

## Ontology-Based Semantics

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**Mihai Ciocoiu**

Dept. of Computer Science  
and Institute for Systems Research  
University of Maryland  
College Park, MD 20742  
mihaic@cs.umd.edu

**Dana S. Nau**

Dept. of Computer Science  
and Institute for Systems Research  
University of Maryland  
College Park, MD 20742  
nau@cs.umd.edu

### Abstract

We consider the problem of providing semantics for declarative languages, in a way that would be useful for enabling automated knowledge exchange. If we only have one (first order) language, we can formalize what we mean for a set of sentences to be a translation of another, by requiring that the two sets share the same models. However, in order to formalize translation for the case where the two sets of sentences are of different languages, we need a different notion of semantics, capable of overcoming the language barrier. We introduce *Ontology-Based Semantics* with this purpose in mind. We show how ontologies can be used to make implicit assumptions explicit, and how they are integrated in our semantics in order to restrict the set of models a set of sentences has. We show how Ontology-Based Models can be used to formally define knowledge translation for the different language case in a similar way ordinary models can be used to define translation for the one language situation. We also provide a syntactical characterization of knowledge translation, that can be used as an effective procedure to check translatability, and we prove it to be sound and complete with respect to our semantic definition of translation.

## 1 INTRODUCTION

The ability to translate knowledge from/to different representation languages is an important ingredient for building powerful AI systems, by easing the difficult and time-consuming task of knowledge base construction, and facilitating knowledge sharing among

existing ones. In this paper we consider the problem of using formal ontologies for providing semantics to declarative languages, in a way that would be useful for enabling automated knowledge exchange.

Using formal ontologies has been proposed [Gruber 1991] as a solution for managing the inherent heterogeneity present in knowledge from different sources. Different approaches vary in their definition of what a formal ontology is, ranging from taxonomic hierarchies of classes [Campbell et al.], to vocabularies of terms defined by human-readable text, together with sets of formal constraining axioms [Gruber 1993]. Another distinction is the level of commitment of the communicating agents with respect to the shared ontology, varying from having all agents commit to a single common ontology — the standardization approach — to having a network of mediators and facilitators that enable translation among agents' different ontologies [Shave 1997, Gray et al. 1997].

For our purpose we will adopt the logical theory view of an ontology, and the constraining axioms will play a crucial role in defining our semantics. While we allow the communicating agents to have their own declarative languages and ontologies, we will require the existence of a common ontology expressive enough to interpret the concepts in all agents' ontologies. We will also require for a declarative language  $L$  that a function  $\sigma$  can be specified that converts sentences of  $L$  to sentences of a first order language  $\mathcal{L}$ . (of course this limits the method's applicability to languages that are not (strictly) more expressive than FOL). Here are some examples of questions we want to address:

- Consider a declarative language  $L$  that has a construct like (**mother Bill Anne**). What would models of this construct look like?
- Consider now the problem of translating sets of sentences among two declarative languages  $L_1$

and  $L_2$ . What exactly do we mean when we say that a set  $S_2$  of  $L_2$  sentences is a translation of a set  $S_1$  of  $L_1$  sentences?

For the simplest case, in which we have just one first-order language  $L$  and we are considering two sets of  $L$ -sentences  $S_1$  and  $S_2$ , the obvious solution is to say that  $S_2$  is a *translation* of  $S_1$  if it has the same set of consequences (i.e.,  $Cn(S_2) = Cn(S_1)$ ), or equivalently, if  $S_1$  and  $S_2$  share the same set of models (i.e.,  $\mathfrak{A} \models S_1 \Leftrightarrow \mathfrak{A} \models S_2$  for all  $\mathfrak{A}$ ). One could extend this further by saying that  $S_2$  is a *partial translation* of  $S_1$  if the consequences of  $S_2$  are also consequences of  $S_1$  (i.e.,  $Cn(S_2) \subseteq Cn(S_1)$ ), or equivalently, if all models of  $S_1$  are models of  $S_2$  (i.e.,  $\mathfrak{A} \models S_1 \Rightarrow \mathfrak{A} \models S_2$  for all  $\mathfrak{A}$ ).

Unfortunately, a direct extension of this idea for sets of sentences of two *different* first order languages  $L_1$  and  $L_2$  will not work the way we would like. One problem is that for intuitively similar concepts (and thus ones that we would like translatable) their representations in the two languages might use combinations of functions/predicates of different arities, such as functions in one language and predicates in the other (or any combinations thereof). Thus, models of sets of sentences in the different languages will be different, even if the sets of sentences are intuitively equivalent. For two different arbitrary declarative languages, defining translation is even harder, since we don't even have the notion of a model.

What this paper tries to do is to specify a way for defining models of sets of sentences of arbitrary declarative languages, so that these models can be used to define translation in the same fashion as for the one language situation above.

