Building a General Approach to Feature Recognition of Material Removal Shape Element Volumes (MRSEVs) *

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Abstract

Automated recognition of features from CAD models has been attempted for a wide range of application domains in mechanical engineering. However, the absence of a clear mathematical formalism for the problem has made it difficult to develop a general approach—and thus most of these methods are limited in scope.

In this paper, we develop a formalization of the problem of recognizing a class of machinable features expressed as MRSEVs (a PDES/STEP library of machining features) [11], and an algorithm for solving this problem. Some of the characteristics of this approach are

- the procedure handles a large variety of hole and pocket features, along with accessibility constraints for those features;
- the procedure is provably complete, even if the features intersect with each other in arbitrarily complex ways.

1 Introduction

Although many approaches have been developed for recognizing machinable features in solid models of mechanical parts, the scope of each approach is often limited by the feature definitions and the object classes of their individual domains. It is often unclear what specific classes of objects, features, and feature interactions can be handled by various approaches, making it difficult to evaluate their overall utility.

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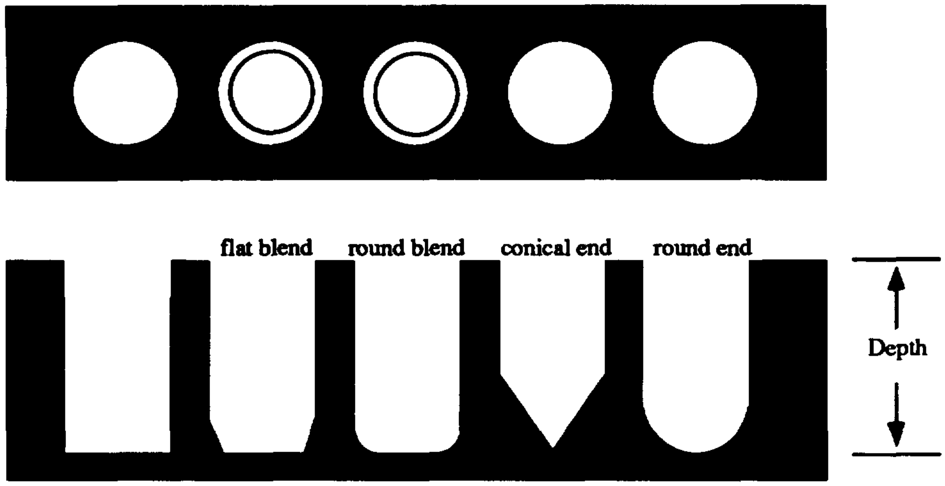
As a first step toward addressing this difficulty, we have developed a formalization of the problem of recognizing a subset of the set of all machinable features expressible as MRSEVs (Material Removal Shape Element Volumes) [11]. MRSEVs are volumetric features corresponding to machining operations on 3-axis milling machines. MRSEVS can be defined using EXPRESS (the official PDES information modeling language) and PDES form features. Kramer has already done this for a subset of the MRSEV library, and has defined the rest using an EXPRESS-like language.

The features in our class include a large variety of hole and pocket features, along with accessibility constraints for those features. Based on this formalization, we have developed a procedure for solving the problem of recognizing every solid that can be described as the difference between an arbitrary piece of stock and an arbitrary set of machinable features. The procedure is provably complete over the set of all solids in our class, even if the features intersect with each other in arbitrarily complex ways. For example, our procedure can handle each of the objects, some of which have appeared previously in [7, 2], in Fig. 8 without any difficulty.

This paper describes our formalism and feature recognition procedure. Section 2 describes the place of this work in relation to other feature recognition research. Section 3 defines the class of MRSEVs that we are interested in, how they are used to generate descriptions of mechanical parts, and the feature recognition problem for this domain. Section 4 presents a few basic procedures for solid models in this domain, and theorems deriving sufficient conditions for guaranteeing recognizability. Section 5 gives our procedure for finding feature models of parts and proves the procedure's completeness and its ability to handle arbitrarily complex feature interactions. Finally, section 7 gives conclusions and future directions for work in this area.



Attribute of hole	Type
location	point
orientation	vector
depth	positive real number
radius	positive real number
end	end condition

Figure 1: MRSEV Holes.

