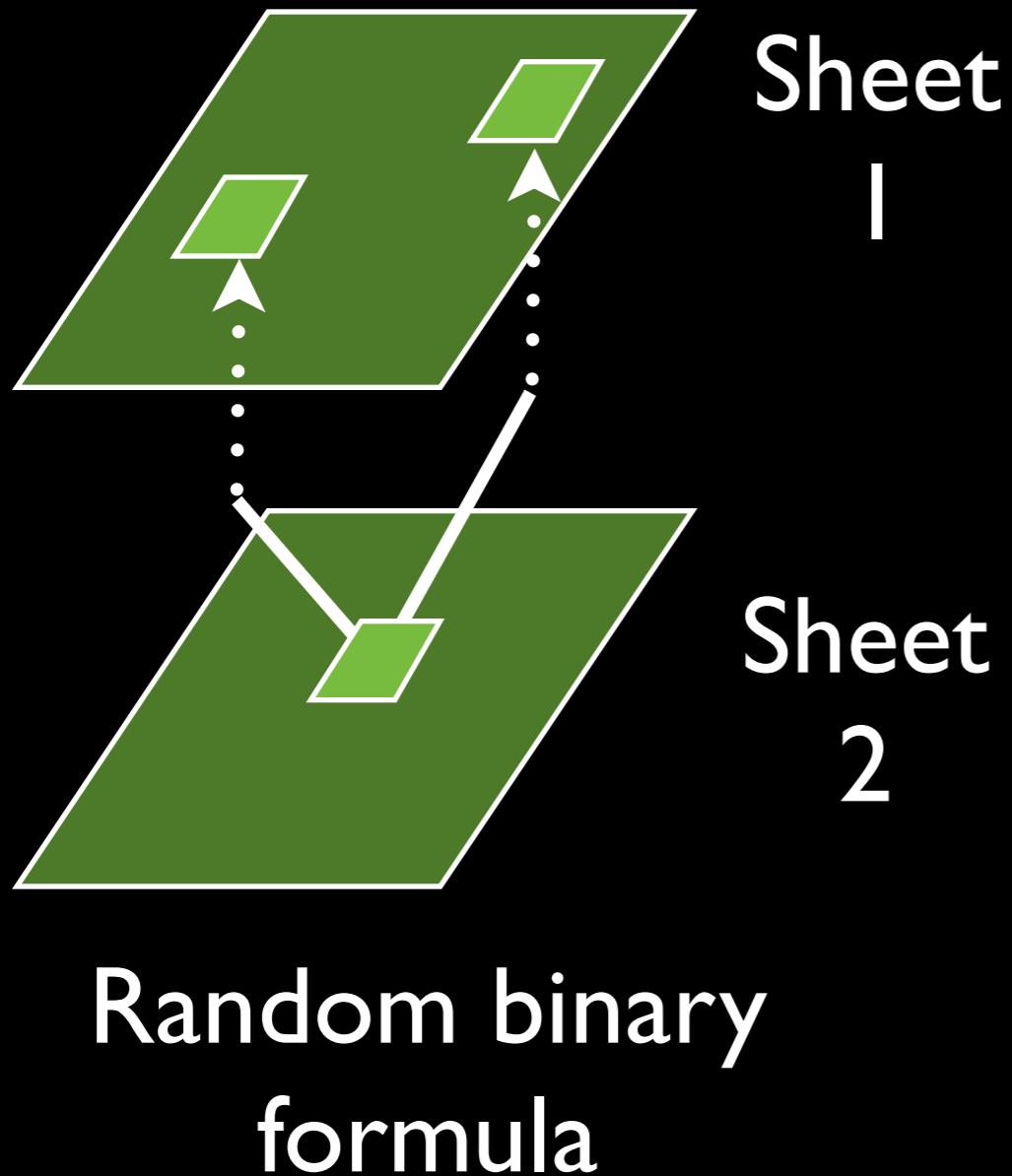
Switch	Baseline time (s)	Adapton speedup	SAC speedup
updown1	3.28E-02	22.4	2.47E-03
updown2	3.26E-02	24.7	4.28