## 2 IMPLICIT ASSUMPTIONS

Consider a declarative language  $L$  that has a construct like **(mother Bill Anne)**. What would we want models of this construct to look like?

One option would be to follow a database approach: use a closed world assumption, proclaim that we are speaking of a universe with only two persons (**Anne** and **Bill**), and that the motherhood relation holds only for **(Bill, Anne)**. In this case, we would have a single model  $\mathfrak{A} = (\{\text{Bill, Anne}\}, \{\langle \text{Bill, Anne} \rangle\})$ . However, this approach would not allow us to define partial translation the way we would like, and would prohibit translating bits and pieces of information from more expressive languages into less expressive ones.

Instead, we would prefer the semantics of **(mother Bill Anne)** to be “the universe includes **Anne** and **Bill** and maybe other persons and the motherhood relation holds for **(Bill, Anne)** and maybe for some other pairs,” like the semantics for FOL is usually defined.

The first temptation would be to define the relation  $\sigma$  so that:

1.  $\sigma$  maps an  $L$  construct such as **(mother ?x ?y)** into a predicate such as **Mother(x, y)** of the first order language  $\mathcal{L}$ ;
2. the models of a set of  $L$  sentences (e.g.,  $S_1 = \{\text{mother Bill Anne}\}$ ) are in fact the models of the set  $\Sigma_1 = \{\sigma(s) : s \in S_1\}$ ; (e.g., the models of  $\{\text{Mother(Bill, Anne)}\}$ ).

This would allow other wanted structures, that include different persons, to be considered, like  $\mathfrak{B} = (\{\text{Bill, Anne, Cathy}\}, \{\langle \text{Bill, Anne} \rangle, \langle \text{Anne, Cathy} \rangle\})$ . However, it would also allow unwanted models to creep in, such as:

$$\mathfrak{C} = (\{\text{Bill, Anne}\}, \{\langle \text{Bill, Anne} \rangle, \langle \text{Anne, Anne} \rangle\}).$$

$$\mathfrak{D} = (\{\text{Bill, Anne, Cathy}\}, \{\langle \text{Bill, Anne} \rangle, \langle \text{Bill, Cathy} \rangle\}).$$

$$\mathfrak{E} = (\{\text{Bill, Anne, Cathy}\}, \{\langle \text{Bill, Anne} \rangle, \langle \text{Anne, Cathy} \rangle, \langle \text{Cathy, Bill} \rangle\}).$$

The problem is that there are implicit assumptions in the language  $L$ , such as the fact that a person cannot be her own mother, that one cannot have two different mothers, etc. These assumptions need to be made explicit in order to define the correct set of models for **(mother Bill Anne)**.

Making such assumptions explicit has become known in AI as building the domain ontology [Gruber 1991], and it holds the promise of enabling knowledge exchange.

The above considerations lead us to explicitly constructing a motherhood ontology  $\Omega$  into  $\mathcal{L}$  and taking the models of the  $L$  sentence **(mother Bill Anne)** to be the models of the  $\mathcal{L}$  theory having as axioms the sentence's image through  $\sigma$  together with the ontology itself, i.e. the models of  $\Omega \cup \{\text{Mother(Bill, Anne)}\}$ . Will this do?

Well, it will give us the models we wanted, but will not help much with translation. If some other language defines in its ontology a motherhood relation, then conceivably an automated translation procedure would identify the motherhood predicates as mutually translatable, but it would also translate to mother-

hood all predicates that satisfy the motherhood ontology. For example, the successor predicate that holds between an integer number and its successor satisfies all of the motherhood axioms, and so an automated translation procedure will consider a (partial) translation of  $(s \text{ ?}x \text{ ?}y)$  to  $(\text{mother ?}x \text{ ?}y)$ . (The translation will only be one-way, since the successor relation satisfies additional constraints, being in fact a bijective function, as opposed to the motherhood, which is only surjective.)

A much better idea is to *share* the motherhood ontology. Sharing ontologies will greatly simplify the task of a semantic-based automated translation procedure and also have the additional benefit of simplifying the process of writing ontologies, by enabling the reuse of already-existing components. In the next section we will present a way to define semantics that makes use of the ontology sharing idea.

### 3 ONTOLOGY-BASED SEMANTICS

#### 3.1 LOGICAL RENDER

As we mentioned earlier, we are interested in providing semantics for *declarative languages*, which for our purpose are languages  $L$  such that a function  $\sigma$  can be specified that converts sentences of  $L$  to sentences of a first order language  $\mathcal{L}$ .

We call such a function  $\sigma$  a *logical rendering* function, and the image  $\Sigma$  of a set  $S$  of  $L$  sentences through  $\sigma$  the *logical render* of  $S$  through  $\sigma$ .

Coming back to our motherhood example, the *logical rendering* function  $\sigma$  will convert instances of  $(\text{mother ?}x \text{ ?}y)$  to corresponding instances of the  $\mathcal{L}$  predicate  $\text{Mother}(x, y)$ .