2 Related Work

The graph procedure approaches of [3, 8] provide an excellent level of computational formality. However, while they have known algorithmic properties, they appear difficult to extend to realistic manufacturing problems. Additionally, graph-based methods and the graph grammars of [14, 17] are prone to combinatorial difficulties [13]. The recent work in [5] describes recognition techniques that attempt to combat the combinatorial problems by abstracting an approximation of the geometric and topological information in a solid model and finding features in the approximation.

The feature interaction problem has been the focus of numerous research efforts, notably the heuristic approaches of [8, 19]. In [9, 10], an algebra of features is developed for the computation of alternate feature interpretations for parts. The work of [4] included the formalization of a feature description language and employed frame-based reasoning algorithms to extract machining features for computer aided process planning. [18] illustrates the need for extracting user defined features types that may arise in specific applications. Each of these goals would benefit from a general feature recognition formalism.

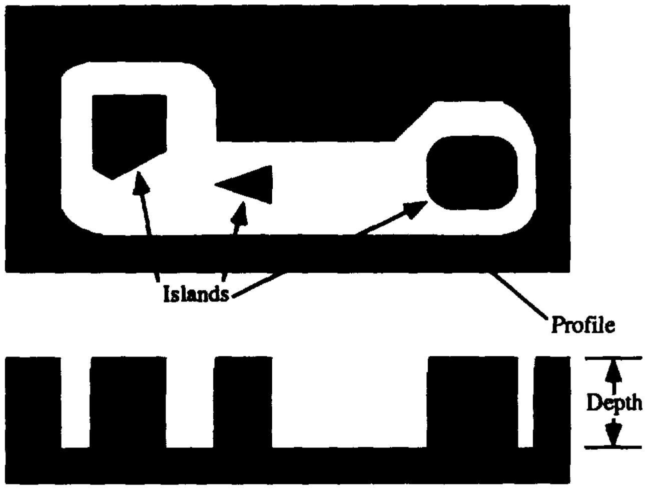
[6] was a seminal work in employing expert systems on the feature recognition problem. Large expert systems, such as [1] for part coding, have practical applicability but do not present a framework for their feature recognition area. In this case, there may be no formal means of specifying the capabilities of the system due to the subjective nature of the part coding problem. [12] presents an early effort to use grammars to parse solid models of parts for group coding.

Perhaps the most comprehensive and formal approach to date has been attempted in [19]. This method provides a computationally rigorous way of recognizing a class of realistic manufacturing features via artificial intelligence techniques in combination with queries to a solid modeler.[1] The work, however, stopped short of proving completeness of the approach and, while providing techniques for handling interacting features, does not formalize the complete class of interactions within its capabilities; arbitrarily complex feature interactions may pose problems.

3 Definitions

A *solid object* is a Lebesgue measurable, manifold subset of three-dimensional Euclidean space with a set of boundary faces consisting of planar, cylindrical, toroidal, conical and spherical surfaces. These are the only surfaces present in MRSEVs defined below, hence this set contains any object that they describe.

[1] A more detailed outline of this method can be found in [16].



Attribute of pocket	Type
location	point
orientation	vector
depth	positive real number
profile	edge loop
islands	set of one or more islands
Attribute of island	**Type**
location	point
profile	edge loop

Figure 2: MRSEV Pocket with islands.

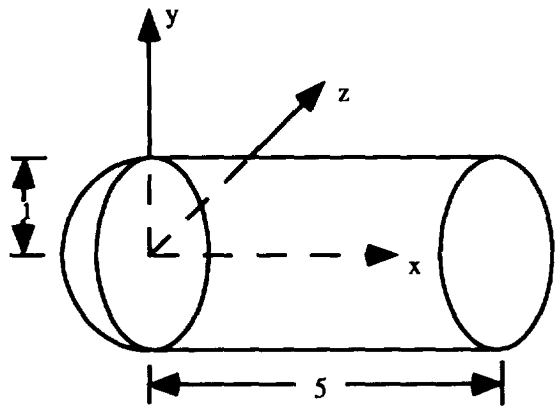


Figure 3: A particular hole instance as described in the text.

