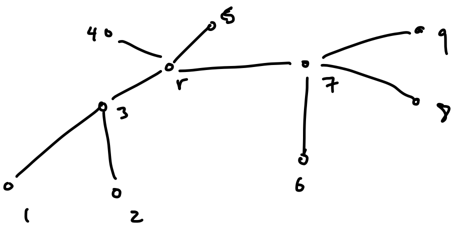
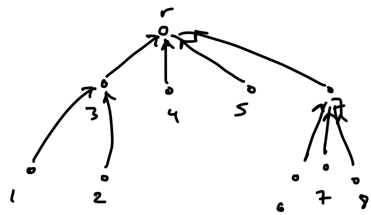
https://share.goodnotes.com/s/X4PUyLj4d4eEZ3P8FR6dim

Tree rooting problem: application of the "Euler Tour Technique"

Input: unrooted tree T, designated root r

Output: $\forall v \in T$, $v \neq r$ then parent $p(v) \in T$ of v in T rooted at r

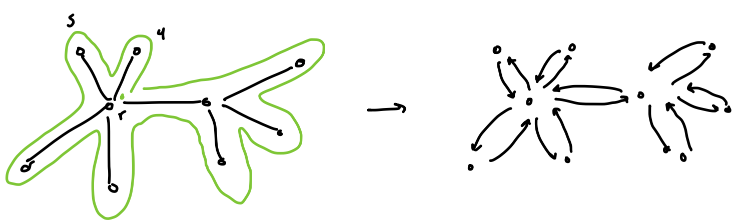




$p[1] = 3$ $p[2] = 3$

This problem is solvable using Euler Tour Technique + List Ranking

$(r,4), (4,r), (r,5) (5,r) \ldots$



T undirected

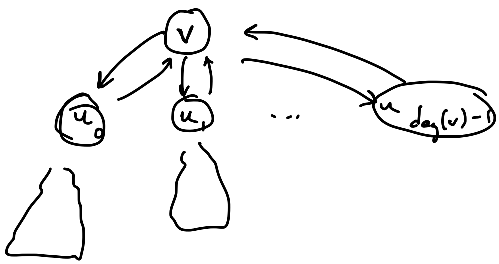
T' directed

where for each (u,v) edge, we introduce two directed edges $\{(u,v), (v,u)\}$

Given a tree T, an Eulerian circuit (or Euler tour) is E: a walk on the directed tree T' that (1) visits all the vertices and (2) doesn't repeat any edge.

For each $v \in V$, fix an ordering on

$$N(v) = \{u_0, u_1, \dots, u_{deg(v)-1}\}$$

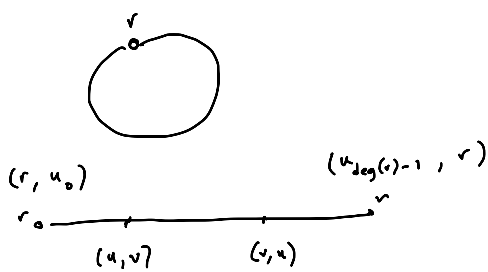


$\forall$ nodes $v \in V$:

$\forall$ arcs $\{(u_i, v) \mid u_i \in N(v)\}$:

$$succ((u_i, v)) = (v, u_{i+1 \bmod deg(v)})$$

ET construction in $O(n)$ work, $O(\log n)$ depth.



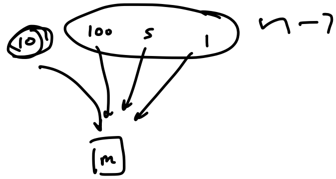
Second copy is the orientation toward the root.

Priority-write (&m, v):

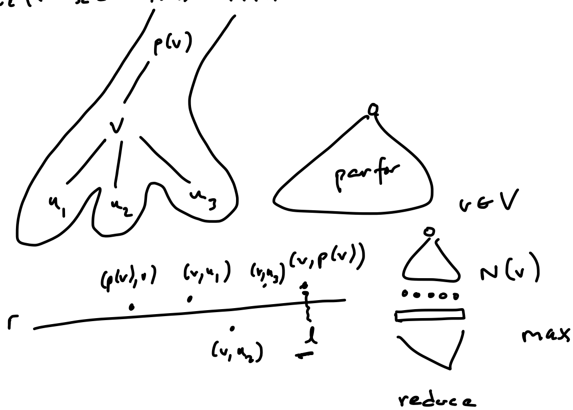
if (m→val > v):

atomically update m→val = v

we'll see this later



CRCW-PRAM with Priority-write



Random-Incremental.

