Strategy Generation in Multi-Agent Imperfect-Information Pursuit Games

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ABSTRACT

We describe a formalism and algorithms for game-tree search in partially-observable Euclidean space, and implementation and tests in a scenario where a multi-agent team, called *tracking* agents, pursues a *target* agent that wants to evade the tracking agents. Our contributions include—

- A formalism that combines geometric elements (agents' locations and trajectories and observable regions, and obstacles that restrict mobility and observability) with game-theoretic elements (information sets, utility functions, and strategies).
- A recursive formula for information-set minimax values based on our formalism, and a implementation of the formula in a game-tree search algorithm.
- A heuristic evaluation function for use at the leaf nodes of the game-tree search. It works by doing a quick lookahead search of its own, in a relaxed version of the problem.
- Experimental results in 500 randomly generated trials. With the strategies generated by our heuristic, the tracking agents were more than twice as likely to know the target agent's location at the end of the game than with the strategies generated by heuristics that compute estimates of the target's possible locations.

Categories and Subject Descriptors

I.2.8 [**Artificial Intelligence**]: Problem Solving, Control Methods, and Search

General Terms

Algorithms, Theory

Keywords

Visibility-based pursuit-evasion games, multi-agent planning, game tree search

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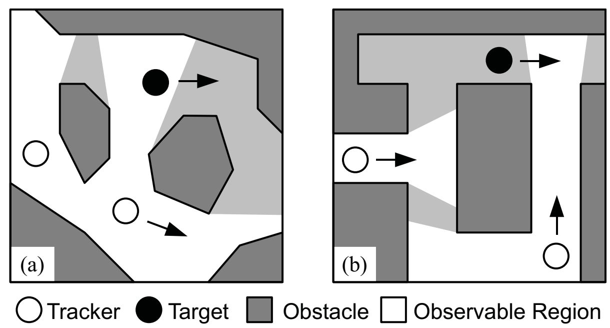


Figure 1: Example pursuit scenarios where (a) a tracker team is able to follow its target more closely by temporarily sacrificing visibility, (b) an evasive target escapes more easily by temporarily revealing its location.

1. INTRODUCTION

We consider multi-agent pursuit scenarios in which there is a team of tracker agents and a moving target agent. The objective of the trackers is to observe the target as much as possible and/or to have the target in at least one tracker's observation range at the end of the game. The target may move evasively, or along a trajectory not known in advance. Movement and observability may be limited by characteristics of the domain, such as solid obstacles or walls, or limited by the agents' own movement and sensor capabilities.

In many multi-agent pursuit scenarios the target will employ strategies to evade detection or obstruct the visibility of the trackers. To be successful, the tracker team must devise a strategy that can recover from a loss of visibility very quickly, and on occasion sacrifice visibility to maintain a strategic advantage later in the game. As illustrated in Fig. 1, maintaining visibility on a target for as long as possible may not result in the optimal pursuit strategy, nor will simply finding a target be enough to ensure that it can continue to be tracked.

Generating effective strategies for a continuous space, imperfect-information domain poses several computational difficulties: 1) the agent team must reason about the geometry of the environment and the solid obstacles in the world since these usually determine an agent's ability to observe the target being pursued; 2) simple heuristic search and optimization approaches do not always work due to the uncertainty that arises from the lack of full observability of the target and lack of perfect knowledge about the target's objectives and its strategy; 3) the pursuer agents must generate plans very quickly and act on those plans since most pursuit scenarios occur in real time.

Previous work on pursuit and evasion games has often simplified these challenges by addressing one of two separate sub-problems: generating pursuit strategies that maintain visibility on a target in a perfect information domain [5, 24, 22], or pre-computing patrol strategies that maximize the likelihood of finding an unseen moving target [13, 38, 21]. Many of the techniques used to address imperfect information either omit game-theoretic considerations, or are formalized for discrete rather than continuous domains [1, 4].

In this paper, we describe a formalism and algorithms for game-tree search in partially-observable Euclidean space, to address each of the challenges outlined above. Our contributions include—

- A new formalism for imperfect-information games in n-dimensional Euclidean space. The formalism allows us to specify continuous and discrete geometric characteristics, obstructed observability due to solid objects, and game-theoretic information sets, utility functions, and strategies (Section 2).
- A recursive formula for computing minimax values at each information-set, and a game-tree search algorithm based on that formula (Section 3).
- A heuristic evaluation function for use at the leaf nodes of the game-tree search. The heuristic function does a quick lookahead of its own in a relaxed version of the problem, taking advantage of the interplay among the problem's geometric and game-theoretic aspects (Section 4.3).
- Experimental results that show our heuristic's effectiveness in 500 randomly-generated imperfect-information pursuit games. The tracking agents' *success rate* (i.e., the fraction of problems on which the tracing agents knew the target agent's location at the end of the game) was more than twice as high with our heuristic than with two other heuristics, both of which use estimates of the possible target locations (Section 5).

The structure of this paper follows the outline provided above. A detailed description of related work is provided in Section 6.