The set of features that we will consider in this paper is based on the library of material removal shape element volumes (MRSEVs), which was developed by Kramer [11] as a means of categorizing the shapes of volumes to be removed by machining operations on a 3-axis machine tool. MRSEVs are volumetric features, some of the benefits of which have been explained in [15]. The MRSEV hierarchy provides a framework for describing a large class of volumes of interest to machining.

Kramer's primary MRSEV types include linear swept features (shapes resulting from sweeping a closed profile of edges along a straight line perpendicular to the plane of the profile [11]), ramps, edge-cut features, and rotational pockets. For the purpose of this paper we confine our domain to the linear swept features, i.e., holes, pockets, and pockets with islands. Figures 1 and 2 present our illustrations of pocket and hole MRSEVs.[2]

The MRSEVs are parameterized solids; a *feature* is a specific instance of one of these MRSEVs, resulting from a specific choice of parameter values. For example, suppose we choose the following parameter values:

$$\begin{aligned}\textbf{location} &= (-1,0,0);\\ \textbf{orientation} &= (1,0,0);\\ \textbf{depth} &= 5;\\ \textbf{radius} &= 1;\\ \textbf{end} &= \text{round end}.\end{aligned}$$

This would define the round-bottomed hole illustrated in Fig. 3. Geometrically, this hole consists of the point set $(S_1 \cap S_2 \cap S_3) \cup S_4$, where

$$\begin{aligned} S_1 &= \{(x,y,z) : x \leq 5\};\\ S_2 &= \{(x,y,z) : y^2 + z^2 \leq 1\};\\ S_3 &= \{(x,y,z) : x \geq 0\};\\ S_4 &= \{(x,y,z) : x^2 + y^2 + z^2 \leq 1\}.\end{aligned}$$

The initial workpiece, $\mathbf{WP}_0$, is a solid object of raw stock material to be acted upon by a set of machining operations generating MRSEVs. The machined part (or just *part*) is a solid object **part** produced as a result of subtracting a finite set F of MRSEVs from an initial workpiece (hence **part** $\subseteq \mathbf{WP}_0$). The *delta volume* is the regularized difference of the initial workpiece and the part: $\Delta = \mathbf{WP}_0 -^* \mathbf{part}$.

Given a **part** and an initial workpiece $\mathbf{WP}_0$, we assume that the solid objects are bounded. Let **min** be the minimum value such that given a point in $\mathbf{WP}_0$ and a vector, a line of length **min** centered at the point and oriented on the trajectory of the vector would extend outside the workpiece in both directions. For example, given a hole of depth **min** with its center located within $\mathbf{WP}_0$, neither end-face of the hole is contained in $\mathbf{WP}_0$.

In this paper, we will only consider parts that satisfy the following restrictions:

- for any hole in F, a subface of its cylindrical face or ending surfaces are present in Δ;

[2] These are basically the same as Kramer's definitions, with one exception. Kramer allowed his pockets to have certain kinds of transition surfaces between a pocket's sides and its bottom, which he referred to as *bottom blends*. For the purposes of this paper we do not allow bottom blends on pockets.

- for any pocket in F, either a subface of its bottom face is present in Δ, or else it is a through pocket and at least two of its non-parallel planar side faces are present in Δ.

A *feature model* of an initial workpiece $\mathbf{WP}_0$ and a **part** is a set of features $FM = \{M_1, M_2, M_3, \ldots, M_n\}$ such that

$$i. \quad \forall M_i \in FM, M_i \cap^* \mathbf{part} = \emptyset$$
$$ii. \quad \mathbf{WP}_0 -^* \mathbf{part} \subseteq \bigcup FM$$

We say FM is a feature model of **part** and $\mathbf{WP}_0$. There may be many feature models of **part** and $\mathbf{WP}_0$. FM is an element of this class of feature models of **part** and $\mathbf{WP}_0$—it *describes* Δ as a set of features.

A feature M is *recognizable* in Δ if it is part of a feature model that describes Δ, and there exists a computable method of recognizing M from Δ.

The feature recognition problem for the parts and features defined above is as follows:

Definition 3.1 Feature Recognition
INPUT: $\mathbf{part}, \mathbf{WP}_0$
OUTPUT: *return a feature model FM of* $\mathbf{WP}_0$ *and* **part**.