Spreadsheet Experiments

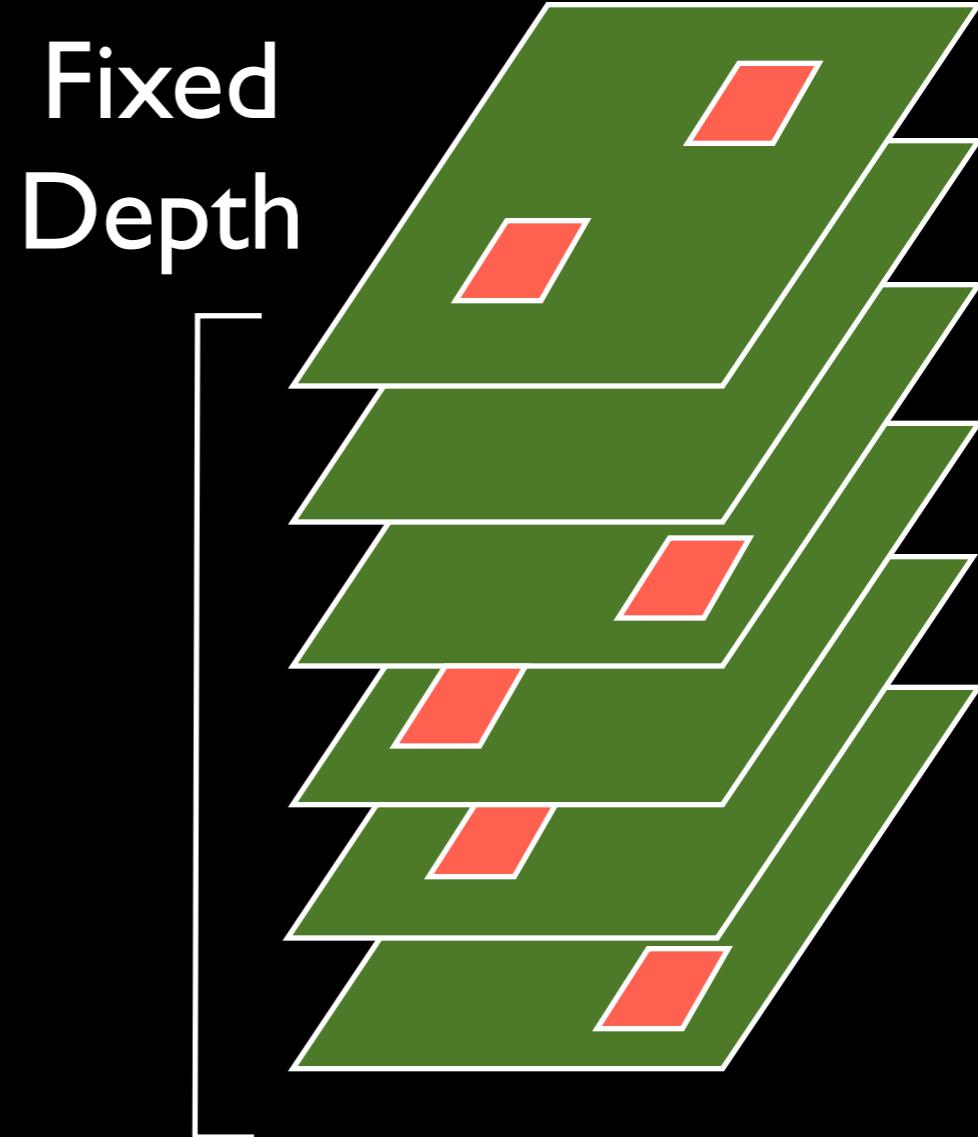
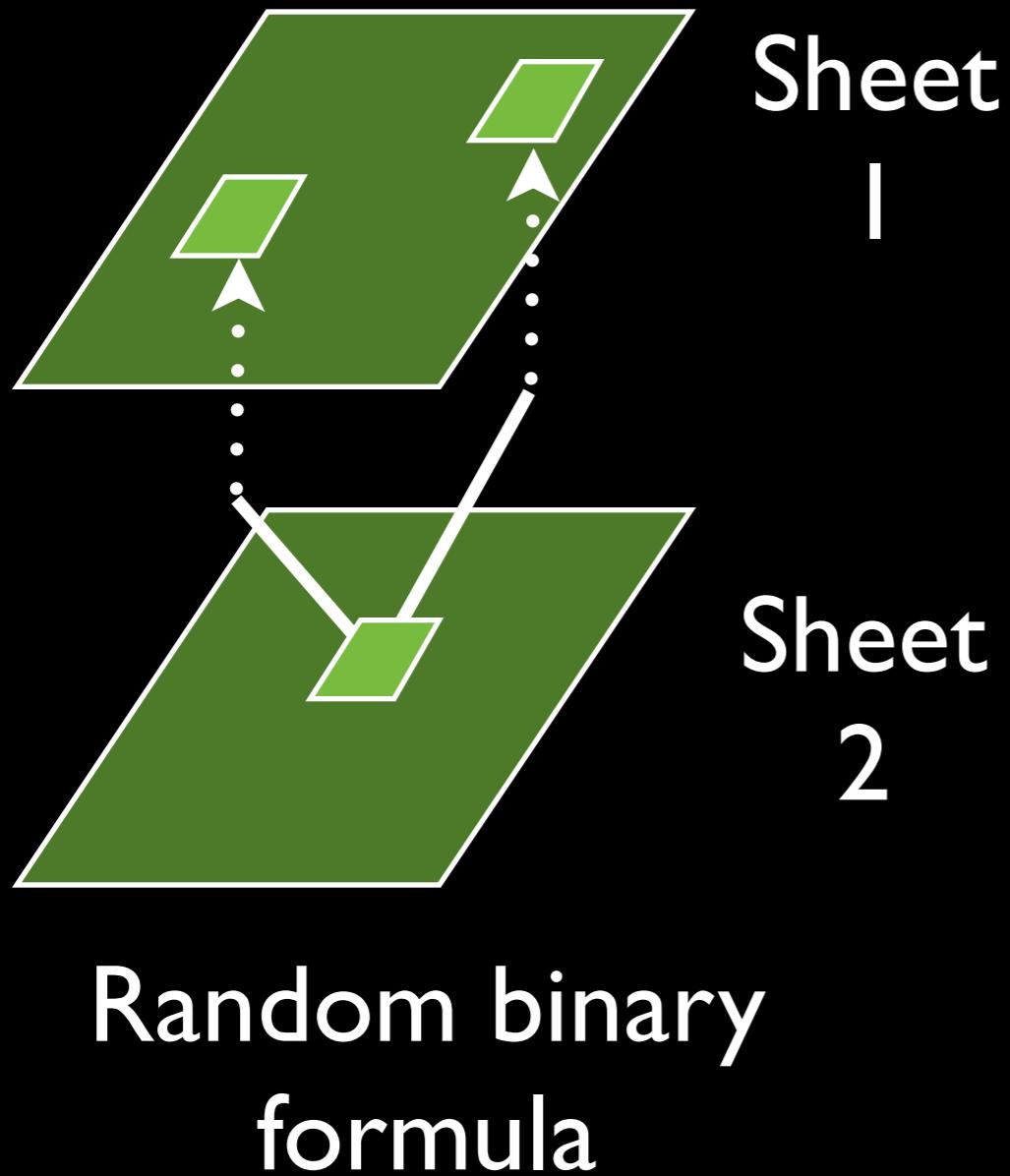