#### 3.2 INTERPRETATIONS

To simplify our definitions we will restrict ourselves to first order languages  $\mathcal{L}$  that contain no function symbols. Note that this is not a reduction of expressivity, since any formula of a first order language  $\mathcal{L}$  that includes function symbols can be converted to a formula of a language  $\mathcal{L}'$  similar to  $\mathcal{L}$  but which has no function symbols and has additional  $(n + 1)$ -ary predicates corresponding to each  $n$ -ary function of  $\mathcal{L}$ .

Our notion of interpretation is the restriction to predicate calculus of the standard mathematical one, as it appears in [Enderton 1972].

**Definition** An *interpretation*  $\pi$  of a function-free language  $\mathcal{L}$  into a theory  $T$  of language  $\mathcal{L}_\Omega$  is a function

on the set of parameters of  $\mathcal{L}$  such that:

1.  $\pi$  assigns to  $\forall$  a formula  $\pi_\forall$  of  $\mathcal{L}_\Omega$  in which at most one variable  $v_1$  occurs free, such that  $T \models \exists v_1 \pi_\forall$ .
2.  $\pi$  assigns to each  $n$ -place predicate symbol  $P$  a formula  $\pi_P$  of  $\mathcal{L}_\Omega$  in which at most  $n$  variables  $v_1, \dots, v_n$  occur free.

**Definition** An *interpretation*  $\varphi^\pi$  of a  $\mathcal{L}$ -formula  $\varphi$  is recursively defined in the obvious way: i.e. if  $\varphi$  is an atomic formula  $P$ , its interpretation is the formula  $\pi_P$  applied to the same set of constants/variables; otherwise  $(\neg\varphi)^\pi$  is  $(\neg\varphi^\pi)$ ,  $(\varphi \rightarrow \psi)^\pi$  is  $(\varphi^\pi \rightarrow \psi^\pi)$ ,  $(\forall x\varphi)^\pi$  is  $\forall x(\pi_\forall(x) \rightarrow \varphi^\pi)$ , etc.

**Definition** An *interpretation*  $\pi$  of a theory  $T_0$  of language  $\mathcal{L}$  into a theory  $T$  of language  $\mathcal{L}_\Omega$  is an interpretation  $\pi$  of the language  $\mathcal{L}$  into  $T$  such that for all  $\mathcal{L}$ -sentences  $\varphi$ ,  $\varphi \in T_0 \Rightarrow \varphi^\pi \in T$ .

#### 3.3 EXPLANATIONS

**Definition** Given a domain ontology  $\Omega$  expressed as a set of  $\mathcal{L}_\Omega$  sentences, a theory  $T$  of  $\mathcal{L}_\Omega$  is called a *domain theory* for  $\Omega$  iff  $T \models \Omega$ .

**Definition** Given an interpretation  $\pi$ , a logical rendering function  $\sigma$  and an ontology  $\Omega$  (expressed as a set of  $\mathcal{L}_\Omega$  sentences), an  $\mathcal{L}_\Omega$  *explanation* of a set  $S$  of  $L$  sentences is a  $\mathcal{L}_\Omega$  domain theory  $T$  for  $\Omega$ , such that if  $\Sigma$  is the logical render of  $S$  through  $\sigma$ ,  $T \models \Sigma^\pi$ .

Intuitively, an  $\mathcal{L}_\Omega$  *explanation* of a set  $S$  of sentences of language  $L$  is a theory of  $\mathcal{L}_\Omega$  that has among its axioms the interpretation of the rendering of  $S$ , with the concepts that appear in them “explained” by ontology  $\Omega$ . Going back to our example, an explanation of  $(\text{mother Bill Anne})$  is a theory that has  $\text{Mother}(\text{Bill}, \text{Anne})$  and the motherhood ontology as axioms.

#### 3.4 ONTOLOGY-BASED MODELS

Given a model  $\mathfrak{A}$  of an explanation  $T$  of a set of  $L$  sentences  $S$ , we can extract from it a model  ${}^\pi\mathfrak{A}$  of the render  $\Sigma$  of  $S$ , that has the desired property of obeying the additional constraints imposed by the ontology  $\Omega$ . Namely, let

1.  $|{}^\pi\mathfrak{A}| =$  the set defined in  $\mathfrak{A}$  by  $\pi_\forall$ ;
2.  $P^{\pi\mathfrak{A}} =$  the relation defined in  $\mathfrak{A}$  by  $\pi_P$ , restricted to  $|{}^\pi\mathfrak{A}|$ .

${}^\pi\mathfrak{A}$  is called an *Ontology-Based Model* of  $S$  (written  ${}^\pi\mathfrak{A} \models_{\sigma, \pi}^\Omega S$ ).