We define a feature recognition procedure to be *complete* if it returns a feature model of **part** and $\mathbf{WP}_0$ whenever they are describable by a feature model.

4 Preliminaries

4.1 Primitive Procedures

The following procedures will help us in recognizing instances of the MRSEVs we described earlier. These procedural primitives will be used to build the recognition procedure for the MRSEVs defined in section 3. We do not give pseudo-code for these procedures, because the specific details will depend on what technique one uses to represent solid models. Instead, for each procedure we give an outline illustrating that it is within the abilities of solid modeling systems.

Procedure 4.1 MAXIMUM ENCLOSING CYLINDER
INPUT: *s, a subset of a cylindrical surface, a solid model,* **part**.
OUTPUT: *a cylinder C_s such that C_s is the largest cylinder having s as a subface and an empty interesction with the* **part**.

Consider the cylinder C_m of height **min** with the same axis and radius as the surface s and centered along the axis such that the surface s contains one or more points in the cylindrical side face of C_m equidistant from C_m's planar side faces. Recall, that because C_m is of height

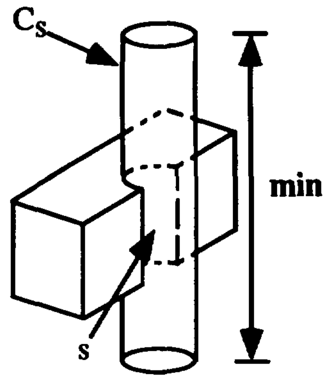


Figure 4: MAXIMUM ENCLOSING CYLINDER.

min and has its center somewhere within the stock we are guaranteed that the both end faces of C_m lie outside the stock. We define C_s to be the largest cylinder with the same radius and axis as s, centered along s, such that $C_s \subseteq C_m$ and $C_s \cap^* \mathbf{part} = \emptyset$. This can be determined by examining the maximum and minimum points on the edges of $\mathbf{part} \cap^* C_m$ with respect to a plane perpendicular to the axis of s. □

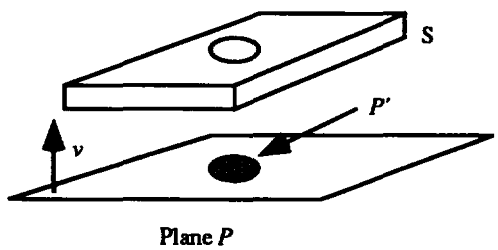


Figure 5: PROJECT OBJECT ONTO PLANE.

Procedure 4.2 PROJECT OBJECT ONTO PLANE
INPUT: *a plane P, an object S, and a normal vector to P, v.*
OUTPUT: *subplane P' of P such that $P -^* P'$ is the projection of S in the direction of v onto P. P' may be zero or more disjoint faces and that the faces may not be of finite size.*

The plane P and vector v define a half-space; the intersection of this half-space with S yields S', the portion of S "above" P in the direction of v. A transformation computes the projection of S' onto P, leaving a set $\overline{P}$, of two dimensional objects with boundaries composed of straight, circular and elliptical edges. Therefore, $P' = P -^* \overline{P}$, is a set of disjoint faces such that for each face f, $f \cap^* \mathbf{part} = \emptyset$ for any transformation of f by v. □

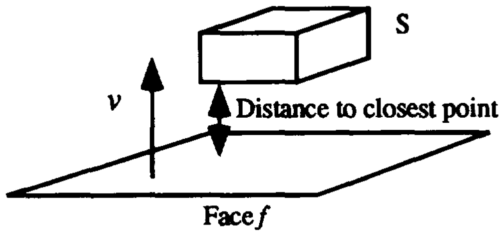


Figure 6: DISTANCE TO CLOSEST POINT.