Random binary
formula

Spreadsheet Experiments

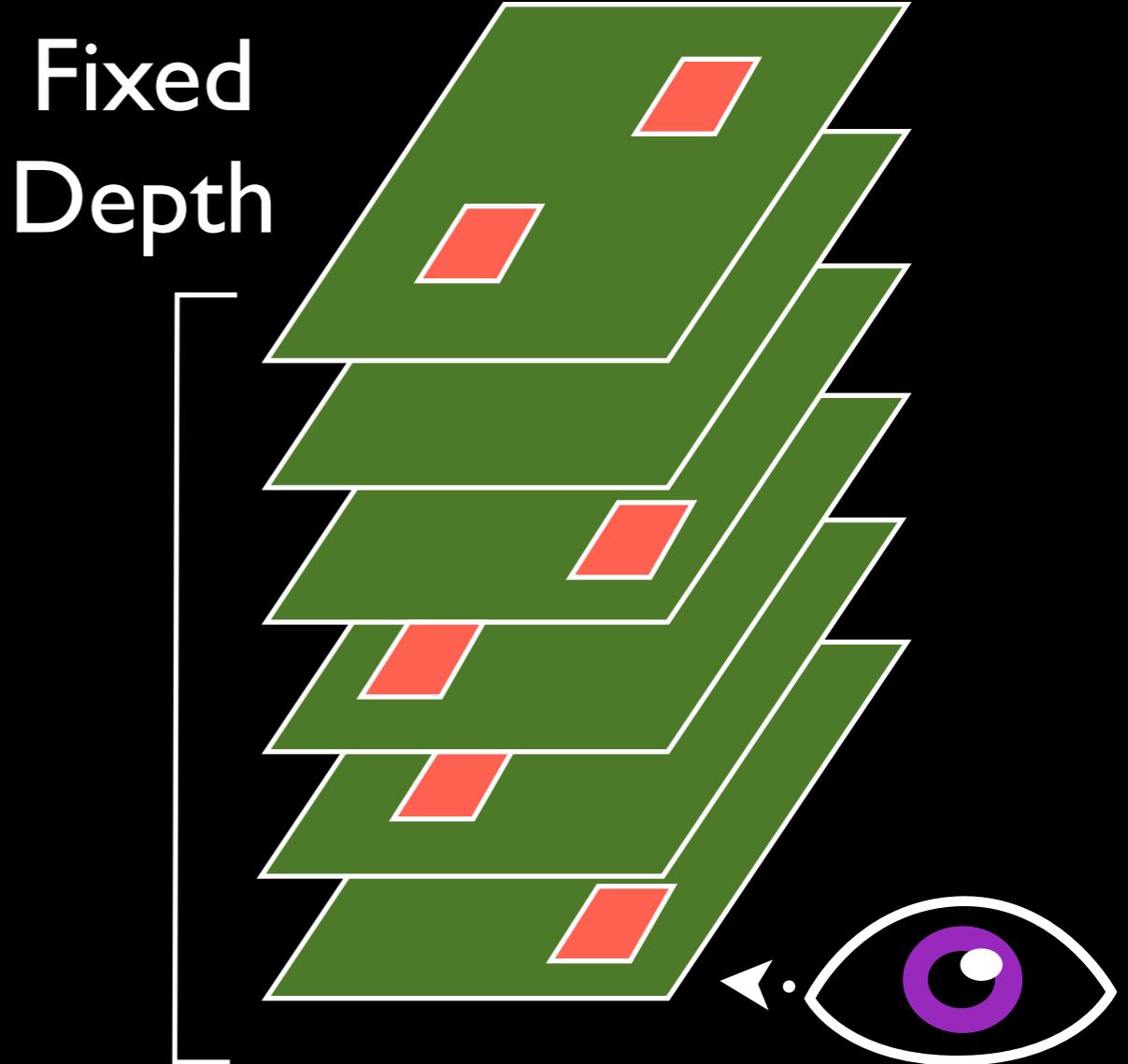
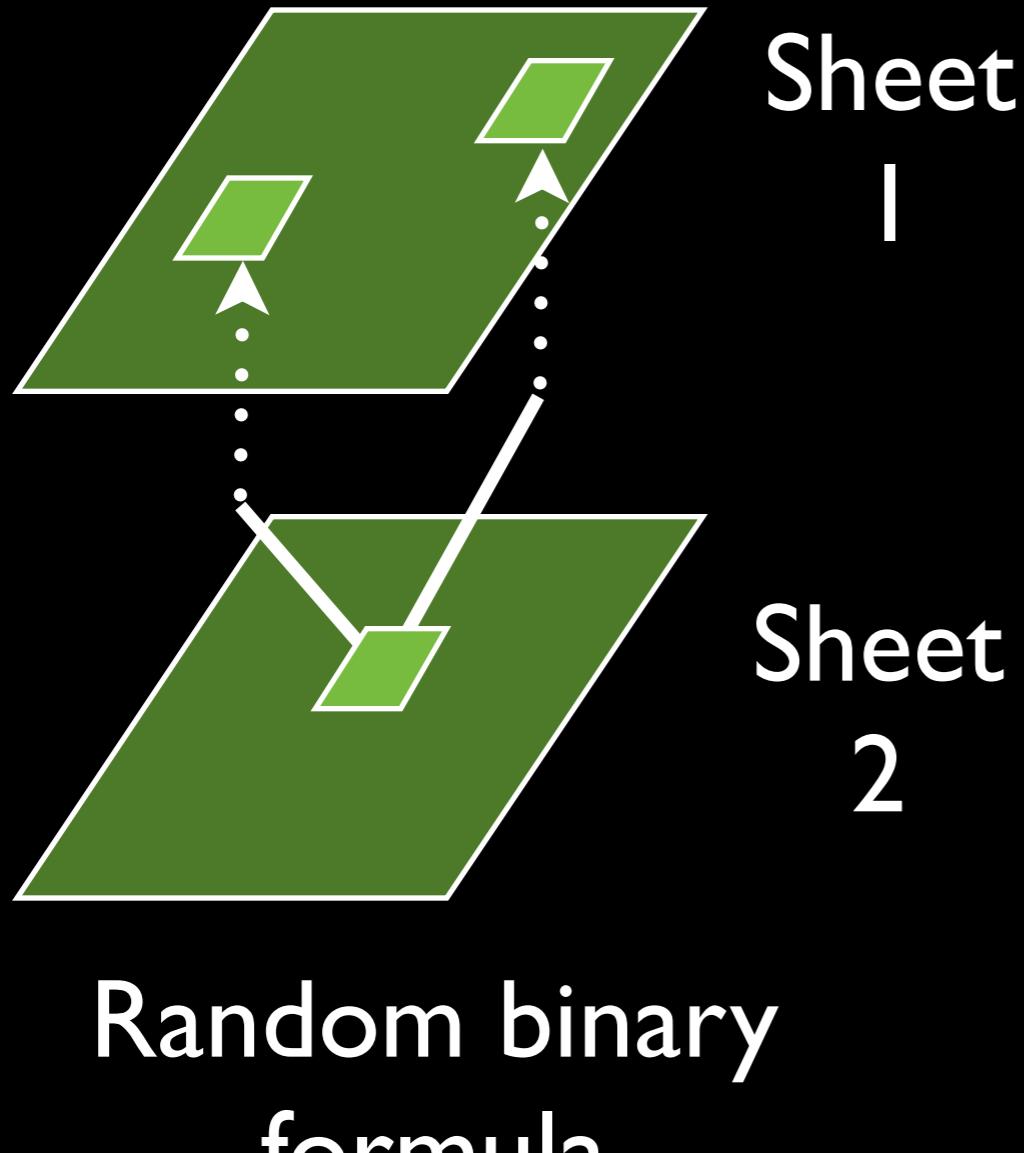