2. FORMALISM

2.1 World Models

Let $\mathbb{R}^n$ be an n-dimensional Euclidean space that contains obstacles $o_1, \ldots, o_k$ modeled as geometric solids.[1] Let $\boldsymbol{r} = (r_1, \ldots, r_k)$ be a set of agents controlled by an entity called the *tracker*, and r_0 be a single agent controlled by the *target*. Let $T = (t_0, t_1, \ldots, t_{end-1}, t_{end})$ be a finite sequence of time points that may be either uniformly or non-uniformly spaced. A *trajectory* for r_i is a function ℓ_i that gives a location $\ell_i(t) = (x_{i1}, \ldots, x_{in})$ for r_i at every time point $t \in [t_0, t_{end}]$, where t_0 and t_{end} are the *starting* and *ending* times.[2]

Observability and reachability play important roles in deciding how to move during pursuit tasks. An agent r's *observed region* at location p is $V_r(p) = \{$all locations that r can observe from $p\}$; see Fig. 2 for an example. An agent's *reachability* at location p

[1]In principle, each o_i could be any regular, semi-analytic set [16].

[2]For simplicity of presentation we use a fixed ending time here, but in general it may vary. For example, it could be the time at which a robotic agent runs out of power and can no longer function.

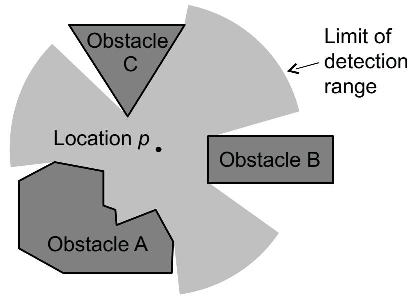


Figure 2: An observable region.

is defined as $R_r(p, \delta) = \{$all locations r can reach from point p in time $\delta\}$, where δ is some time interval $(t_i, t_{i+1}] \subseteq [t_0, t_{end}]$. This function describes the movement capabilities of agent r in the problem domain.

At any given time t_c, the set of all time points at which the tracker has observed the target's location is

$$\hat{T}_0(t_c) = \{t \in [t_0, t_c] : \ell_0(t) \in V_1(\ell_1(t)) \cup \ldots \cup V_k(\ell_k(t))\},$$

where each V_i is r_i's observed region. Hence the tracker has observed a *partial* trajectory $\hat{\ell}_0 : \hat{T}_0(t_c) \to \mathbb{R}^n$ for the target, where $\hat{\ell}_0(t) = \ell(t)$ for every $t \in \hat{T}_0(t_c)$. Given the tracker's observations, the set of all possible locations for the target is

$$\hat{L}_0(t_c) = \{\ell_0(t_c) : \ell_0 \text{ is a trajectory for } r_0\text{, and } \ell(t) = \hat{\ell}_0(t) \text{ for every } t \in \hat{T}_0(t_c)\}.$$

Similarly, if the tracking agents' trajectories are $\boldsymbol{l} = (\ell_1, \ldots, \ell_k)$ and the target observes partial trajectories $\hat{\boldsymbol{l}} = (\hat{\ell}_1, \ldots, \hat{\ell}_k)$, then

$$\hat{L}_i(t_c) = \{\ell_i(t_c) : \ell_i \text{ is a trajectory for } r_i \text{ and } \ell(t) = \hat{\ell}_i(t) \text{ for every } t \in \hat{T}_i(t_c)\};$$
$$\hat{\boldsymbol{L}} = (\hat{L}_1, \ldots, \hat{L}_k).$$

2.2 Imperfect-Information Game Tree Search

In what follows, we assume the tracker wants to be as sure as possible of the target's location at time t_{end}, hence wants to minimize the volume $\mathrm{Vol}(\hat{L}_0(t_{end}))$ of the set $\hat{L}_0(t_{end})$ of possible target locations at the ending time t_{end}; and the target wants to maximize this same volume. Thus, a *multi-agent tracking problem* is a zero-sum game in which the utility function is $\mathrm{Vol}(\hat{L}_0(t_{end}))$.

Given a multi-agent tracking problem, we now discuss how to compute a strategy for the agent team over a finite sequence of time points $T = (t_0, t_1, \ldots, t_{end})$. At each time t, the target's *information set*[3] $I_0(t)$ includes t, the target's location $\ell_0(t)$, and the set $\hat{\boldsymbol{L}}(t)$ of possible tracker locations. The tracker's information set $I_r(t)$ includes t, the locations $\boldsymbol{l}(t) = \{\ell_1(t), \ldots, \ell_k(t)\}$ of the tracker's agents, and the set $\hat{L}_0(t)$ of possible target locations.

A *pure strategy* is a function that, given an agent's current information set, returns the agent's next *move* (i.e., its change in location between t_j and t_{j+1}). For example, if the target has a pure strategy σ_0, then $\ell_0(t_{j+1}) = \ell_0(t_j) + \sigma_0(I_0(t_j))$ for each t_j; and if the tracker has a pure strategy $\boldsymbol{\sigma} = (\sigma_1, \ldots, \sigma_k)$, then $\boldsymbol{l}(t_{j+1}) = \boldsymbol{l}(t_j) + \boldsymbol{\sigma}(I_r(t_j))$ for each t_j.

[3]An *information set* [26, 27] is the game-theoretic analog of a belief state. The main difference is that unless one has a good predictive model of the opponent's strategy (e.g., [2]), uncertainty about the other agents' actions can make it difficult or impossible to put a probability distribution over the states of an information set.