Euler Tour Technique (ETT)

- dynamic tree : Euler Tour Trees
- pre/post/in-order traversals
- height of a node
- subtree sums

Tree $O(n \log(n))$

↓

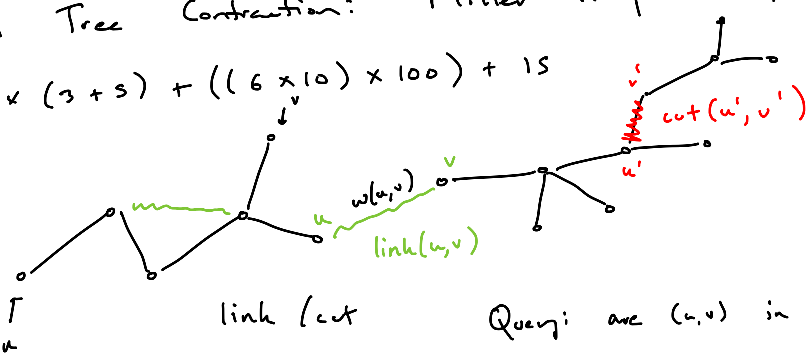
linked-list (Euler tour)

↓

List-Ranking

Parallel Tree Contraction: Miller-Reif (80s)

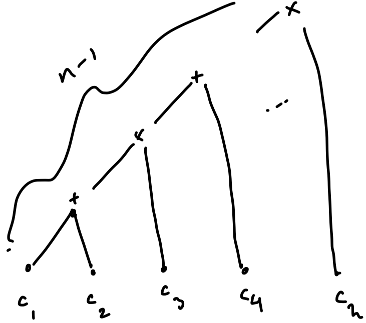
- $2 \times (3+5) + ((6 \times 10) \times 100) + 15$



link / cut

Query: are (u,v) in the same tree?

Tree contraction yields an efficient soln. to dynamic tree problem (RC-trees).



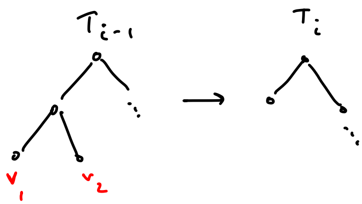
Look at this more general problem of <u>tree contraction</u>.

<u>Defn</u>: Tree Contraction Sequence: $T_1, T_2, \dots, T_k$

(1) $T_1 = T$

(2) $V(T_i) \subseteq V(T_{i-1})$

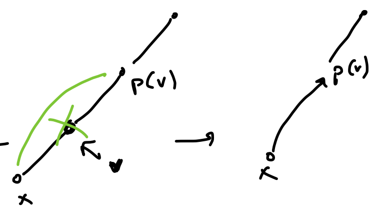
(3) $|V(T_k)| \in O(1)$



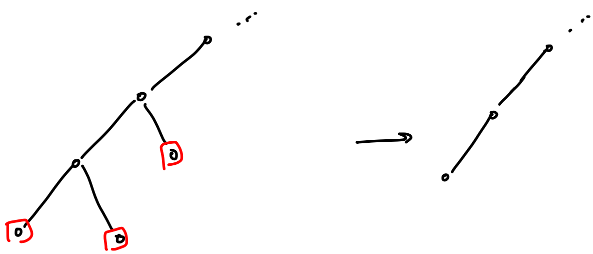
(4) If $v \in T_{i-1} - T_i$ (i.e. v is removed in T_i) then either:

(a) v is a leaf of T_{i-1} or

(b) v has exactly one child in T_{i-1}, x, and $p(v) \in T_{i-1}$ is the parent of x in T_i