Spreadsheet Experiments



I. Random Mutations

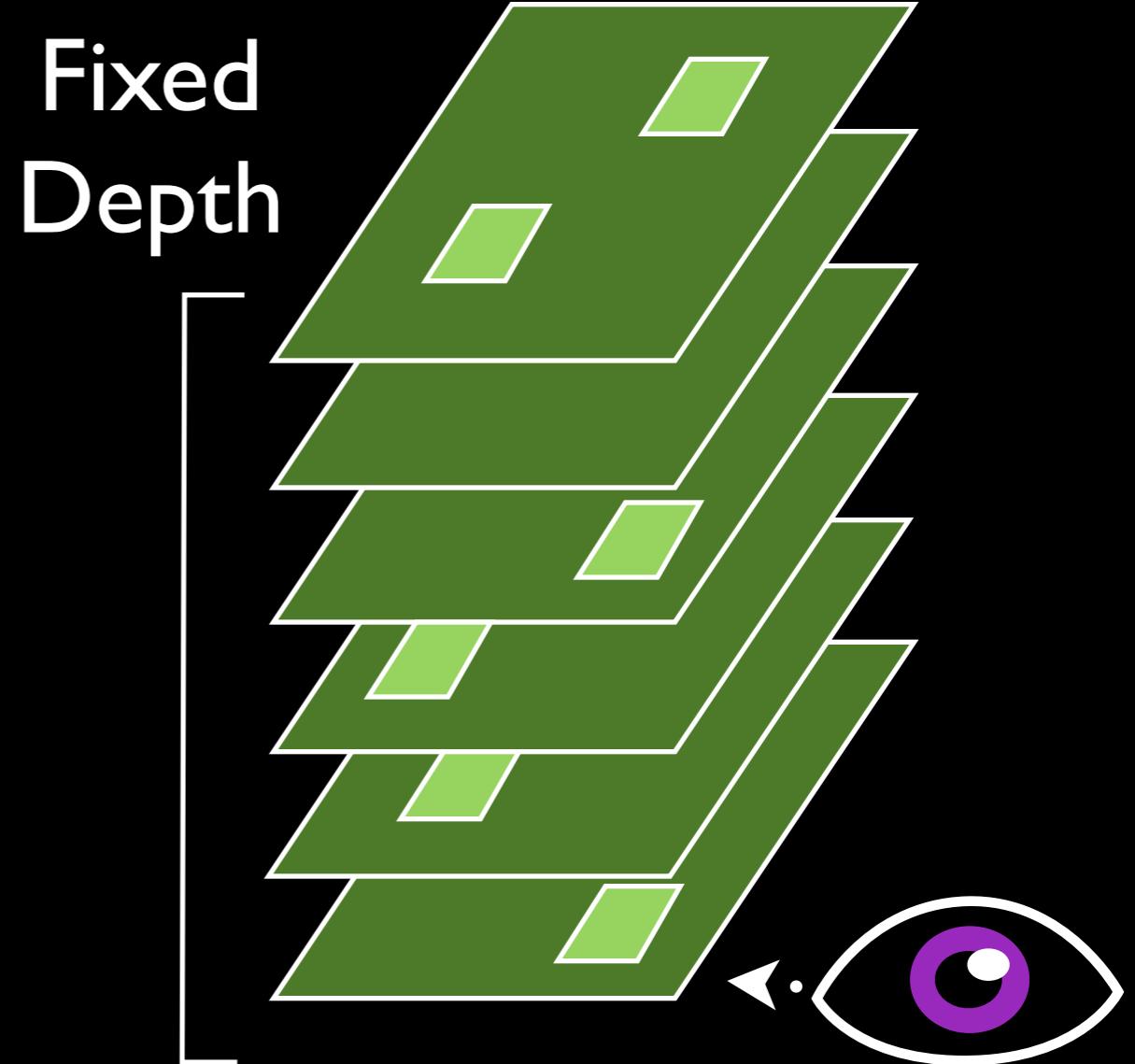
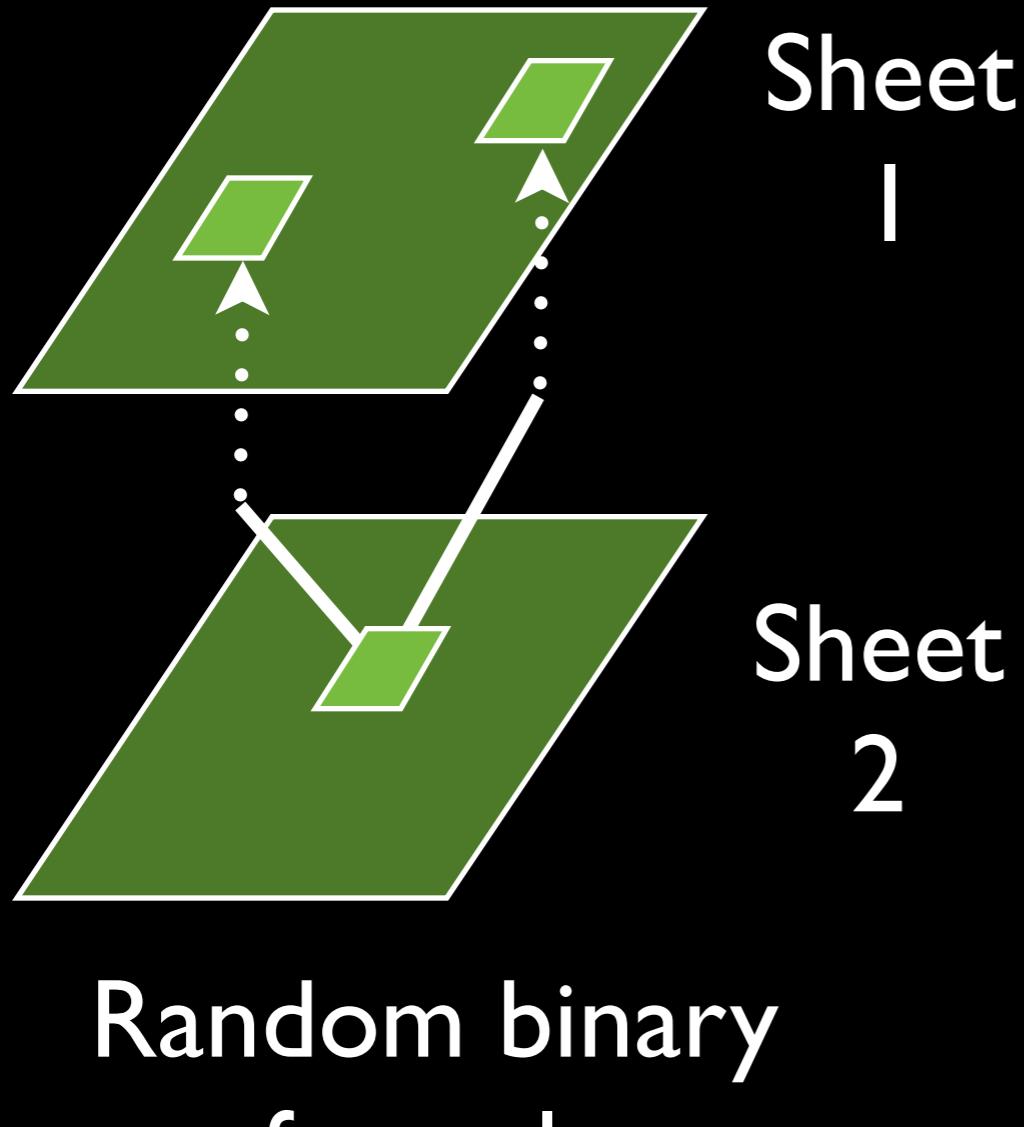
Spreadsheet Experiments



