

1. (10) Let

$$f(x) = x^2 - 4.$$

Suppose our guess at the zero of this nonlinear equation is 2.5. If we apply Newton's method, what would the next guess be?

Answer: We compute $f'(x) = 2x$, so

$$\begin{aligned} x &\leftarrow x - \frac{x^2 - 4}{2x} \\ &= 2.5 - \frac{(2.5)^2 - 4}{5}. \end{aligned}$$

Note: This works out to be 2.05. Note that the guess 2.5 has one significant figure, and after 1 iteration we have 2: very fast convergence. This is the basis for the algorithm usually used in the `sqrt` function in languages like C++, Fortran, and Matlab. The initial guess is obtained by saling and then table look-up.

2. (10) Suppose we want to solve the problem

$$\min_x \frac{1}{2}x_1^2 + \frac{3}{2}x_2^2 + x_1 - x_2 + 5$$

and we have the initial guess $x_1 = 2, x_2 = 3$. What is the steepest descent direction from this guess?

Answer: The gradient of the function is

$$\begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix} = \begin{bmatrix} x_1 + 1 \\ 3x_2 - 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \end{bmatrix},$$

so the steepest descent direction is

$$-\begin{bmatrix} 3 \\ 8 \end{bmatrix}.$$