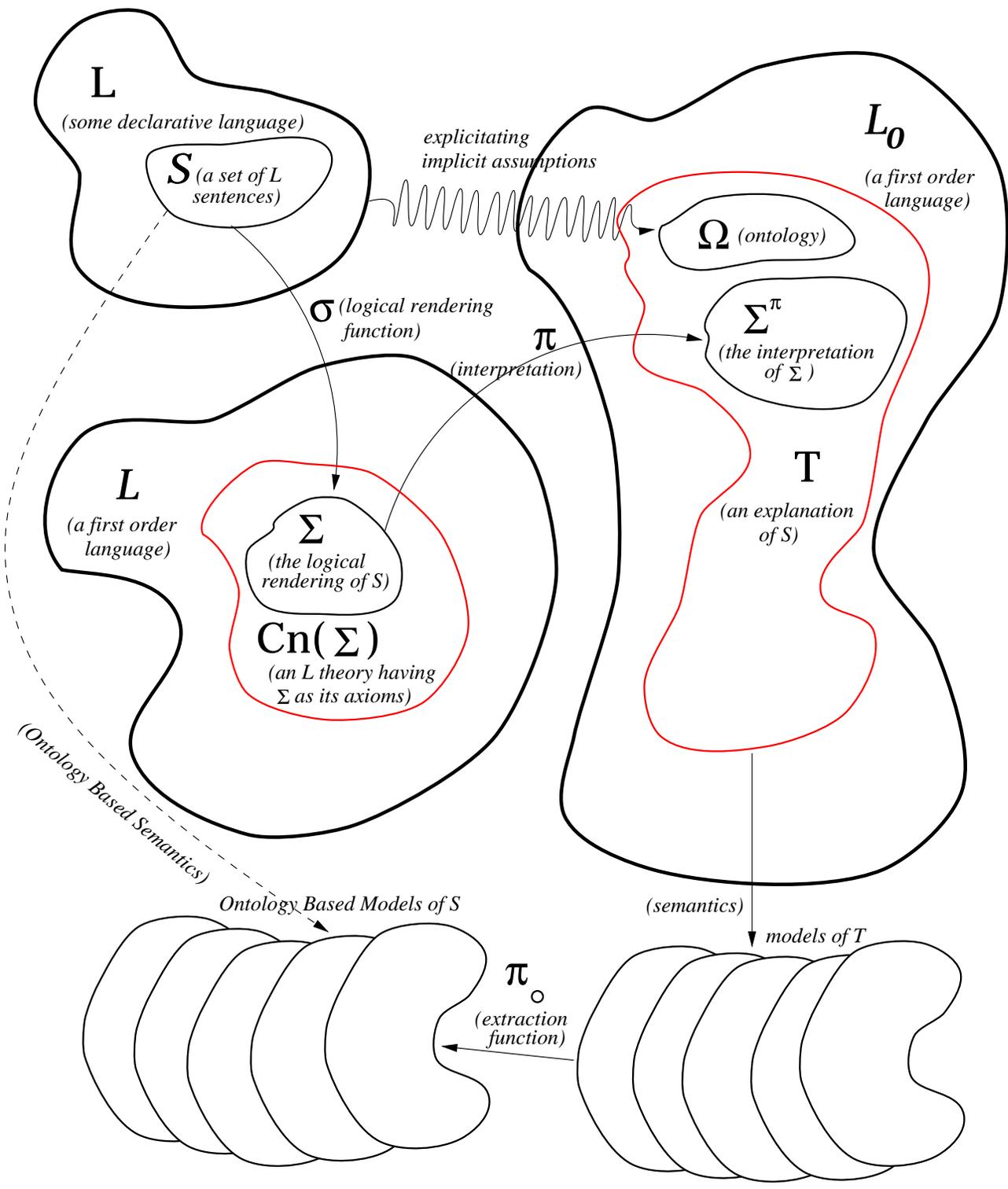


Figure 1: Ontology-Based Models

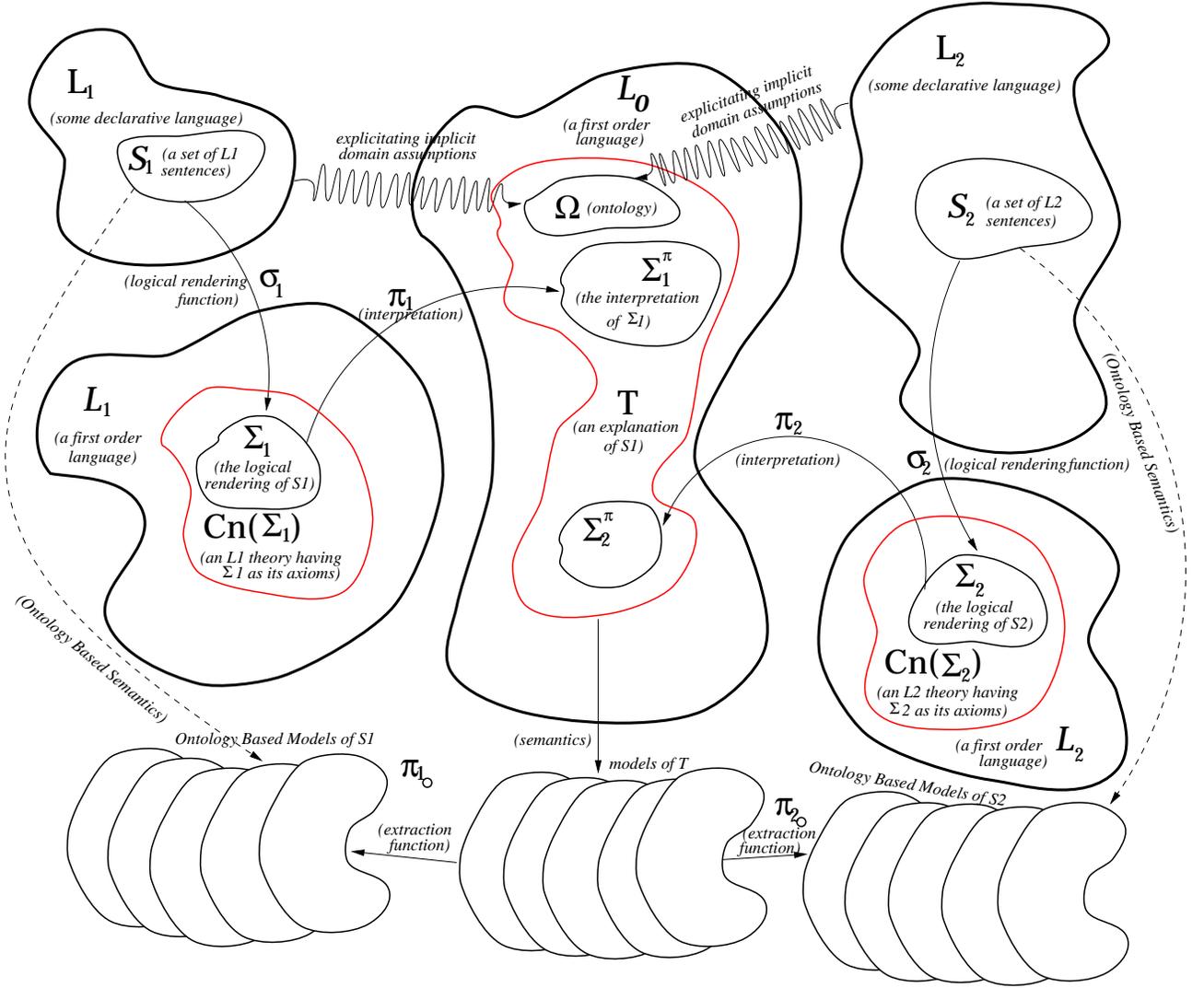


Figure 2: Ontology-Based Translation

## 4 ONTOLOGY-BASED TRANSLATION

Ontology-Based Models allow a definition of translation for the different language situation in the same way that ordinary models allowed it for the single-language case.

Suppose we are given two declarative languages  $L_1$  and  $L_2$ , a domain ontology  $\Omega$  (expressed as a set of sentences in language  $\mathcal{L}_\Omega$ ), rendering functions  $\sigma_1$  and  $\sigma_2$ , and interpretations  $\pi_1$  and  $\pi_2$ . Then a set  $S_2$  of  $L_2$  sentences is an *Ontology-Based Partial Translation* of a set  $S_1$  of  $L_1$  sentences iff for every model  $\mathfrak{A}$  of every explanation  $T$  of  $S_1$ ,  $\pi_1 \mathfrak{A} \models_{\sigma_1, \pi_1}^\Omega S_1 \Rightarrow \pi_2 \mathfrak{A} \models_{\sigma_2, \pi_2}^\Omega S_2$ .

Similarly, suppose we are given two declarative languages  $L_1$  and  $L_2$ , a domain ontology  $\Omega$  (expressed as a set of sentences in language  $\mathcal{L}_\Omega$ ), rendering functions  $\sigma_1$  and  $\sigma_2$ , and interpretations  $\pi_1$  and  $\pi_2$ . Then a set  $S_2$  of  $L_2$  sentences is an *Ontology-Based Translation* of a set  $S_1$  of  $L_1$  sentences iff  $S_2$  is an *Ontology-Based Partial Translation* of  $S_1$  and  $S_1$  is an *Ontology-Based Partial Translation* of  $S_2$ .