I. Random
Mutations

2. Observe
last sheet

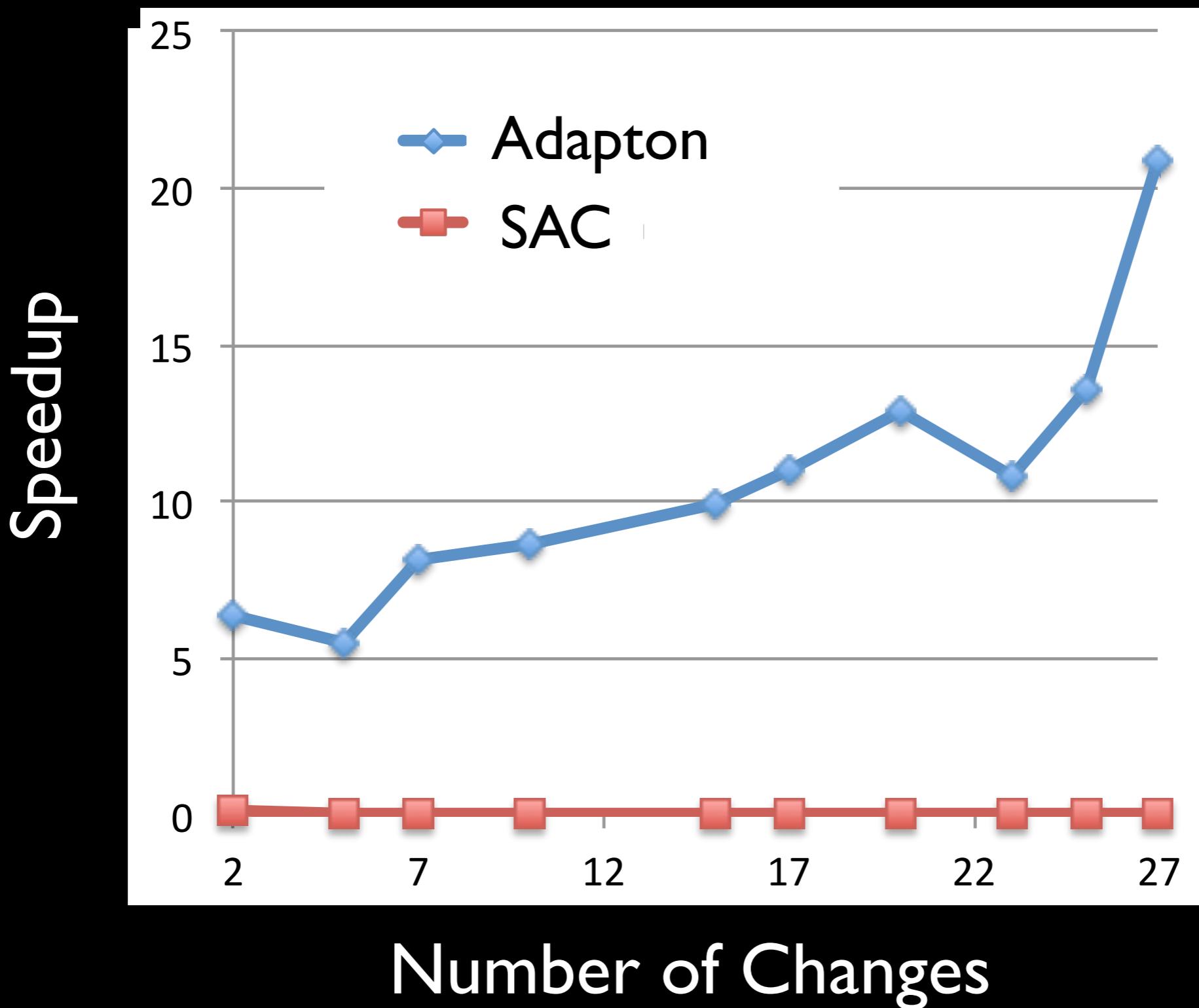
Spreadsheet Experiments



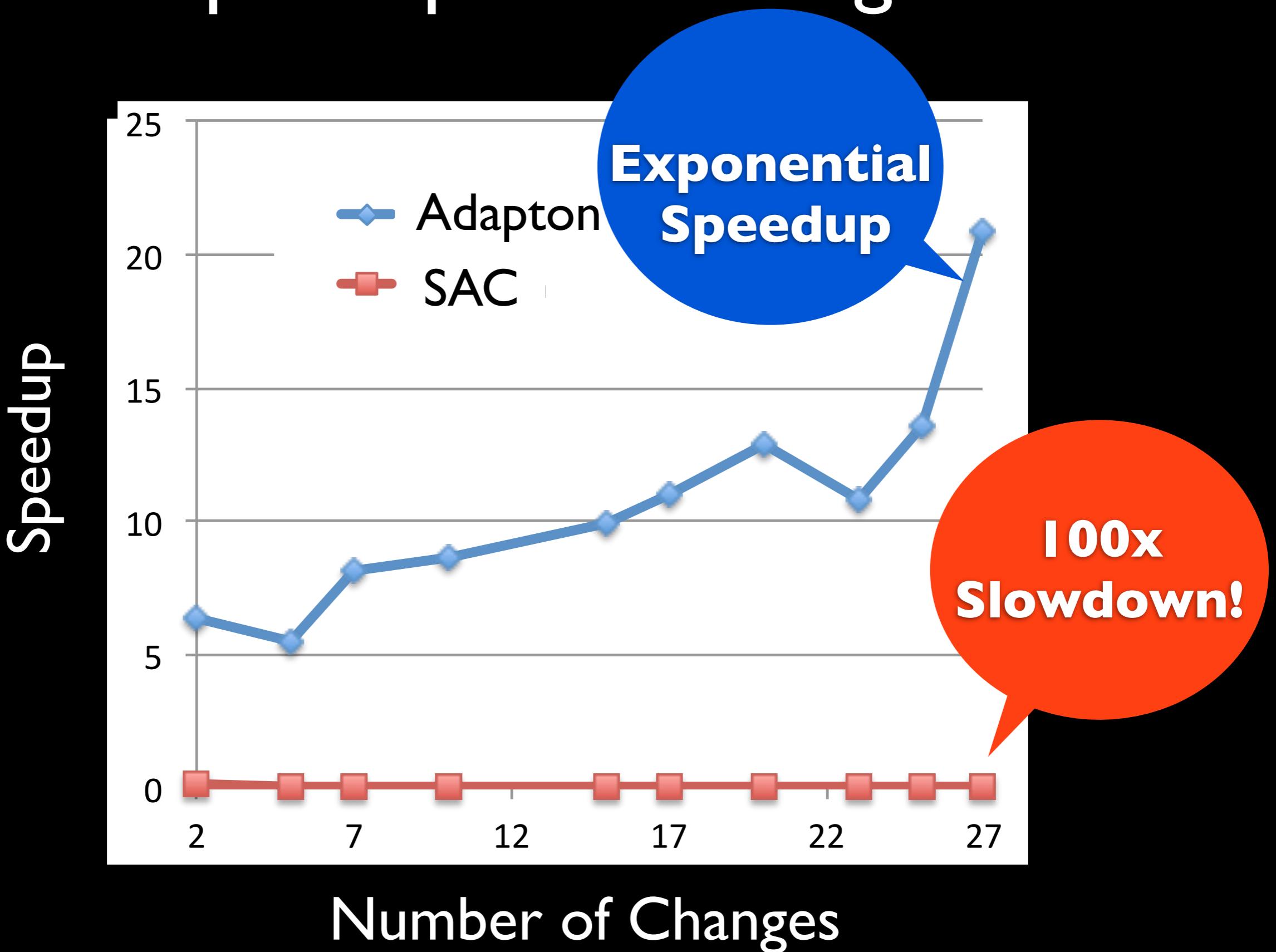
1. Random Mutations
2. Observe last sheet