Procedure 4.3 DISTANCE TO CLOSEST POINT
INPUT: *a planar face f, a vector normal to f, v, and an object S.*
OUTPUT: *the distance from f on vector v to a point p on the surface of S such that, for all other points p' on S on vector v from f, the distance from f to p' is greater than or equal to the distance to p.*

Let S' be the intersection of the swept solid with bottom face f and height equal to **min** with the **part**. The solid resulting from the intersection, if one exists, can be transformed into a coordinate system having v as an axis. The point on each face of S' closest to f on vector v can be calculated by geometric based on the surface type (i.e. calculating the closest point when the face of S' is planar is a different formula than the case when it is cylindrical). The smallest of these distances is the distance from f to S on vector v. □

4.2 Basic Properties

Using the fact that these procedures are computable through queries to a solid modeler, the conditions for immediate recognizability of each MRSEV feature type are formulated in the following lemmas. Note that in presenting these lemmas, we are actually building the general feature recognition procedure for this class of MRSEVs.

Lemma 4.1 *Let M be a MRSEV hole having a subface of one of its faces as a face in Δ, then there exists a recognizable MRSEV hole M' such that $M \subseteq M'$.*

Proof: Let f be a face of M in Δ, to find an instance of a MRSEV hole requires **location**, **orientation**, **radius**, **depth**, and **hole end**. We will show how suitable values for these attribute can be found from Δ and the **part**.

Case 1: f is conical, toroidal, or spherical

If f is one of these surface types then it must be the end surface for the hole—hence providing a value for **hole end**. Also, each of these surfaces provides an **orientation** and a **location** for the hole.[3] The **depth** is determined from the hole bottom of which f is a subface; it must extend past the initial workpiece $\mathbf{WP}_0$. Hence the **depth** may be arbitrarily set to **min**. Optimally, we want the largest **radius** value possible and an additional computation is required. Consider the plane P passing through the hole bottom and perpendicular to the hole axis. With P' returned by PROJECT OBJECT ONTO PLANE, project the portion of the object in the direction of the hole opening onto the plane. Determine which face of the one or more in P' contains the axis of the hole. Let **radius** be the radius of the largest circle that can be inscribed in that face of P' and centered at **location**.

Case 2: f is cylindrical

Let C_s be the smallest cylinder containing f as a subface as computed by MAXIMUM ENCLOSING CYLINDER. C_s gives us values for the **radius** and **orientation**. If the cylinder C_m centered in C_s of height **min** with the same radius and orientation as C_s does not intersect the **part** then C_s is creatable by a through hole. In this case the **hole end** and **location** attributes can be arbitrarily set to any values instantiating such a through hole.

If C_m does intersect the **part** then it does so in one direction of the **axis**[4]. In the direction of C_m's intersection with the **part** will be a planar **hole end**—any other kind of hole end would have been found in case 1.

Using the circular cross section of C_s as a face, determine the DISTANCE TO CLOSEST POINT of the **part** in the direction of the hole bottom. This provides the **depth** and **location** of the deepest hole that may have created face f. Therefore we have found an instance of a flat bottomed hole.

Case 3: f is planar

We need not consider planar surfaces created by MRSEV holes. The only such planar surface must be a hole bottom and, if not recognized already by case 2, it must also be recognizable as the bottom of a MRSEV pocket.

[3] In the case of a partial spherical surface, there may be many possible choices of orientation and location, any one of which may be chosen.

[4] If C_m intersected the **part** in both directions of the axis, the hole would be inaccessible.

Hence in each case we have determined attributes for an MRSEV hole M' recognizable from Δ and at least as large as the hole M that contained f as a subsurface. □

Lemma 4.2 *Let M be a MRSEV pocket having a subface of its bottom face as a subset of the boundary of Δ, then there exists a recognizable MRSEV pocket M' such that $M \subseteq M'$.*

Proof: Let f be the face of Δ containing a subface of the bottom of M. An instance of a MRSEV pocket requires finding values for the **location**, **orientation**, **depth**, **profile** and **islands** attributes.

Let v be the normal vector[5] to the surface of f, P be the plane containing f, and S be the solid object that is the intersection of the **part** with the half-space above P in the direction of f. Compute the set of subfaces of plane P, P' using PROJECT OBJECT ONTO PLANE. Consider the subface f' of $P' \cap^* \Delta$ containing f. Note that f' is a finite bounded face.

The face f' determines a **location** for the MRSEV pocket bottom and vector v provides an **orientation**. The **depth** can be set to **min** because the result of the projection implies that there is no subset of the **part** above f' in the direction v. The outside edge loop of f' gives a value for the **profile** of the MRSEV pocket and, lastly, any interior edge loops define the locations for any **islands**.