## 5 AN EXAMPLE TRANSLATION

Suppose we have a declarative language  $L_1$  that has a construct like  $(GM ?x ?y)$  whose intended semantics is “ $?y$  is a grandmother of  $?x$ ”, and another declarative language  $L_2$  that has a construct  $Y \text{ anc } X$  whose intended semantics is “ $Y$  is an ancestor of  $X$ ”. In or-

der to provide ontology-based semantics for those two languages, we must first build a domain ontology  $\Omega$  (in our case a family ontology), and provide the logical rendering ( $\sigma_1$  and  $\sigma_2$ ) and interpretation functions ( $\pi_1$  and  $\pi_2$ ).

### 5.1 A FAMILY ONTOLOGY

As primitive concepts, our toy family ontology will have the concepts of *Male* and *Female*. As a primitive relation, it will have the parenthood relation  $Parent(x, y)$ , which holds if  $y$  is a parent of  $x$ . As a defined relation it will introduce  $Ancestor(x, y)$  by the following two axioms:

$$\begin{aligned} (\forall x)(\forall y) \text{ Parent}(x, y) &\rightarrow \text{Ancestor}(x, y) \\ (\forall x)(\forall y)(\forall z) \text{ Parent}(x, z) \wedge \text{Ancestor}(z, y) &\rightarrow \\ &\text{Ancestor}(x, y) \end{aligned}$$

Suppose we also write the following axioms in order to constrain the possible interpretation of the primitive concepts:

$$\begin{aligned} (\forall x) \neg(\text{Male}(x) \wedge \text{Female}(x)); \text{ i.e., } \text{Male} \text{ and } &\text{Female} \text{ are disjoint concepts.} \\ (\forall x) \neg \text{Parent}(x, x); \text{ i.e., One cannot be his/hers own} &\text{parent.} \\ (\forall x)(\forall y) \neg(\text{Parent}(x, y) \wedge \text{Ancestor}(y, x)); \text{ i.e., One} &\text{cannot be a parent of one of his/hers ancestors.} \\ (\forall x)(\forall y)(\forall z) (\text{Parent}(x, y) \wedge \text{Parent}(x, z) \wedge &\text{Female}(y) \wedge \text{Female}(z)) \rightarrow y = z; \text{ i.e.,} \\ \text{One's Female parent is unique.} & \\ (\forall x)(\forall y)(\forall z) (\text{Parent}(x, y) \wedge \text{Parent}(x, z) \wedge &\text{Male}(y) \wedge \text{Male}(z)) \rightarrow y = z; \text{ i.e., One's} \\ \text{Male parent is unique.} & \end{aligned}$$

### 5.2 LOGICAL RENDERING AND INTERPRETATION FUNCTIONS

The logical rendering functions for this simple example would just convert the given constructs into the corresponding first-order atomic sentences, i.e. the logical render of  $(GM \text{ ?x ?y})$  would be the  $\mathcal{L}_1$  atomic sentence  $GM(x, y)$  and the logical render of  $Y \text{ anc } X$  would be the  $\mathcal{L}_2$  atomic sentence  $Anc(x, y)$ .

The interpretation of  $GM(x, y)$  will be a  $\mathcal{L}_\Omega$  formula having at most two free variables, in our case  $(\exists z) \text{ Parent}(x, z) \wedge \text{Parent}(z, y) \wedge \text{Female}(y)$ .

Finally, the interpretation of  $Anc(x, y)$  must be a  $\mathcal{L}_\Omega$  formula having at most two free variables, in our case  $Ancestor(x, y)$ .

### 5.3 EXAMPLE TRANSLATION

In this section we show how the syntactic characterization of Ontology-Based Translation can be used to verify translatability for a given set of ground sentences of two different languages.

Consider the  $L_1$  theory  $S_1 = \{(GM \text{ Bill, Anne}), (GM \text{ Anne, Cathy})\}$ , and the  $L_2$  theory  $S_2 = \{\text{Anne anc Bill, Cathy anc Anne, Cathy anc Bill}\}$ . Then  $S_2$  is an ontology-based partial translation of  $S_1$ , but  $S_1$  is *not* an ontology-based partial translation of  $S_2$ . Indeed, the  $\mathcal{L}_1$  and  $\mathcal{L}_2$  logical renders of  $S_1$  and  $S_2$  are:

$$\begin{aligned} \Sigma_1 &= \{GM(\text{Bill, Anne}), GM(\text{Anne, Cathy})\} \text{ and} \\ \Sigma_2 &= \{Anc(\text{Bill, Anne}), Anc(\text{Anne, Cathy}), \\ &\quad Anc(\text{Bill, Cathy})\} \end{aligned}$$

and their  $\mathcal{L}_\Omega$  interpretations are:

$$\begin{aligned} \Sigma_1^{\pi_1} &= \{(\exists z) \text{ Parent}(\text{Bill, } z) \wedge \\ &\quad \text{Parent}(z, \text{Anne}) \wedge \\ &\quad \text{Female}(\text{Anne}), \\ &\quad (\exists z) \text{ Parent}(\text{Anne, } z) \wedge \\ &\quad \text{Parent}(z, \text{Cathy}) \wedge \text{Female}(\text{Cathy})\} \text{ and} \\ \Sigma_2^{\pi_2} &= \{\text{Ancestor}(\text{Bill, Anne}), \\ &\quad \text{Ancestor}(\text{Anne, Cathy}), \\ &\quad \text{Ancestor}(\text{Bill, Cathy})\} \end{aligned}$$