A *mixed strategy* is a probability distribution over pure strategies.[4] Let σ_0 and $\boldsymbol{\sigma} = (\sigma_1, \ldots, \sigma_k)$ be any mixed strategies for the target and tracker. Then the *expected utility* given $\boldsymbol{\sigma}$ and σ_0 and I is the expected value $E^*(\boldsymbol{\sigma}, \sigma_0, I) = E[\text{Vol}(L_0(t_{end})) \mid \boldsymbol{\sigma}, \sigma_0, I]$. The Minimax Theorem [40] does not apply *per se*, but an approximation of it applies, i.e., there are mixed strategies $\boldsymbol{\sigma}^*$ and σ_0^*, and a *minimax value* $E^*[I]$ that corresponds intuitively to the information set I's optimal expected utility, such that

$$\begin{aligned} E^*[I] &\approx \min_{\boldsymbol{\sigma}} \max_{\sigma_0} E[\text{Vol}(L_0(t_{end})) \mid \sigma_0, \boldsymbol{\sigma}, I] \\ &\approx \max_{\sigma_0} \min_{\boldsymbol{\sigma}} E[\text{Vol}(L_0(t_{end})) \mid \sigma_0, \boldsymbol{\sigma}, I]. \end{aligned}$$

From this we can show[5] that for any information set $I_r(t_j)$ for the tracker,

$$E^*[I_r(t_j)] \approx \tag{1}$$
$$\begin{cases} \text{Vol}(L_0(t_j)), & \text{if } t_j = t_{end}, \\ \min_{\boldsymbol{m}} \max_{m_0} E^*[I_r(t_{j+1}) | I_r(t_j), \boldsymbol{m}, m_0], & \text{otherwise;} \end{cases}$$

where $\boldsymbol{m}$ and m_0 are probability distributions over the moves available to $\boldsymbol{r}$ and r_0 in $I_r(t_j)$. Conceptually, Eq. (1) is an analog of the well-known recursive formula for minimax values in perfect-information games such as chess, but generalized to accommodate both geometry and imperfect information.

Eq. (1) can be modified to accommodate *limited-depth lookahead* and a *heuristic evaluation function* $h(I)$ that gives an estimate of $E^*(I)$. In particular, if we look ahead to some time point $t_d \leq t_{end}$, then

$$E^*[I_r(t_j)] \approx \tag{2}$$
$$\begin{cases} \text{Vol}(I_r(t_j)), & \text{if } t_j = t_{end}, \\ h(I_r(t_j)), & \text{if } t_j = t_d < t_{end}, \\ \min\limits_{\boldsymbol{m}} \max\limits_{m_0} E^*[I_r(t_{j+1}) | I_r(t_j), \boldsymbol{m}, m_0], & \text{otherwise.} \end{cases}$$

The above equation provides the basis for the strategy-generation algorithm in the next section.

3. ALGORITHM

This section develops an algorithm to perform a game-tree search with limited-depth lookahead in partially-observable multi-agent tracking problems. Since the number of possible trajectories is infinite in multi-agent tracking problems, and thus the reachability at any particular point may be infinite, we will assume that the problem space has been discretized, so that $R_r(p, \delta)$ is finite for all p. The geometric-modeling literature gives a variety of decomposition and tesselation techniques for accomplishing this.

A multi-agent tracking problem can be formally described by the tuple $\langle T, R, V, \ell_0(t_0), \boldsymbol{l}(t_0)\rangle$, where T is a set of time points, R and V describe reachability and observability in the problem domain, and $\ell_0(t_0)$ and $\boldsymbol{l}(t_0)$ are the initial locations of the agents. We assume that each of the tracker's agents share a common information set I, removing the need to compute separate game trees for each tracker agent. If one of the trackers spots the target, they will all have that information.

We now describe how to compute the minimax value for a multi-agent tracking problem. Algorithm 1 shows a high-level description of our game-tree search procedure. The goal of our algorithm is to produce a strategy $\sigma(I(t_i))$ that specifies the next move for the tracker by using a depth-limited game tree search. Because of the high computational complexity of game-tree search in imperfect-information games, we have modified Eq. (2) to incorporate a "paranoid" model of the target's behavior, i.e., an assumption that the target always knows the tracker's location and strategy, and will choose the moves that are worst for the tracker.

Algorithm 1 Depth-limited minimax search for a team of trackers following a single target.

minimax(I, t_j, t_d)
  if $t_j = t_{end}$ **then return** $Vol(I)$
  if $t_j = t_d$ **then return** $h(I)$
  $\alpha = \infty$
  for all $\boldsymbol{l}' \in \Gamma_{track}(t_j)$ **do**
    $\beta = -\infty$
    for all $\hat{L}_0' \in \hat{\Gamma}_{target}(t_j)$ **do**
      $I' = \langle \boldsymbol{l}', \hat{L}_0' \rangle$
      $\beta = \max(\beta, minimax(I', t_{j+1}, t_d))$
    $\alpha = \min(\alpha, \beta)$
  return α

In Section 2 we defined $\ell_0(t_i)$ as the location of the target at time t_i, and the set $\boldsymbol{l}(t_i) = \{\ell_1(t_i), \ldots, \ell_k(t_i)\}$ as the location of the trackers. Additionally, we defined $\hat{L}_0(t_i)$ as the set of possible target locations in the tracker's information set. In order to perform a game-tree search, we must be able to compute $\ell_0(t_{i+1})$, $\boldsymbol{l}(t_{i+1})$ and $\hat{L}_0(t_{i+1})$ in terms of t_i.