Hence, for any MRSEV pocket M having a subface of its bottom face as a subface of Δ, there exists a MRSEV pocket M' at least as large as the pocket M that it is recognizable in Δ. □

Lemma 4.3 *Let M be a MRSEV pocket with subfaces of two or more non-parallel planar side faces as subfaces of the boundary of Δ, then there exists a recognizable MRSEV pocket M' such that $M \subseteq M'$.*

Proof: Let f_1 and f_2 be the faces of Δ containing subfaces of two non-parallel planar side faces of M. Again, we wish to find values for the **location**, **orientation**, **depth**, **profile** and **islands** attributes of a MRSEV pocket M'.

Because f_1 and f_2 are non-parallel, it is known they intersect at a line, l. Pick a point p on l and consider the plane P passing through p and perpendicular to l. There are two cases:

[5] The vector v points away from the interior of the part

Compute P_1' and P_2' with PROJECT OBJECT ONTO PLANE. P_1' is the projection of the **part** onto P in one direction on l, P_2' is the projection from the other direction. The set of planar faces given by $P_1' \cap^* P_2' \cap^* \Delta$ are the cross-sections of through pockets parallel to line l. Consider the face f_p in this set that has edges from the projection of f_1 and f_2 onto P.

Line l provides an **orientation**; because it is a through pocket the **depth** may be set to **min** and the **location** can be set to an arbitrary place outside Δ. Face f_p provides a **profile** and, again because it is a through pocket, there can be no **islands**. □

Theorem 4.1 *Given any feature model of a* **part** *and* $\mathbf{WP}_0, FM$, *for all MRSEV features $M \in FM$, there exists a feature M' recognizable in Δ such that $M \subseteq M'$.*

Proof: Let FM be a feature model for a **part** and $\mathbf{WP}_0$ and let $M \in FM$. M is either a MRSEV hole or pocket feature.

Case 1: M is a hole

By our previous assumptions, it is known that a subface of one of the faces of M is a subface of a face in Δ. Therefore, by Lemma 4.1 there exists a recognizable feature M' such that $M \subseteq M'$.

Case 2: M is a pocket

M has a portion of its bottom face in Δ or is a through pocket with at least two of its non-parallel planar side faces present in Δ. The former case has been proven in Lemma 4.2 and the latter in Lemma 4.3.

Hence, if **part** and $\mathbf{WP}_0$ have a feature model FM we can recognize a set of features forming another feature model of **part** and $\mathbf{WP}_0$, FM', such that $\forall M \in FM, \exists M' \in FM'$ such that $M \subseteq M'$.□

5 MRSEV Recognition Procedure

Figure 7 gives the feature recognition procedure as determined by the lemmas of the previous section.

Our claim is that this procedure is complete; that is it returns a feature model for every **part** and $\mathbf{WP}_0$ for which one exists. The proof of this is a consequence of theorem 4.1.

Procedure 4.4 MRSEV FEATURE RECOGNITION
INPUT: **part** *and* $\mathbf{WP}_0$ *for which a feature model exists*
OUTPUT: *a set F, a feature model of* **part** *and* $\mathbf{WP}_0$

$F = \emptyset$
While $\Delta -^* \bigcup F \neq \emptyset$ do
  Pick a face f from the **boundary**$(\Delta -^* \bigcup F) \cap^*$ **boundary**(Δ)
  If f is cylindrical, toroidal, conical, or spherical find
    an instance of a hole M' creating f; Lemma 4.1
    proves this is possible.
  Else if f is planar then either
    Case 1: a subface f' of f is a subface of the bottom of a
      pocket. Lemma 4.2 proves we can find a pocket M' having f' as a subface
      of its bottom face.
    Case 2: a subface f_1' of f is a subface of a
      side face of a pocket. Hence, one of the other faces, f_2,
      in Δ must also belong to that pocket.
      Lemma 4.3 says we can find an instance of a pocket M' subsuming the one
      which created f_1 and f_2.
  Call the feature recognized M'
  Let $F = F \cup \{M'\}$
EndWhile

Figure 7: Recognition procedure.