Speedup vs # Changes

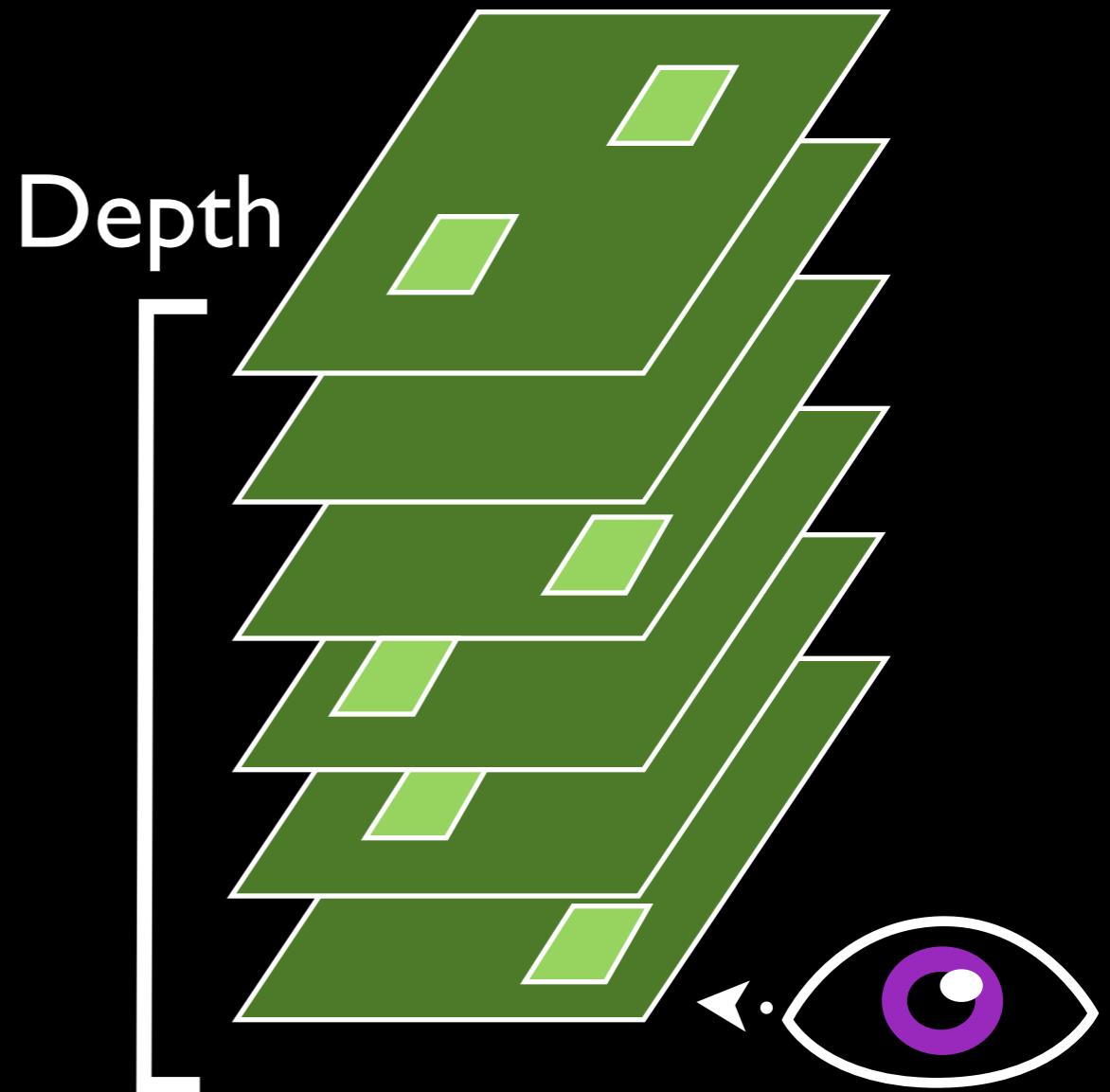
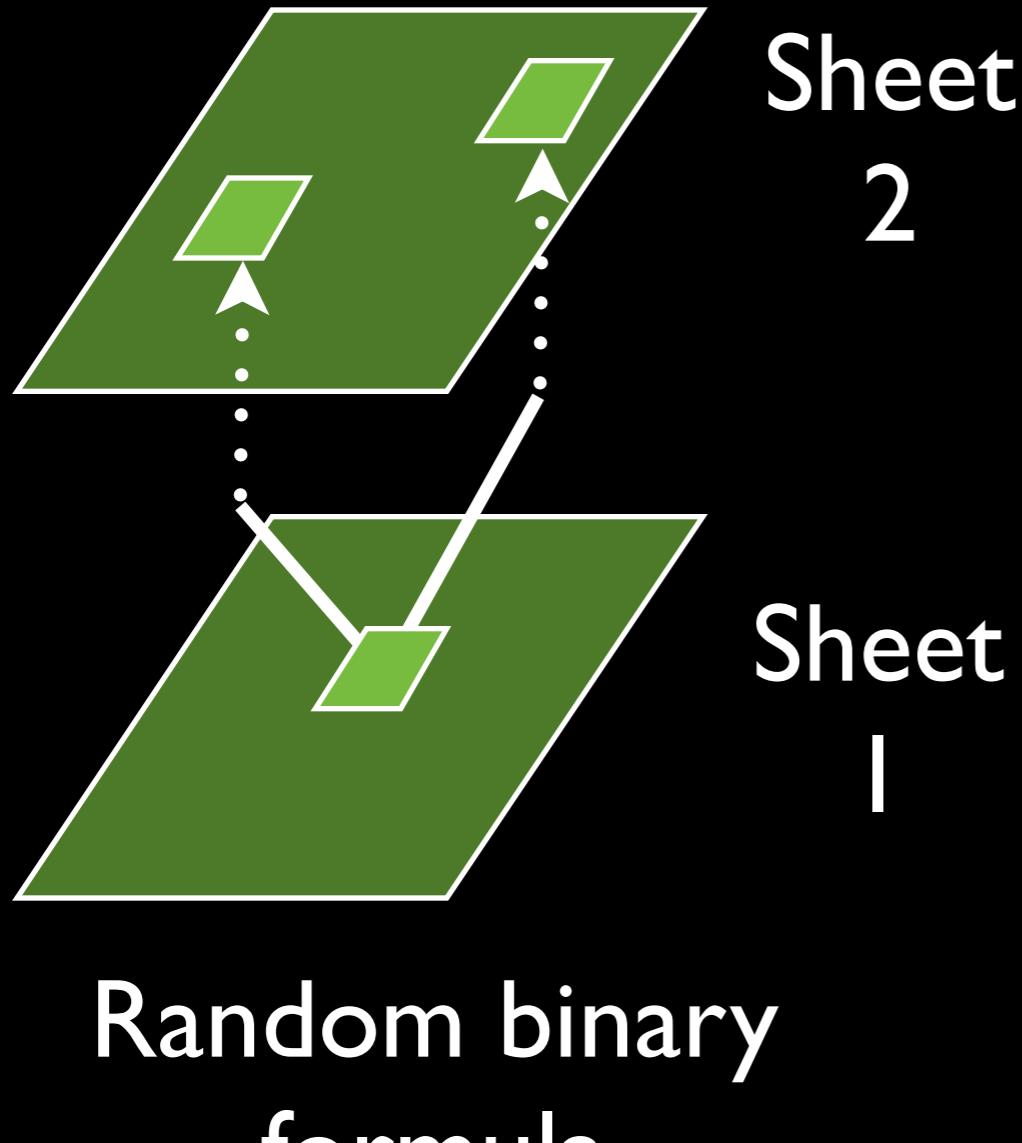
(15 sheets deep)