The set of possible positions for agent r at time t_{i+1} can be determined by

$$\gamma_r(t_i) = R_r(\ell_r(t_i), \delta_i)$$

where $\delta_i = (t_i, t_{i+1}]$. Regardless of the strategy used by the target, we can now guarantee that $\ell_0(t_{i+1}) \in \gamma_0(t_i)$. We can provide a similar definition for the set of tracker locations, such that $\boldsymbol{l}(t_{i+1}) \in$

$$\Gamma_{track}(t_i) = \gamma_1(t_i) \times \ldots \times \gamma_k(t_i)$$

Using $\gamma_0(t_i)$ and $\Gamma_{track}(t_i)$ we can now plot the possible trajectories of the target and trackers. However, to compute the minimax value for a particular state, we will also need some way to determine $\hat{L}_0(t_{i+1})$. To do this, we can start by generalizing *reachability* to operate on a set of locations

$$\mathcal{R}(\hat{L}_0(t_i)) = \textstyle\bigcup_{p \in \hat{L}_0(t_i)} R_0(p, \delta_i).$$

If a tracker knows the exact location of the target at time t_i, then it follows from the above definition that $\mathcal{R}(\hat{L}_0(t_i)) = \gamma_0(t_i)$. Otherwise, $\mathcal{R}(\hat{L}_0(t_i))$ represents the "expanded" set of potential target locations at time t_{i+1}. In either case, only some of those locations are likely to remain hidden from the trackers.

Intuitively, if the target is not observable at time t_i, then it must hold that $\hat{L}_0(t_i) \cap V_r(\ell_r(t_i)) = \emptyset$ for each tracker. For simplicity, we will generalize *observability* for the set of all tracker locations

$$\mathcal{V}(\boldsymbol{l}(t_i)) = \textstyle\bigcup_{r=1}^{k} V_r(\ell_r(t_i)).$$

Thus, we can make the following determination:

$$\hat{L}_0(t_{i+1}) = \begin{cases} \mathcal{R}(\hat{L}_0(t_i)) \setminus \mathcal{V}(\boldsymbol{l}(t_{i+1})), & \text{if } \ell_0(t_{i+1}) \notin \mathcal{V}(\boldsymbol{l}(t_{i+1})) \\ \ell_0(t_{i+1}), & \text{otherwise} \end{cases}$$

We can now update $\hat{L}_0(t_i)$ for any set of moves selected from $\gamma_0(t_i)$ and $\Gamma_{track}(t_i)$. This is sufficient for performing a game-tree search, but some additional work is required to produce a proper strategy.

[4] As a special case, any pure strategy σ can be viewed as a mixed strategy such that $P(\sigma) = 1$ and $P(\sigma') = 0 \ \forall \sigma' \neq \sigma$.

[5] We omit the proof due to lack of space. The approach is similar in spirit to the information-set search algorithm in [27], but it reasons over Euclidean space rather than sets of histories, and accounts for simultaneous actions by the agents.

Algorithm 2 Algorithm for computing the *relaxed lookahead (RLA)* heuristic.

$h^d_{RLA}(I, t_j)$
  $v = 0$
  $S_0 = \hat{L}_0(t_j)$
  $S = \boldsymbol{l}(t_j)$
  for $i = 0 \ldots d$ **do**
    $v = v + |S_0 \setminus \mathcal{V}(S)|$
    $S_0 = \mathcal{R}(S_0)$
    $S = \mathcal{R}(S)$
  return $v/(d+1)$

3.1 Reasoning about a hidden target

Consider any time point t_i at which the target is *hidden* (i.e., not observable by the tracker). Recall from Section 2 that the tracker's strategy $\boldsymbol{\sigma}$ is a function of the tracker's information set $I_r(t_i)$, which includes (among other things) the set $\hat{L}_0(t_i)$ of all possible target locations that are consistent with the tracker's observations up to time t_i.

At time t_{i+1}, the target can stay hidden by remaining somewhere in $\hat{L}_{hide}(t_{i+1}) = \mathcal{R}(\hat{L}_0(t_i)) \setminus \mathcal{V}(\boldsymbol{l}(t_{i+1}))$, or it can reveal itself at some location in $\hat{L}_{reveal}(t_{i+1}) = \mathcal{R}(\hat{L}_0(t_i)) \cap \mathcal{V}(\boldsymbol{l}(t_{i+1}))$. There are no other possible locations for the target to be at t_{i+1}, and the exact location is determined by the target's strategy. Thus in the game tree, the set of possible "next positions" for the target is

$$\hat{\Gamma}_{target}(t_i) = \{\{p\} : p \in \hat{L}_{reveal}(t_{i+1})\} \cup \{\hat{L}_{hide}(t_{i+1})\}$$

We can now compactly represent the possible moves of the target and tracker using the sets $\hat{\Gamma}_{target}(t_i)$ and $\Gamma_{track}(t_i)$ respectively. This also provides a "worst-case" target model that depends only on the tracker's information set, eliminating a large amount of redundant work from the tree.