Corollary 5.1 *(*Completeness*)* *Suppose* **part** *and* $\mathbf{WP}_0$ *can be described by a feature model, then the procedure* MRSEV FEATURE RECOGNITION *returns a feature model of* **part** *and* $\mathbf{WP}_0$.

Proof: The procedure, using the results and subroutines from section 4, finds features M' such that there does not exist a feature M'' where $M' \subseteq M''$. We must show that the set F returned by MRSEV FEATURE RECOGNITION is a feature model. Therefore we must show:

$i.$ $\forall M_i \in F, M_i \cap^* \mathbf{part} = \emptyset$
$ii.$ $\mathbf{WP}_0 -^* \mathbf{part} \subseteq \bigcup F$

The way recognizable features are instantiated by the procedure satisfies *(i)*. To show *(ii)*, recall that there exists a feature model FM of $\mathbf{WP}_0$ and **part**. By theorem 4.1, it is known that for every feature $M \in FM$ there exists a feature M' in F such that $M \subseteq M'$. Therefore $\mathbf{WP}_0 -^* \mathbf{part} \subseteq \bigcup FM \subseteq \bigcup F$

Hence the procedure returns an feature model for any $\mathbf{WP}_0$ and **part** that have one. □

The result that the procedure can find a feature model for any $\mathbf{WP}_0$ and **part** with arbitrarily complex feature interactions is similar.

Corollary 5.2 *(*Feature Intersection Independence*)* *Suppose* **part** *and* $\mathbf{WP}_0$ *can be described by a feature model and for every such model the features intersect. The procedure* MRSEV FEATURE RECOGNITION *returns a feature model of* **part** *and* $\mathbf{WP}_0$.

Proof: Again, we know a feature model exists: call it FM. The fact that the features intersect in an inconceivably pathological manner does not alter the fact that the procedure, using the results and subroutines from section 4, finds features M' such that there does not exist a feature M'' where $M' \subseteq M''$. We must show that the set F returned by MRSEV FEATURE RECOGNITION is a feature model.

As before, we get *(i)* for free. To show *(ii)*, recall by theorem 4.1, it is known that for every feature $M \in FM$ there exists a feature M' in F such that $M \subseteq M'$. Therefore $\mathbf{WP}_0 -^* \mathbf{part} \subseteq \bigcup FM \subseteq \bigcup F$

Hence the procedure returns an feature model for any $\mathbf{WP}_0$ and **part** regardless of how the features describing $\mathbf{WP}_0$ and **part** interact. □

6 Examples

To illustrate that this procedure can function in realistic machining situations, Fig. 8 presents some examples

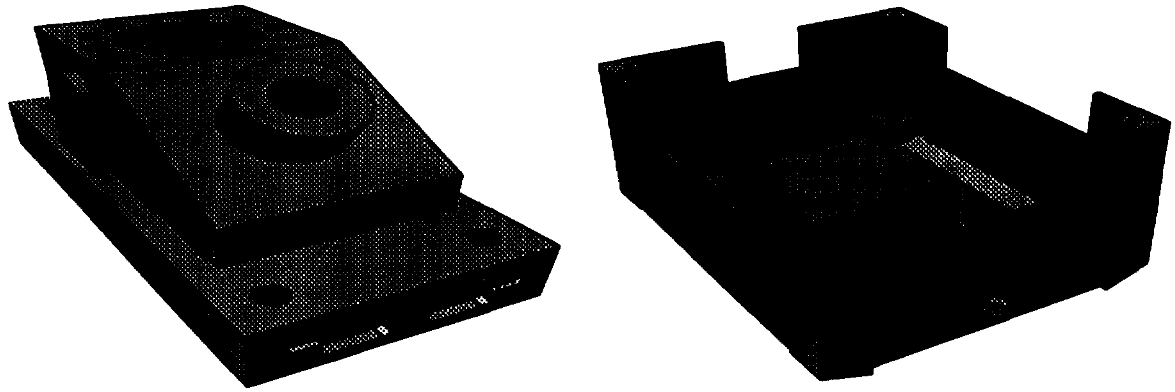


Figure 8: Examples from [7] of parts recognizable with this procedure.

from the domain described by these MRSEVs. Both of these figures appeared previously in [7, 2].