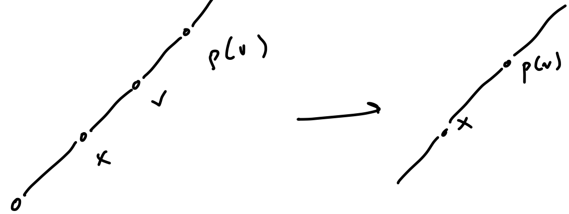
(a)



T_{i-1}

"rake" operation (raking leaves)

(b)



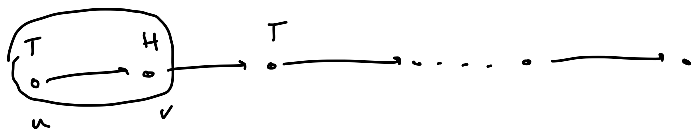
"compress" v.

$T_1 \ \ldots \ T_k$ $\quad$ k = length of tree contraction sequence

$$\log(n) - 1 \le \underbrace{k}_{\text{length}} \le \underbrace{n-1}$$

$$k \in O(\log n)$$

Idea:

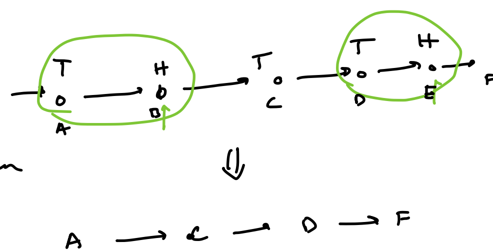


compress($\bar{v}$)

interleave $\underbrace{\text{raking all leaves}}_{\text{enough to handle balanced trees in } O(\log n) \text{ steps.}}^{(i)}$ with $\underbrace{\text{compress operation}}_{\text{necessary for long chains}}^{(ii)}$

Contract : // ~~one~~ two contraction steps

- rake all the leaves
- $\forall$ unary nodes, flip coins.
  - if coin(u) = T, coin(succ(u)) = H then compress(succ(u))



Lemma: # nodes in T decreases by a constant factor in expectation after applying Contract.

- T_0 = set of leaves (out-degree 0)
- T_1 = set of unary nodes (out-degree 1)
- T_2 = set of binary nodes (out-degree 2)

$$V = T_0 \cup T_1 \cup T_2$$

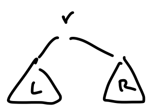
Claim: $|T_0| = |T_2| + 1$

Pf: strong induction on n (BC holds for $n=1$)

$\overset{r}{\bullet}$

(a) if r is unary, done by IH.

(b) if r is a binary node.



$$|T_0^L| = |T_2^L| + 1$$

$$|T_0^R| = |T_2^R| + 1$$

$$\underbrace{|T_0^L| + |T_0^R|}_{T_0} = \underbrace{\left((|T_2^L| + |T_2^R| + 1\right) + 1}_{T_2} \quad \square$$

Claim: Random Coin Tossing in Contract will generate an independent set $I_1 \subseteq T_1$ of size

$$|I_1| \geq \frac{1}{4}|T_1| \quad \text{in expectation}$$

Consider $v \in T_1$. X_v = I.R.V. that's 1 iff v is compressed

$v \in I_1$

$$\overset{T}{\circ} \longrightarrow \overset{H}{\underset{v}{\circ}}$$

$P[v \in I_1] = \frac{1}{4}$ $\quad$ (coin(pred(v)) = T, coin(v) = H)

$$E[|I_1|] = E\left[\sum_{v \in T_1} X_v\right] = \frac{1}{4}|T_1| \quad \checkmark$$

Let n = # nodes before Contract

n' = # nodes after:

Claim: $E[n'] \le \frac{3n}{4}$

$$E[n'] \le |T_2| + \frac{3}{4}|T_1|$$

$$\le \frac{3}{4} + \frac{3}{4}|T_1| + \frac{3}{2}|T_2|$$

$$= \frac{3}{4}\left(1 + |T_1| + 2|T_2|\right)$$

$$= \frac{3}{4}\left(|T_0| + |T_1| + |T_2|\right)$$

$$= \frac{3}{4}n$$