The minimax value can be computed recursively (see Algorithm 1) using a heuristic evaluation function $h(I)$ given some finite search depth t_d. The algorithm's performance can be improved by incorporating alpha-beta pruning, or other traditional game-tree pruning techniques.

4. HEURISTIC FUNCTIONS

In Algorithm 1's second **if-then** statement, the algorithm requires a heuristic evaluation function h to apply to the leaf nodes of its search. This section defines three different heuristic functions, the *RS*, *MD*, and *RLA* heuristics, which we will evaluate experimentally later in Section 5.

4.1 Region Size (*RS*)

The size of $\hat{L}_0(t)$ will vary throughout the game, but when it becomes large, it often indicates that the target is in danger of being lost. A simple heuristic is to just compute the current size of $\hat{L}_0(t)$, and use that to approximate the minimax value:

$$h_{RS}(I(t_j)) = |\hat{L}_0(t_j)|$$

This *region size* heuristic is a good approximation of the minimax value only towards the end of the game. Since the heuristic does not use any look-ahead, there is no guarantee that at time t_{j+1} the evaluation won't rapidly shift in favor of the target or tracker.

4.2 Maximum Distance (*MD*)

An even more obvious heuristic is to just measure the distance between each of the trackers and the target. If we compute the trackers' A^* distance[6] to each point in $\hat{L}_0(t_j)$, we can define

$$h_{MD}(I(t_j)) = \tfrac{1}{k}\textstyle\sum_{r=1}^{k} \max_{p\in\hat{L}_0(t_j)} d(\ell_r(t_j), p)$$

This heuristic computes the average of the *maximum distance* that the target could be from each of the trackers at time t_j. When used as part of a single-ply minimax search, each tracker will independently move in the direction of the target, treating it as though it's as far away as possible.

4.3 Relaxed Lookahead (*RLA*)

Both of the above heuristics reason about the target's behavior locally – i.e., by performing a one-step look-ahead – but do not look ahead farther to predict long-term future consequences of the target's strategies in the game. Because of the geometric properties of a multi-agent tracking problem, it is possible to estimate the size of $\hat{L}_0$ several rounds in advance, without needing to perform a full game-tree search. This allows us the benefit of lookahead, without the requiring an expensive computational effort.

While there may be a number of ways to estimate the size of $\hat{L}_0$ several rounds in advance, we will extend the definition of $\hat{L}_0(t_j)$ to include all points reachable by the target d rounds into the future, excluding those which could be seen by a tracker under any potential circumstance:

$$\begin{aligned}\hat{L}^d_0(t_c) = \{\ell_0(t_{c+d}) : \ell_0(t_c) \in \hat{L}_0(t_c) \wedge \\ \nexists \boldsymbol{l}(t_{c+d}) \text{ s.t. } \ell_0(t_{c+d}) \in V(\boldsymbol{l}(t_{c+d}))\}.\end{aligned}$$

This relaxation of the problem serves as a very rough approximation of the value of $\hat{L}_0(t_{j+d})$. Using this approximation, we can define a *relaxed lookahead* heuristic,

$$h^d_{RLA}(I(t_j)) = \tfrac{1}{d+1}\textstyle\sum_{i=0}^{d} |\hat{L}^i_0(t_j)|,$$

which estimates the average size of $\hat{L}_0(t_j)$ over the next d rounds. Note there is a special case where $h^0_{RLA}(I(t_j)) = h_{RS}(I(t_j))$.

A computable version of this heuristic is shown in Algorithm 2, which has a runtime complexity that is polynomial in the total number of vertices searched. This algorithm can be accelerated by caching the function $d(p_1, p_2)$, and a similar function $v(p_1, p_2)$ to compute the shortest distance from p_1 that a tracker needs to travel to see p_2. Each require only a polynomial amount of space to store, but can reduce the computational overhead significantly.

In our implementation of this heuristic, we evaluate each location explicitly, such that $h^d_{RLA}(I(t_j))$ is the sum of

$$\Delta t(p, I(t_i)) = \min_{r} \max_{\ell\in\hat{L}_0(t_i)} [v(\ell_r(t_i), p) - d(\ell, p)]$$

summed over all points p that the target can reach in d time steps. To make this computation equivalent to Algorithm 2, we need to restrict the values of $\Delta t(p, I(t_i))$ to the range $[0, d]$. By doing this, we attribute equal weight to any target locations that are unobservable for d or more time steps, which may include locations that are not observable at all.

A proof is omitted, but this formulation allows for more efficient use of the cached values, while still producing the same result. In practice, it is also useful to incorporate a tie-breaker in both the *region size* and *relaxed lookahead* heuristics: if either heuristic results in a tie, we use the *max distance* heuristic to determine which is better. This is particularly useful if any of the trackers are isolated, and cannot directly affect the size of $\hat{L}_0$ in the near future.

[6]This can be accelerated by computing the A^* distances in advance, or by caching $d(p_1, p_2)$ during run-time.

Figure 3: An example state-of-the-world in the *gridworld* domain. Shaded areas represent the region $\hat{L}_0$ for a target that has been lost.

5. EXPERIMENTAL RESULTS

To evaluate the heuristics presented earlier, we ran a series of experiments in the pursuit-evation domain illustrated in Fig. 3. Strategies for the tracker agents were computed using the depth-limited minimax search shown in Algorithm 1 with the *RS*, *MD* and *RLA* heuristics. We used a similar algorithm to compute three different approximations of the "worst-case" target model described in Section 3.1: our *anti-RS*, *anti-MD* and *anti-RLA* target models approximate the worst-response to the trackers' action using a depth-limited minimax search with the corresponding heuristic function.