Speedup vs # Changes

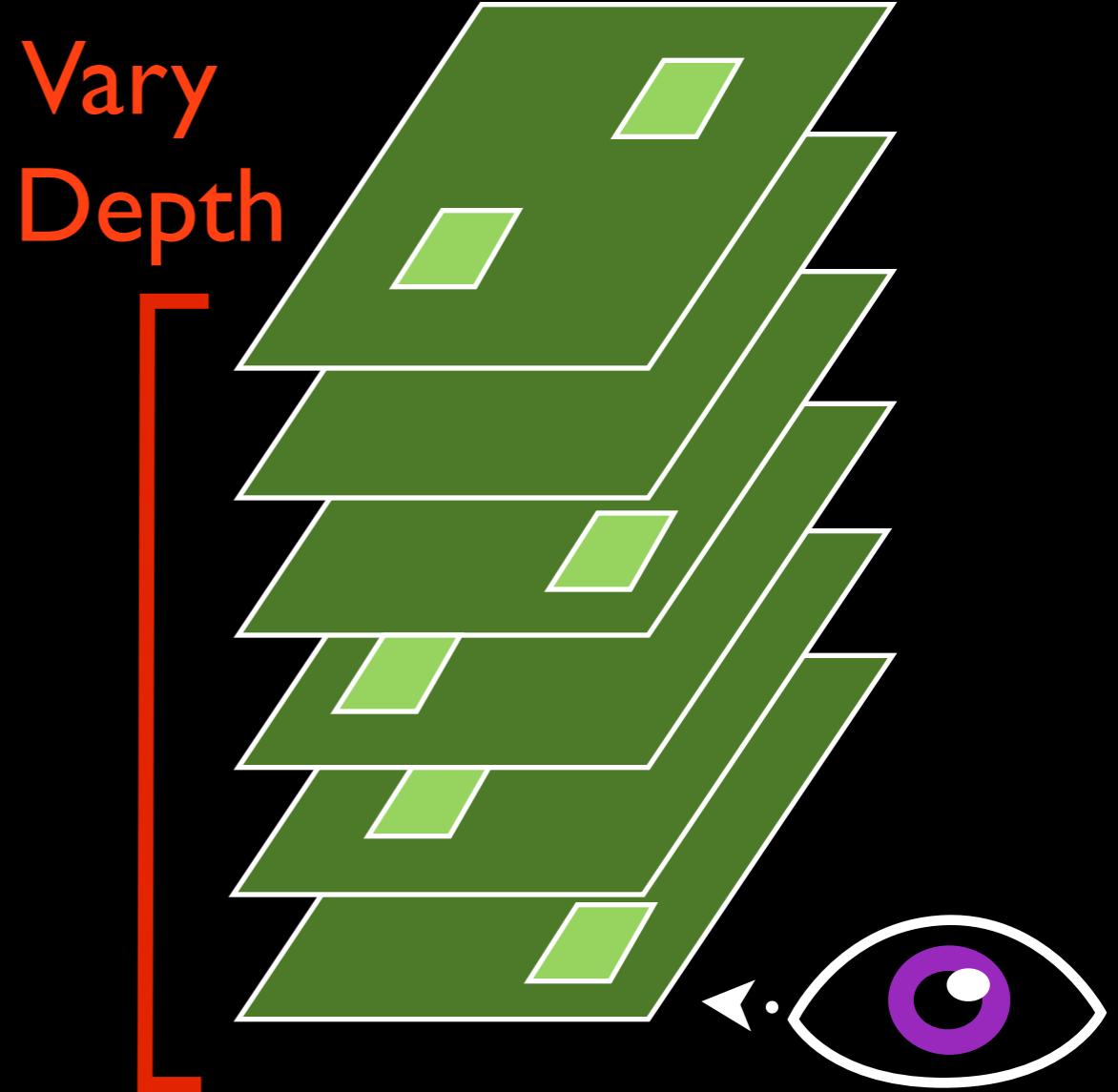
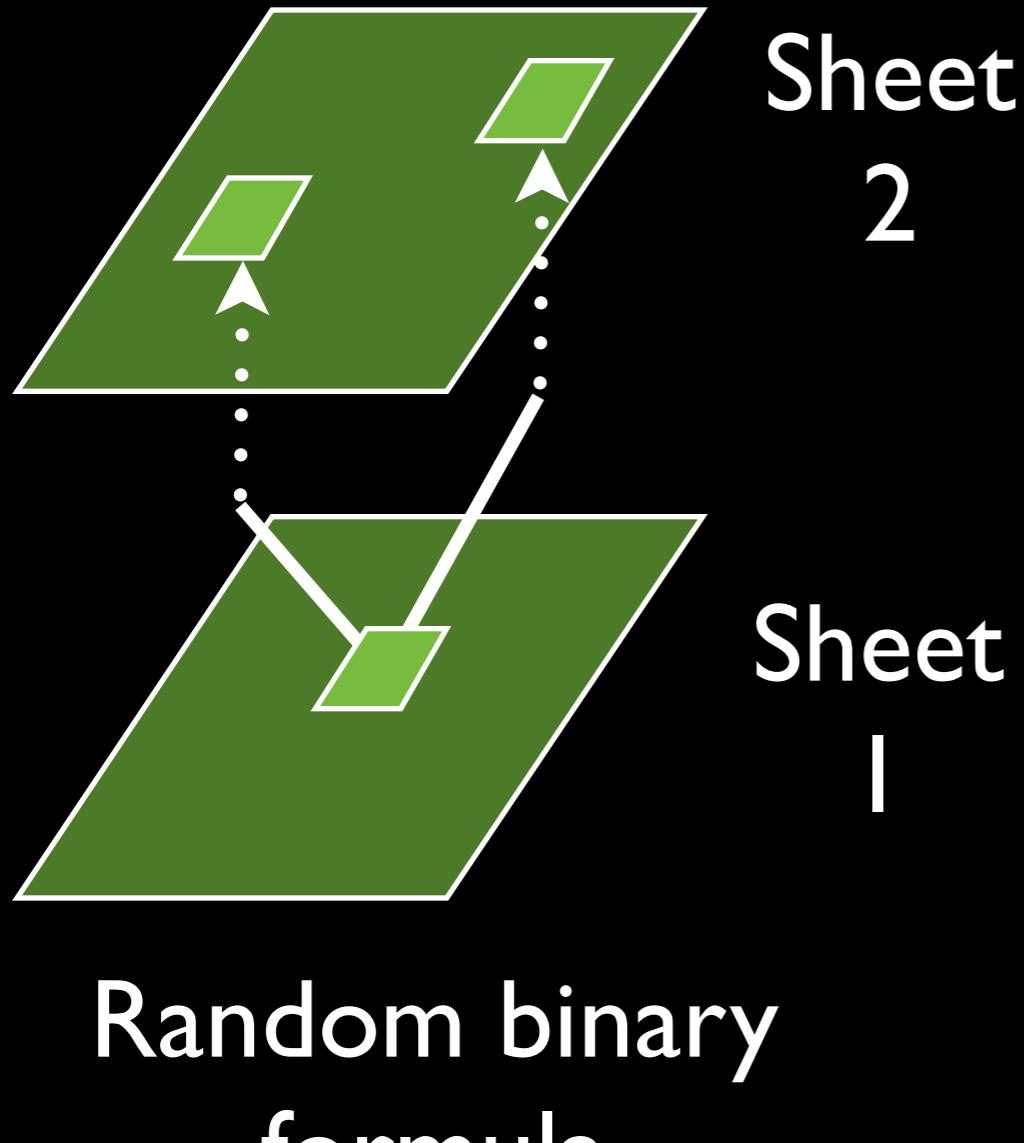


Spreadsheet Experiments



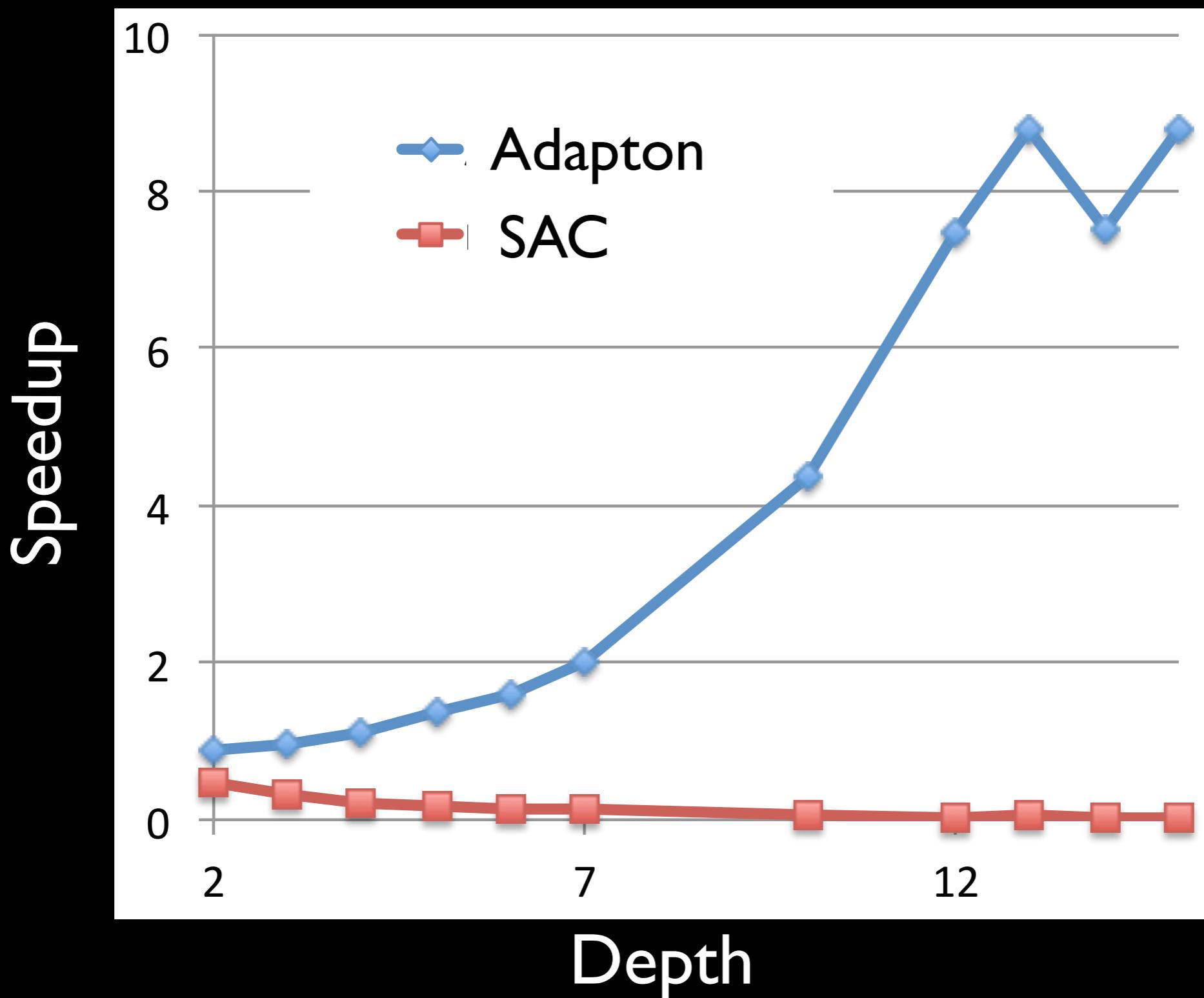
1. Random Mutations
2. Observe last sheet