It is easy to show that  $(\Sigma_1^{\pi_1} \cup \Omega) \vdash \Sigma_2^{\pi_2}$ , and this is a necessary and sufficient condition (see Section 6 for a proof) for  $S_2$  to be an ontology-based partial translation of  $S_1$ . However, the reverse is not true.  $S_1$  is not an ontology-based partial translation of  $S_2$ . To see this, consider an explanation  $T$  of  $S_2$ ,

$$\begin{aligned} T &= Cn(\Omega \cup \{\text{Parent}(\text{Bill, Anne}), \\ &\quad \text{Parent}(\text{Anne, Cathy})\}) \end{aligned}$$

and a model of it:

$$\begin{aligned} \mathfrak{A} &= (\{\text{Anne, Bill, Cathy}\}, \\ &\quad \text{Parent}^{\mathfrak{A}} = \{\langle \text{Bill, Anne} \rangle, \langle \text{Anne, Cathy} \rangle\}, \\ &\quad \text{Male}^{\mathfrak{A}} = \{\text{Bill}\}, \\ &\quad \text{Female}^{\mathfrak{A}} = \{\text{Anne, Cathy}\}, \\ &\quad \text{Ancestor}^{\mathfrak{A}} = \{\langle \text{Bill, Anne} \rangle, \langle \text{Anne, Cathy} \rangle, \\ &\quad \langle \text{Bill, Cathy} \rangle\}). \end{aligned}$$

Note that  $\pi_2 \mathfrak{A} \models_{\sigma_2, \pi_2}^\Omega S_2$ , but  $\pi_1 \mathfrak{A} \not\models_{\sigma_1, \pi_1}^\Omega S_1$ , so  $S_1$  is not an ontology-based partial translation of  $S_2$ . Also note that neither  $S'_1 = \{(GM \text{ Bill Cathy})\}$  nor any other

non-void set  $S_1$  of  $L_1$  sentences could be an ontology-based partial translation of  $S_2$ , since we can always construct models  $\mathfrak{A}$  of  $S_2$  explanations such that  $\pi_1 \mathfrak{A}$  are not ontology-based models of  $S_1$ , by violating the sex/parenthood constraints necessary for the grandmotherhood relation of  $L_1$ . (Such a class of models would be the ones which interpret the motherhood relation by the void set.)

In practice, an automated inference procedure could be used on the domain ontology and the logical rendering and interpretation functions, in order to precompile a generic rule of the form “ $?y$  anc  $?x$  is an ontology-based translation of (GM  $?x$   $?y$ )”, and then use it for generating efficient direct translators among the two languages.

## 6 A SYNTACTIC CHARACTERIZATION

**Theorem 6.1** (*Soundness and Completeness of Syntactic Characterization*)

A set  $S_2$  of  $L_2$ -sentences is an ontology-based partial translation of a set  $S_1$  of  $L_1$ -sentences (with respect to rendering functions  $\sigma_1$  and  $\sigma_2$  and interpretations  $\pi_1$  and  $\pi_2$ ) iff

$$(\Sigma_1^{\pi_1} \cup \Omega) \vdash \Sigma_2^{\pi_2}$$

where  $\Sigma_1$  and  $\Sigma_2$  are the logical renders of  $S_1$  and  $S_2$  through  $\sigma_1$  and  $\sigma_2$ .

**Lemma 6.2** *If  $\pi \mathfrak{A}$  is an ontology-based model of a set of sentences  $S$  with respect to logical rendering function  $\sigma$  and interpretation  $\pi$ ,  $\pi \mathfrak{A} \models_{\sigma, \pi}^{\Omega} S$ , then  $\mathfrak{A}$  is a model of the interpretation  $\Sigma^{\pi}$  of the logical render  $\Sigma$  of  $S$ , i.e.  $\mathfrak{A} \models \Sigma^{\pi}$ .*

**Proof** (Lemma 6.2) Suppose  $\mathfrak{A} \not\models \Sigma^{\pi}$ . By definition of an ontology-based model, there must exist an  $\mathcal{L}_{\Omega}$  explanation  $T$  of  $S$ , such that  $T \models \Omega$  and  $T \models \Sigma^{\pi}$ , and a model  $\mathfrak{B}$  of  $T$  such that  $\pi \mathfrak{B} = \pi \mathfrak{A}$ . (If such a model doesn't exist, then  $\pi \mathfrak{A}$  cannot be an ontology-based model of  $S$ .)