Results for each experiment were averaged over a set of 500 independent trials, each using randomly-generated locations for a target and two trackers. Each trial lasted for 50 times steps, and performance was measured by the size of $\hat{L}_0$ at the end of the game. In our scenario generation process, we did not consider scenarios that are likely to be unsolvable (i.e., scenarios in which the target was not observable by at least one tracker agent initially), and scenarios that are likely to be trivial (i.e., scenarios in which the target and the tracker agents were positioned too closely).

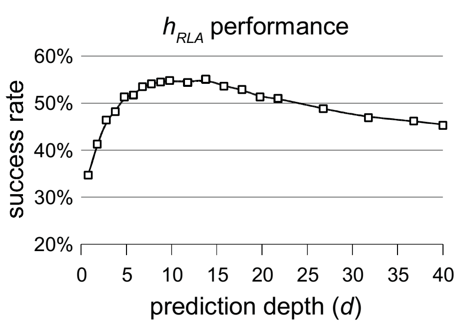


Figure 4: *RLA*'s success rate at various prediction depths.

***RLA*'s prediction-depth parameter.** Recall that the *relaxed lookahead (RLA)* heuristic has a parameter d, the *prediction depth*, which is how far ahead to look in a relaxed version of the game. To see what values of d would work well in the gridworld domain, we did experimental tests with d ranging from 1 to 40, on 500 randomly generated gridworld games. For each game, the tracker performed a 2-ply search using h^d_{RLA} as its heuristic function, playing against a simple 2-ply *anti-RS* target model. Fig. 4 shows the tracker's *success rate* (the fraction of games in which the tracker knows the target's exact location) as a function of *RLA*'s prediction depth in the gridworld domain. Since the curve peaked around $d = 10$, we used $d = 10$ for *RLA* in the experiments reported below.

Comparison of the three heuristics. To compare the *RLA*, *RS*, and *MD* heuristics, the tracking agents were tested at 2-ply, 4-ply, and 6-ply[7] search depths, against the three target models, for a total of 27 different experiments. The target in each experiment operated independently, according to its own 4-ply game-tree search.

In our experiments, the *RLA* heuristic performed much better than the naive *RS* and *MD* heuristics. More specifically:

- Fig. 5 shows the average size of $\hat{L}_0(t_{end})$, the area in which the tracker knew the target might be located. Note that the size was much smaller (i.e., better tracker performance) when the tracker used the *RLA* heuristic than when it used the *RS* and *MD* heuristics, regardless of which target model was used.
- Fig. 6 shows the tracker's *success rate*, the fraction of games in which the tracker knew the target's exact location at the end of the game. The success rate was twice as high when the tracker used the *RLA* heuristic than when the tracker used the *RS* and *MD* heuristics, regardless of which target model was used.

Performance versus search depth. The following table gives the average running times[8] of game-tree searches using each heuristic, averaged over the same set of trials as in Figs. 5 and 6.

	2-ply	4-ply	6-ply
RLA	30ms	144ms	2361ms
RS	15ms	74ms	1069ms
MD	15ms	78ms	1306ms

Although searching deeper into the game-tree produces strategies that are slightly more successful, the most dramatic results come from changing the heuristic, which requires considerably less computational effort for an equivalent gain in performance. In particular, let us compare a 2-ply search with *RLA* to a 4-ply or 6-ply search using *RS* or *MD*. Of these five searches, the above table shows that the 2-ply *RLA* search has the smallest running time, and Figs. 5 and 6 show that it also has the best performance.

Example strategy execution. Fig. 7 shows an example of the trajectories generated by our implementation. This is a simple problem with two tracking agents using the h^{40}_{RLA} heuristic[9] against the corresponding *anti-RLA* target model.

Notice that, from the beginning, the two trackers split up and attempt to block the target from separate directions. Tracker r_1, which takes the longer route, has no immediate benefit from moving away from the target. It is only because of the deep lookahead provided by h^{40}_{RLA} that this occurs. Had the tracker taken a more greedy approach, such as with the *max distance* heuristic, then it would have missed the opportunity to block an escape route.

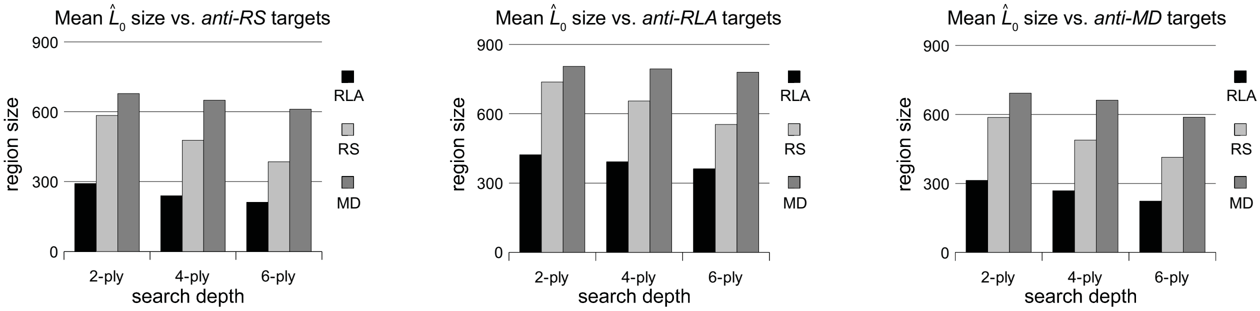
In addition to splitting up, tracker r_2 waits at a key location from time t_{18} to t_{31}, blocking two paths. This behavior is not specific to RLA, but also appears using the *region size* heuristic. If the agent leaves that location prematurely, it risks increasing the size of $\hat{L}_0$ from either one direction or the other. In other words, by being impatient, it would allow the target to escape back the way it came.