Spreadsheet Experiments



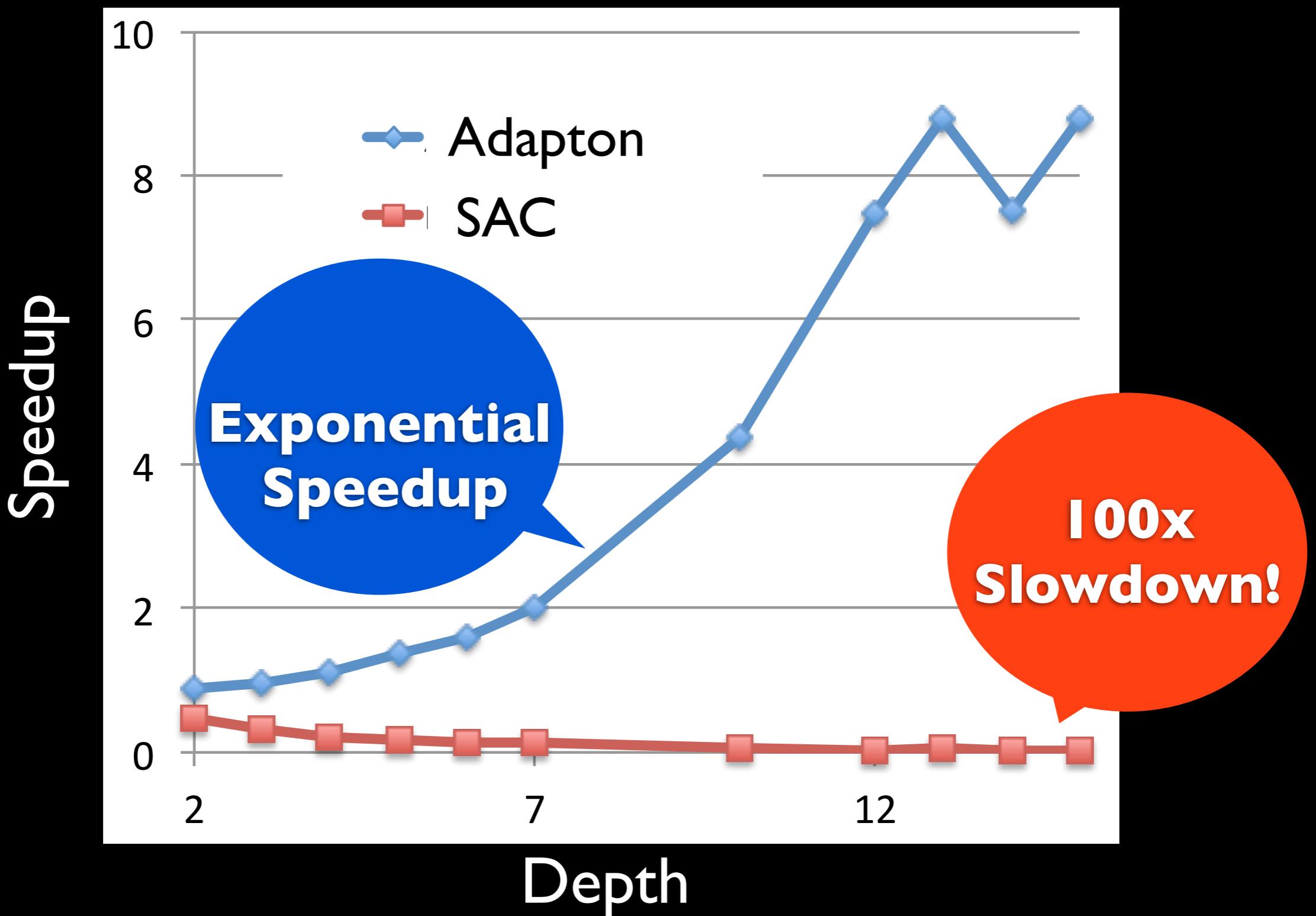
- I. Random Mutations
2. Observe last sheet

Speedup vs Sheet Depth (10 changes between observations)



Speedup vs Sheet Depth

(10 changes between observations)



Paper and Technical Report

- *Formal semantics* of Adapton
- *Algorithms* to implement Adapton
- More empirical data and analysis

Aside: Formal Semantics

- ▶ **CBPV + Refs + Layers** (*outer versus inner*)
- ▶ Syntax for **traces** and **knowledge** formally represents **DCG** structure
- ▶ Formal specification of change propagation
- ▶ **Theorems:**
 - **Type soundness**
 - **Incremental soundness**
("from-scratch consistency")

Summary

- ▶ **Adapton:** Composable, Demand-Driven IC
 - **Demand-driven** change propagation
 - Reuse patterns:
Sharing, swapping and switching
- ▶ Formal specification (see paper)
- ▶ Implemented in OCaml (and Python)
- ▶ Empirical evaluation shows **speedups**

<http://ter.ps/adapton>

	pattern	input #	LazyNonInc baseline		ADAPTON vs. LazyNonInc		EagerTotalOrder vs. LazyNonInc	
			time (s)	mem (MB)	time	spdup	mem	ovrhd
filter	lazy	1e6	1.16e-5	96.7	12.8	2.7	2.24	8.0
		1e6	6.85e-6	96.7	7.80	2.7	1.53	8.0
quicksort		1e5	0.0741	18.6	2020	8.7	22.9	144.1
mergesort		1e5	0.346	50.8	336	7.8	0.148	96.5
filter	swap	1e6	0.502	157	1.99	10.1	0.143	17.3
		1e6	0.894	232	2.36	6.9	0.248	12.5
fold(min)		1e6	1.04	179	472	9.1	0.123	33.9
fold(sum)		1e6	1.11	180	501	9.1	0.128	33.8
exptree		1e6	0.307	152	667	11.7	10.1	11.9
updown1	switch	4e4	0.0328	8.63	22.4	14.0	0.00247	429.9
updown2		4e4	0.0326	8.63	24.7	13.8	4.28	245.7
filter	batch	1e6	0.629	157	2.04	10.1	4.11	9.0
		1e6	1.20	232	2.21	6.9	3.32	6.6
fold(min)		1e6	1.43	179	4350	9.0	3090	8.0
fold(sum)		1e6	1.48	180	1640	9.1	4220	8.0
exptree		1e6	0.308	152	497	11.7	1490	9.7