University of Maryland
CMSC652 — Complexity Theory
Professor Jonathan Katz

Problem Set 3

Due at the *beginning* of class on Oct. 6

1. In class we gave two definitions of $\mathcal{P}/\text{poly}$ (see also the lecture notes for lecture 4). Prove that they are equivalent.

2. Let $\mathcal{B}_0'$ be any *complete* bounded fan-in basis. (A basis $\mathcal{B}$ is complete if any function can be computed by some circuit over $\mathcal{B}$.) Show that if a function can be computed by a circuit family of size $s(n)$ and depth $d(n)$ over $\mathcal{B}_0$, then it can be computed by a circuit family of size $O(s(n))$ and depth $O(d(n))$ over $\mathcal{B}_0'$.

3. (a) Prove that the parity function on n bits *requires* circuits of depth $\Omega(\log n)$ to compute over the basis $\mathcal{B}_0$.

   (b) Prove the following theorem regarding the circuit depth required by most functions:

   > Let $\varepsilon > 0$ and $d(n) = (1-\varepsilon) \cdot n$. Then for n large enough there exists an n-ary function not computable by circuits of depth at most $d(n)$ (over $\mathcal{B}_0$).

   Note: You should *not* need to look at any other sources to solve this problem...