In practice, not all scenarios play out as well as this one. However, the combination of a short game-tree search, with a long-term predictive lookahead, help explain the performance benefits detailed in the previous sections.

[7]We define a *ply* as a single move by either player.

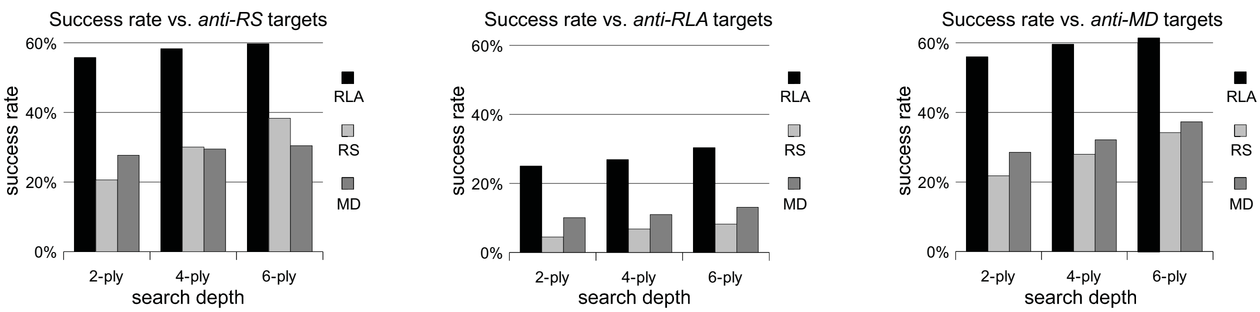
[8]The experiments were run using *Java VM* version 1.6, on a 2.4GHz processor with 1GB of available memory.

[9]For this example, the values of Δt used to compute the *RLA* heuristic were restricted to the range [0,1].



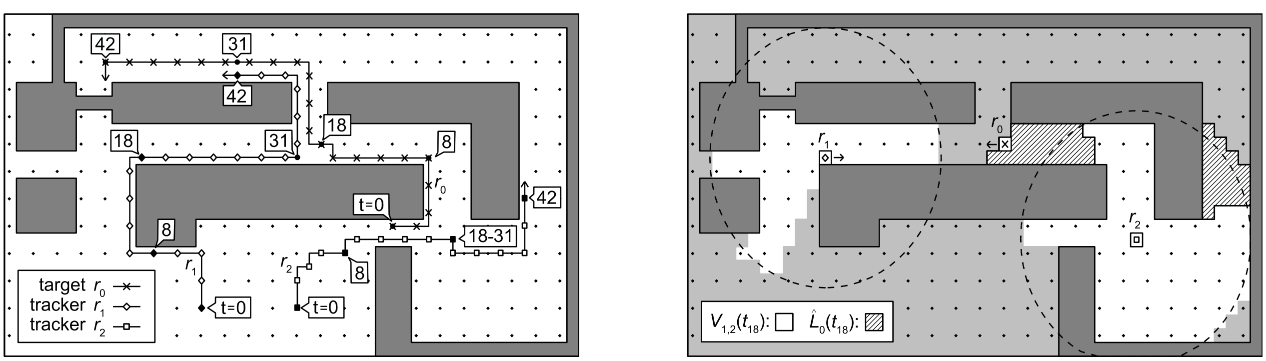
(a) (b) (c)

Figure 5: Average region size $Vol(\hat{L}_0(t_{end}))$ after 50 rounds for *RS*, *RLA*, and *MD* tracker agents versus (a) *anti-RS*, (b) *anti-RLA*, and (c) *anti-MD* target models in the *gridwold* domain. Each bar is an average of more than 500 trials.



(a) (b) (c)

Figure 6: Tracking success rate for *RS*, *RLA*, and *MD* tracker agents versus (a) *anti-RS*, (b) *anti-RLA*, and (c) *anti-MD* target models in the *gridwold* domain. Each bar is an average of more than 500 trials.



(a) (b)

Figure 7: (a) Strategies generated by our algorithm on a simple tracking problem; note how trackers r_1 and r_2 move to block two escape routes. (b) The tracker's observable regions $V_1(t_{18})$ and $V_2(t_{18})$ and its set $\hat{L}_0(t_{18})$ of possible target locations, at time t_{18}. The circles show the tracking agents' maximum sensor distances.

6. RELATED WORK

Much of the existing work on visibility-based pursuit-evasion games centers on robot patrolling, or hider-seeker games, where the objective is to find an unseen target moving within some enclosed domain. Versions of this problem exist for both continuous [37, 21, 13, 23, 38] and discrete domains [28, 1, 4, 17] resulting in a variety of different approaches. Distinct from the work in this paper, patrol strategies do not typically specify a pursuit strategy once the target has been found, and thus are often pre-computed.