Since  $T$  is a theory and  $T \models \Sigma^{\pi}$ , it must be the case that  $\Sigma^{\pi} \subseteq T$ . Since  $\mathfrak{B} \models T$ ,  $\mathfrak{B}$  is a model for  $\Sigma^{\pi}$ ,  $\mathfrak{B} \models \Sigma^{\pi}$ . Since  $\mathfrak{A} \not\models \Sigma^{\pi}$ , there must exist a sentence  $\gamma^{\pi} \in \Sigma^{\pi}$  such that  $\mathfrak{A} \not\models \gamma^{\pi}$ . It can be proved by induction on the structure of  $\gamma$  that this cannot be the case.

**Proof** (Theorem 6.1, soundness) Suppose  $(\Sigma_1^{\pi_1} \cup \Omega) \vdash \Sigma_2^{\pi_2}$ . Consider an arbitrary domain theory  $T$ , and an arbitrary model  $\mathfrak{A}$  of  $T$ . If  $\pi_1 \mathfrak{A} \models_{\sigma_1, \pi_1}^{\Omega} S_1$ , then by Lemma 6.2,  $\mathfrak{A} \models \Sigma_1^{\pi_1}$ . Since  $T$  is a domain theory,

$\Omega \subseteq T$ ; and since  $\mathfrak{A}$  is a model of  $T$ ,  $\mathfrak{A} \models \Omega$ ; and thus  $\mathfrak{A} \models (\Sigma_1^{\pi_1} \cup \Omega)$ . Since  $T$  is a theory, then by our supposition that  $(\Sigma_1^{\pi_1} \cup \Omega) \vdash \Sigma_2^{\pi_2}$ , it follows that  $\Sigma_2^{\pi_2} \subseteq T$ , and thus  $T$  is an explanation of  $S_2$ .

Since  $\mathfrak{A}$  is a model of  $T$ ,  $\pi_2 \mathfrak{A}$  is an ontology-based model for  $S_2$  i.e.,  $\pi_2 \mathfrak{A} \models_{\sigma_2, \pi_2}^{\Omega} S_2$ . Since  $T$  and  $\mathfrak{A}$  were arbitrarily chosen, it follows that for every model  $\mathfrak{A}$  of every domain theory  $T$ ,  $\pi_1 \mathfrak{A} \models_{\sigma_1, \pi_1}^{\Omega} S_1 \Rightarrow \pi_2 \mathfrak{A} \models_{\sigma_2, \pi_2}^{\Omega} S_2$ , and thus  $S_2$  is an ontology-based partial translation of  $S_1$ .

**Proof** (Theorem 6.1, completeness) Suppose  $S_2$  is an ontology-based partial translation of  $S_1$ . Consider an arbitrary model  $\mathfrak{A} \models (\Sigma_1^{\pi_1} \cup \Omega)$  and let  $T = Cn(\Sigma_1^{\pi_1} \cup \Omega)$ .  $T$  is an explanation of  $S_1$ ; and since  $\mathfrak{A} \models (\Sigma_1^{\pi_1} \cup \Omega)$ ,  $\mathfrak{A} \models T$ .

By our supposition and the definition of ontology-based translation, it follows that  $\pi_2 \mathfrak{A} \models_{\sigma_2, \pi_2}^{\Omega} S_2$ . By Lemma 6.2 it follows that  $\mathfrak{A} \models \Sigma_2^{\pi_2}$ . Since  $\mathfrak{A}$  was arbitrarily chosen,  $(\Sigma_1^{\pi_1} \cup \Omega) \models \Sigma_2^{\pi_2}$ ; and by completeness of first order deduction,  $(\Sigma_1^{\pi_1} \cup \Omega) \vdash \Sigma_2^{\pi_2}$ .

## 7 CONCLUSIONS

In this paper we have presented a new kind of semantics, called *Ontology-Based Semantics*, intended for facilitating automated knowledge exchange between declarative languages. Our results include the following:

We have shown how domain specific information, encoded as ontologies, is used in constructing *Ontology-Based Models* that restrict the possible interpretations a set of sentences can have.

We have shown how Ontology-Based Models can be used to formally define knowledge translation for the different-language case, in a way similar to how ordinary models can be used to define translation for the one-language situation.

We have provided a syntactical characterization of knowledge translation, that can be used as an effective procedure to check translatability, and we have proved it to be sound and complete with respect to our semantic definition of translation.

The principal benefit of our semantics is that it provides a formal foundation for reasoning about the properties of systems that do automated knowledge translation based on ontology sharing. As part of our collaboration with NIST on their Process Specification Language (PSL) project [Schlenoff et al. 1999, Schlenoff et al. 1999], we plan to develop a system that would be able to automatically generate efficient

translators based on declarative languages' ontology-based semantics, specified as a logical rendering function and a PSL interpretation.

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