An example of the general procedure's feature intersection independence can be found in Fig. 9. This figure depicts a cylindrical part containing a hole with two intersecting keyways and shoulders. The interaction of the criss-crossing keyways within a cylindrical hole could be problematic for many feature recognition methodologies. If delta-volume reduction technique were used, the possibility exists that it may recognize the hole first—thus making the keyways difficult, if not impossible, to recognize. Any methodology attempting to find edge loops to determine the cross-section of the hole would there is no such planar edge loop in the part. The shoulders, while easily described as instances of MRSEV pockets, may confuse a system that cannot deduce the existence of faces not in the final part. In some graph-based recognition schemes, each keyway doubles the number of graph elements needed to describe the hole. While this may not preclude the recognition of the hole, it will require additional computational time to recognize the hole—a task that, in the worst case for some systems, is exponential.

Procedure 4.4 handles this example without any special-case reasoning. If the stock material is the complete cylinder, then the two shoulders both have part of their bottom faces in Δ—hence, by lemma 4.2, the procedure will find MRSEV pockets subsuming all other MRSEV pockets that describe the shoulders. For each of the keyways, it is evident that at least two of their non-parallel planar side faces are present in Δ—hence, by lemma 4.3, they will be recognized as instances of MRSEV pockets. Recognizability of the hole is given by lemma 4.1.

It is important to note that the theorems, while strong enough to guarantee the recognizability of a large class of parts, do not make any statements regarding the MRSEVs instances that will be found. As described, the algorithm is non-deterministic and, depending on its implementation, the MRSEV feature model described above for Fig. 9 is one of many alternate valid models.

7 Conclusions

The main contributions of this paper include the development of a problem definition, a feature recognition procedure based on that definition, and a formal proof of completeness of that procedure. We have explicitly defined what class of parts we handle, and the algorithm is guaranteed to find feature models for all parts in that class, regardless of how complicated the interactions are among the features.

Our approach differs from the iterative Δ-volume subtraction techniques as developed in [6] in that we do not reduce Δ to $\emptyset$. In addition, the key goal of this work is to build a formalization of the feature recognition problem. Then, to demonstrate its benefits, a general feature recognition algorithm for a realistic domain can be built using the formalism as a guide for its design and analysis. Most previous work has focused on introducing new techniques for getting a solution to the problem—not usually the development of a general problem definition.

The primary limitation of our approach as presented here is that it is designed only to handle linearly swept features (i.e., holes and pockets). However, our definitions of holes and pockets are more general than the definitions used in a number of feature recognition systems; for example, the pockets may be complicated swept contours that include corner radii, islands and other characteristics, in order to realistically describe a non-trivial set of mechanical parts.

Near-term goals for future work include completing an incorporation of bottom blends into our current definition of pocket MRSEV, incorporating a more sophisticated definition of accessibility, and implementing our procedure. Medium-term directions include extending our results and procedure to include other MRSEVs,

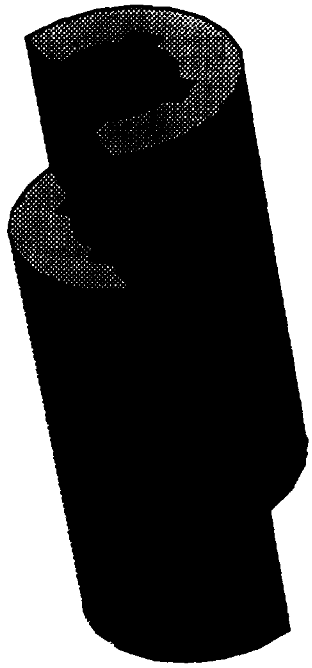


Figure 9: An example part with a variety of feature intersections.

and generalizing these results to encompass a wider variety of feature recognition domains.

As a long-term goal, we hope to develop a general computational paradigm for recognition of machinable features, and mathematical results presented in this paper can be viewed as a first step toward that goal. More powerful results of similar nature will be required to build a satisfactory and useful formalism for a wider class of feature recognition problems. Such a formalism would provide a framework within which to compare and contrast the results of feature recognition research in any application area that can be represented in this class. This would allow conclusions about complexity, features recognized, feature interactions, and completeness of an approach to have significance outside individual application areas.

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