In a continuous domain, patrol strategies can be computed by tesselating the environment into cells, and modeling the pursuer as a visibility ray or cone [37]. Various algorithms exist to compute the route a pursuer should patrol to detect an evader, or to determine that no solution is possible [21]. In most cases, the problem is simplified by assuming the target has unbounded speed [13], or by approximating its movement [38]. As a result, game-theoretical elements often play a less significant role, although some techniques rely on them more heavily than others [23].

Graph-based versions of the hider-seeker game have existed for some time [28], but have not always modeled observability explicitly. Discrete versions of the visibility-based problem have been addressed using techniques from game-theory, including equilibrium concepts, and non-deterministic strategies [1, 4]. A variant of this problem, where the pursuer is free to observe an arbitrary subset of the domain each round, has been shown to be NP-hard [17]. However, this work is again focused primarily on computing patrol strategies, with the additional limitation of only considering games which are finite.

Substantial work exists on visual tracking of a moving target both with and without obstacles [22, 20, 24], however these techniques do not typically consider imperfect information, and cannot extensively model scenarios where visibility has been lost on the target. Some special cases have been investigated in which the observer keeps a given distance from the evader [24] or in which the observer has bounded velocity [25]. Reinforcement Learning (RL) [36] has also been adapted for this task. For example, [18] studies two cases, where the target is adversarial and non-adversarial, and proposed a RL-based learning rule in order to generate actions for the pursuers depending on the rewards they will get based on both their and the evaders movements in the world. One drawback is that the model proposed by this paper is discrete, i.e., it does not handle continuous environments as the approaches mentioned above.

In robotics, many different approaches have been proposed for planning autonomous robot operations for pursuit and evasion scenarios. A widely-known tool is differential game theory [3], which extends sequential game theory to continuous time. Based on this theory, a pursuit and evasion game is modeled using differential equations with continuous variables that describe the dynamics of the game (e.g., the pursuers' and evaders' speeds and the time it takes for a capture). Nash Equilibria have been shown to exist for certain differential game variants of the pursuit-evasion problem [5]. However, differential models are often limited to games where the target remains visible, due to the challenge posed by imperfect information. Furthermore, solving a differential equation system requires significant amounts of computational time and resources in general, and many have argued that it is not feasible for planning actions in real time [23, 39, 34, 12].

Game-tree search has been highly successful in finite perfect-information zero-sum games such as chess [8, 7] and checkers [31], where the minimax theorem [40] applies. The basic idea is to compute approximations of minimax values using a depth-limited version of the minimax algorithm [33]. Numerous techniques have been developed to speed up the computation or make the approximations more accurate; some of the best-known techniques include alpha-beta pruning [19], transposition tables [8], and quiescence pruning [32].

Much work has been done during the past decade to extend game tree search to imperfect-information games such as bridge [35, 15], kriegspiel [27, 30], and poker [14, 6, 9]. One of the biggest problems is computational complexity, which is exponentially worse than in perfect-information games [29]. Some of the more popular approaches for alleviating this complexity involve doing the game tree search over a *simplified game* that has a smaller search space [6, 14], *aggregating* sufficiently similar states into groups that are treated as if they were identical [15, 35], and *Monte Carlo rollout* techniques that search a stochastic sample of the states in the game tree [27].

7. CONCLUSIONS

We have described a formalism that combines geometric and game-theoretic reasoning in imperfect-information multi-agent pursuit games. Based on this formalism, we have presented a recursive formula and search algorithm that are similar to game-tree search in perfect-information games such as chess, but generalized to accommodate both geometry and imperfect information. Our results show how our algorithm can be used to compute strategies that are considerably more sophisticated than simple strategies that just chase after the target.

Our experimental results show how the interplay between the geometric and game-theoretic aspects of our problem domain can be used to compute powerful game-tree-search heuristics. In particular, our *RLA* heuristic function works by creating a relaxed version of the problem that can be searched very quickly, and doing its own lookahead search in this relaxed problem. The relaxation involves analyzing the tracking agents' observability ranges at a particular future time point t, ignoring the dependencies among the trackers' actions, and considering all possible courses of action for the trackers a tracker could take up to time t. In our experimental results on 500 randomly generated problems, the *RLA* heuristic performed twice as well as two other heuristics: the *MD* heuristic, which measures the distance from the target, and the *RS* heuristic, which evaluates the level of uncertainty about the target's location. The difference in performance was so great that *RLA* at 2-ply performed better than *MD* and *RS* at 4-ply and 6-ply.

Future work. In this paper, we reduced the search algorithm's time complexity by making a "paranoid" assumption about the target's behavior. This model has worked quite well in perfect-information games such as chess, but tends to produce strategies that are overly cautious, posing challenges both theoretically [11, 10] and experimentally [27] for imperfect-information games. Consequently, two important tasks will be (1) to develop and test several alternative models of the target's behavior, and (2) to investigate other ways of reducing the search complexity.

Adapting the algorithm to support mixed strategies would improve the effectiveness of the algorithm, but at the cost of a significantly increased running time. An exploration of this tradeoff could help discover when the paranoid assumption does and does not do well.

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